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TREATISE

ON

COMMERCIAL ARITHMETIC

TO WHICH ARE ADDED

PRACTCAL COURSES ON

MENSURATION AND BOOK-KEEPING

DESIGNED FOR

HIGH SCHOOLS AND ACADEMIES

BY THE CHRISTIAN BROTHERS.

FOURTH EDITION

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PREFACE.

Our object, in the publication of this work, is to supply our High Schools and Academies in the Dominion of Canada, with a moderate-sized book containing sufficiently developed and practical Treatises on Commercial Arithmetic, Mensuration, and Book-keeping, saving, thereby, to families, the expense of several text-books, and placing the study of these branches within the reach of the laboring classes.

As decimals follow the same scale as whole numbers, we have chosen to treat them along with the latter; therefore, they will be found introduced in numeration.

Though we have essentially followed the decimal system, yet, the old method has not been neglected.

Amongst its various particular features, this work offers the important advantage of proposing a far greater number of practical questions than any other of the same size. We are also confident that students will find in it all the informations requisite to qualify them for the position of accountants or business men.

Some persons prefer to have the answers placed

immediately after the examples; others, to have them omitted. Both these methods have their utility and their disadvantage. In order, therefore, that the learner may have the benefit of both, we have left out the answers to nearly one-third of the examples or problems. These will be found, together with clear solutions of all the examples, in a Key to this work, prepared for the use of teachers and private learners. The Key will also give the Journal Entries of all the Memoranda of the Book-keeping course.

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COMMERCIAL ARITHMETIC

DEFINITIONS.

1. Arithmetic is the science of numbers.

2. A **Number** is a unit, or a collection of units.

3. A **Unit** is one, or a single thing.

4. Magnitude, or **Quantity,** is any thing that will admit of increase or decrease.

4 *bis.* An **Integral Number,** or **Integer,** is a whole number; as *three*, *eight* dollars, *twenty* horses.

5. Any quantity less than the unit, is a **Fractional Number,** or a **Fraction;** as $\frac{1}{2}$, $\frac{2}{3}$ of a foot.

6. Numbers, in general, are either abstract or concrete.

7. Abstract Numbers are numbers used without reference to any particular thing or quantity. Thus, *five*, *seven*, *fifteen*.

They are divided into three classes:

1st. Those which are not accompanied with subdivisions, as *four*, *eight*, &c.: they are called *abstract integral numbers*.

2nd. Those which are accompanied with decimal subdivisions as *three units fifteen hundredths,—six units two hundred twenty-five thousandths:* they are called *abstract decimal numbers*.

3rd. And lastly, those which contain only decimal subdivisions, as *forty hundredths,—seventy-five thousandths*: they are called *abstract decimal fractions*, or simply *decimals*.

8. Concrete Numbers are numbers used with reference to some particular thing or quantity. Thus, *seven* dollars, *nine* yards.

---

1. *What is* Arithmetic?— 2. *What is a* number?— 3. *Define* unit?— 4. Magnitude *or* quantity?— 5. Fraction?— 6. *How are numbers divided?*— 7. *What are* abstract numbers?— 8. *What are* concrete numbers?

They are also subdivided into three classes:

1st. Those which contain no subdivisions, as *six* yards, *eleven* pounds.

2nd. Those which are accompanied with decimal subdivisions, as *five dollars twenty-five cents.*

3rd. And lastly, those which contain decimal subdivisions only, as *twenty-five cents* ($0.25).

9. A **Simple Number** is either an abstract or a concrete number of but one denomination; as, *two*, *ten* dollars, *fifteen* hats.

10. A **Compound Number** is a collection of concrete units whose subdivisions are not decimals, but represent several denominations, taken collectively; as, *six* pounds *four* shillings *nine* pence, *three* feet *five* inches, etc.

11. A **Power** is the product arising from multiplying a number or quantity by itself, or repeating it any number of times as a factor.

12. A **Root** is a factor repeated to produce a power.

13. A **Demonstration** is the process of reasoning by which a truth or principle is established.

14. An **Operation** is the process of finding, from given quantities, others that are required.

15. A **Problem** is a question requiring an operation.

16. A **Rule** is a direction for performing an operation.

17. **Analysis,** in Arithmetic, is the process of investigating principles, and solving problems, independently of set rules.

18. The **Principal** or **Fundamental Operations** of Arithmetic are, Notation and Numeration, Addition, Subtraction, Multiplication, and Division.

SIGNS.

19. A **Sign** is a symbol employed to indicate the relations of numbers, or quantities, or operations to be performed upon them.

(.) is the *decimal* sign indicating that the number after it is a decimal.

$ means dollar.

---

9. *What is a* simple number?— 10. *What is a* compound number?— 11. *What is a* power?— 12. *What is a* root?— 13. *What is a* demonstration?— 14. *What is an* operation?— 15. *What is a* problem?— 16. *A* rule?— 17. *What is* Analysis?— 18. *What are the* fundamental operations *of Arithmetic*?— 19. *What is a* sign?

+, the sign of *addition*, is read *plus*. Thus, 8 + 7 signifies that 7 is to be added to 8.

—, the sign of *subtraction*, is read *minus*. Thus, 8 — 7 signifies that 7 is to be subtracted from 8.

×, the sign of *multiplication*, is read *multiplied by*. Thus, 9 × 6 signifies that 9 is to be multiplied by 6.

÷, the sign of *division*, is read *divided by*. Thus, 32 ÷ 8 signifies that 32 is to be divided by 8.

=, the sign of *equality*, is read *equal*, or *equal to*. Thus, 8 + 6 = 14, signifies that 8 plus 6 is equal to 14.

( ), a parenthesis, the sign of *aggregation*, indicates that all the numbers, or quantities, included within it, are to be considered as a single one. Thus, (7 + 4) × 3, indicates that the sum of 7 and 4, or 11, is to be multiplied by 3. A vinculum or bar, ———, has the same signification. Thus, $\overline{9 \times 4} \div 3 = 12$.

[ ], *brackets* or *crotchets*, are used to indicate that the operations on the quantities contained within the parenthesis have been performed, but before those indicated by the signs outside the brackets. Thus, [(8 × 7) + 14] ÷ 2 comes to 8 × 7 = 56; 56 + 14 = 70; 70 ÷ 2 = 35.

: is the sign of *ratio*. Thus, 5 : 4 means the ratio of 5 to 4, and is read 5 is to 4.

: : indicates the sign of *proportion*, or the equality of ratios. Thus, 6 : 9 : : 8 : 12, is read, 6 is to 9 as 8 is to 12.

NOTATION AND NUMERATION.

20. Notation is the process ef *expressing* numbers by letters or figures; and,

21. Numeration is the process of *reading* numbers when expressed by figures.

22. Two methods of notation are in common use— the *Roman* and the *Arabic*.

ROMAN NOTATION.

23. The **Roman Notation**, so called from its having originated with the Romans, employs *seven* capital letters to express numbers, viz.:

I	V	X	L	C	D	M
one,	five,	ten,	fifty,	one hundred,	five hundred,	one thousand.

20. *What is* notation?— 21. *What is* numeration?— 22. *How many methods of notation are now in common use?*— 23. *Why is the Roman notation so called.*

It will be seen from the following Table, that all numbers may be expressed by the use of these letters, either by repetitions or combinations.

1st. Every *repetition* of a letter repeats its value; thus, II, represents *two;* III, represents *three;* XX, *twenty*, etc.

2nd. When a letter of any value is placed after one of greater value, it *adds* its own value to the greater; but when placed *before*, its value is to be subtracted; thus, VII represents *seven;* XI represents *eleven;* while IX represents *nine*, or one *less* than ten; XL, *forty*, etc.

3rd. A bar or dash (—) placed over a letter, increases its value a thousand-fold; thus $\overline{V}$ denotes *five thousand;* $\overline{IV}$, *four thousand;* $\overline{X}$, *ten thousand*, etc.

TABLE.

I.....	is	One.	XXVII	is	Twenty-seven.
II.....	"	Two.	XXIX.	"	Twenty-nine.
III.....	"	Three.	XXX...	"	Thirty.
IV.....	"	Four.	XXXVI	"	Thirty-six.
V.....	"	Five.	XL.....	"	Forty.
VI.....	"	Six.	XLIX.	"	Forty-nine.
VII.....	"	Seven.	L.....	"	Fifty.
VIII....	"	Eight.	LX.....	"	Sixty.
IX.....	"	Nine.	LXX...	"	Seventy.
X.....	"	Ten.	LXXXI	"	Eighty-one.
XI.....	"	Eleven.	XC.....	"	Ninety.
XII....	"	Twelve.	XCIV.	"	Ninety-four.
XIII...	"	Thirteen.	C.....	"	One hundred.
XIV....	"	Fourteen.	CCC...	"	Three hundred.
XV.....	"	Fifteen.	CD.....	"	Four hundred.
XVI....	"	Sixteen.	D.....	"	Five hundred.
XVII..	"	Seventeen.	DC.....	"	Six hundred.
XVIII.	"	Eighteen.	CM....	"	Nine hundred.
XIX....	"	Nineteen.	M.....	"	One thousand.
XX.....	"	Twenty.	MC.....	"	Eleven hundred.
XXI....	"	Twenty-one.	MD....	"	Fifteen hundred.
XXII..	"	Twenty-two.	MM....	"	Two thousand.
XXIII.	"	Twenty-three.	MMM.	"	Three thousand.
XXIV.	"	Twenty-four.	$\overline{X}$.....	"	Ten thousand.
XXV...	"	Twenty-five.	$\overline{M}$.....	"	One million.

EXERCISES IN ROMAN NOTATION.

Express the following numbers by letters:

1. Six. *Ans.* VI.
2. Eight.
3. Ten.
4. Thirteen.
5. Fifteen.
6. Seventeen.
7. Nineteen.
8. Twenty-five.
9. Thirty.
10. Forty-six.
11. Fifty-four.
12. Sixty.
13. Sixty-eight.
14. Eighty-four.
15. Ninety-nine.
16. One hundred and six.
17. Four hundred and nineteen.
18. Eight hundred and seventy-five.
19. Nine hundred and sixty-five.
20. Four hundred and forty-one.
21. Four hundred and eighty-seven.
22. Six hundred and ninety-five.
23. One thousand six hundred and fifty.
24. One thousand eight hundred and forty.

ARABIC NOTATION.

24. Arabic Notation employs ten characters, or figures, to express numbers, viz.:

1 2 3 4 5 6 7 8 9 0

One, two, three, four, five, six, seven, eight, nine, cipher.

25. The first nine characters are called *significant figures*, because each has a value of its own. They are sometimes called *digits*, from the Latin word *digitus*, which signifies *finger*. The *cipher* is called *naught*, or *zero*, because it has no value of its own.

26. In order to reduce the numeral *figures* to a small number, we give each a second value according to the place it occupies. Thus, the first represents the *units;* the second, the *tens;* the third, the *hundreds;* the fourth, *units of thousands;* and so on, each succeeding figure to the left belonging to a distinct order, the unit of which is tenfold the value of a unit of the order to the right.

27. Since the value of a number expressed by any figure depends upon the place the latter occupies, it follows that figures have two **Values;** the one *absolute* or simple, that is, the value expressed by a figure standing alone, or, when in a collection, standing in the right-hand place; the other *relative* or *local*, that

24. *How many characters are employed in the* Arabic Notation?— 25. *What are the first nine characters called?*— 26. *How can we represent all possible numbers with the ten figures?*— 27. *How many values have figures?*

is, the value expressed by a figure used in combination with other figures and depending upon the place the figure occupies. The cipher becomes significant when connected with other figures only, by filling a place which otherwise, would be vacant (No. 28).

Thus, in 8042, the simple value of the first figure on the left hand is 8, and its local value 8 units of thousands, because it is a figure of the fourth order; the simple value of the third figure is 4, and its local value 4 tens, because it is a figure of the 2nd order; the simple value of the first is 2, and its local value 2 units; the cipher fills the vacant place of the hundreds.

NUMERATION TABLE.

Hundreds of Sextillions.	Tens of Sextillions.	Sextillions.	Hundreds of Quintillions.	Tens of Quintillions.	Quintillions.	Hundreds of Quadrillions.	Tens of Quadrillions.	Quadrillions.	Hundreds of Trillions.	Tens of Trillions.	Trillions.	Hundreds of Billions.	Tens of Billions.	Billions.	Hundreds of Millions.	Tens of Millions.	Millions.	Hundreds of Thousands.	Tens of Thousands.	Thousands.	Hundreds.	Tens.	Units.
1	2	7,	8	9	4,	2	3	7,	8	6	7,	5	2	3,	6	7	8,	4	7	8,	6	3	8.
Period of Sextillions.			Period of Quintillions.			Period of Quadrillions.			Period of Trillions.			Period of Billions.			Period of Millions.			Period of Thousands.			Period of Units.		

RULE FOR NOTATION.

28. To write in figures any number without difficulty.

Place as many dots or points as the number contains figures; then begin at the left-hand, and write each figure in the place it must occupy, and if there are any vacant places, supply them with naughts or ciphers.

Example. Given the number *four billions six millions twenty thousand five hundred units.*

Write the 4 in the order of the units of billions, the 6 in the order of the units of millions, the 2 in the order of tens of thousands, the 5 in the order of hundreds of units, and put ciphers in the vacant places. Thus

. . . . . .
4 006 020 500

28. *What is the rule for notation?*

RULE FOR NUMERATION.

29. To read numbers represented by figures.

Begin at the right hand, and point off the figures into periods of three places each. The first period is called UNITS; *the second,* THOUSANDS; *the third,* MILLIONS; *the fourth,* BILLIONS; *the fifth,* TRILLIONS, &c. *The last may have but one or two figures.*

Ex. The number 345 678 907 654 326 is read in the following manner: three hundred and forty-five trillions, six hundred and seventy-eight billions, nine hundred and seven millions, six hundred and fifty-four thousands, three hundred and twenty-six units.

EXERCISES IN NUMERATION OF SIMPLE NUMBERS.

READ AND WRITE THE FOLLOWING NUMBERS.

1.	400	7.	800800003	13.	28754105
2.	6004	8.	87974015	14.	1000500
3.	80067	9.	35000918	15.	3008727
4.	670005	10.	30150900	16.	505054045
5.	9006014	11.	708000549	17.	78592835
6.	92100121	12.	4050300	18.	106405021

EXERCISES IN NOTATION AND NUMERATION OF SIMPLE NUMBERS.

EXPRESS BY FIGURES AND READ THE FOLLOWING NUMBERS:

1. Twenty-seven, forty-eight, sixty-five.
2. Seventy-five, ninety-three, eight hundred.
3. One hundred and ten, one hundred and twenty-four.
4. Three hundred and fifty-one, six hundred and two.
5. Four hundred and ninety-one, nine hundred and nine.
6. One thousand and one, three hundred and three.
7. Eight thousand one hundred and twelve, thirty-six thousand.
8. Nine hundred and seventeen thousand five hundred and two.
9. Seven hundred and eighteen thousand three hundred and ten.
10. Two millions six hundred and twenty-five thousand.
11. Seventy-seven millions eight hundred thousand and fifteen.
12. Four hundred millions three thousand four hundred.

EXPRESS THE FOLLOWING ROMAN NUMBERS BY FIGURES.

1.	IV	7.	V	13.	XXXV	19.	CD
2.	X	8.	VII	14.	XL	20.	CMIV
3.	L	9.	IX	15.	XLIX	21.	DCCXXX
4.	C	10.	XI	16.	LXV	22.	CMXLIX
5.	D	11.	XV	17.	XCIX	23.	$\overline{\text{XIX}}$
6.	M	12.	XXIV	18.	CVI	24.	$\overline{\text{MM}}$

29. *What is the rule for numeration?*

DECIMALS.

30. By **Decimals** are meant, parts ten times, a hundred times, a thousand times, etc., smaller than the unit, or which are successively ten times smaller than the other.

31. The parts contained ten times in the unit are called *tenths;* the tenths of tenths, *hundredths*, because they are contained a hundred times in the unit; the tenths of hundredths, *thousandths*, because they are contained a thousand times in the unit; the tenths of thousandths, *ten-thousandths*, &c.

32. A whole number and decimals, in a single expression, constitute a **Mixed Number.**

Thus, 12.54 is a mixed number, and is read twelve, and decimal fifty-four hundredths; 60.208, read sixty, and decimal two hundred and eight thousandths.

NUMERATION TABLE

FOR WHOLE NUMBERS AND DECIMALS.

ASCENDING PROGRESSION. DESCENDING PROGRESSION.

Hundreds of Trillions.	Tens of Trillions.	TRILLIONS.	Hundreds of Billions.	Tens of Billions.	BILLIONS.	Hundreds of Millions.	Tens of Millions.	MILLIONS.	Hundreds of Thousands.	Tens of Thousands.	THOUSANDS.	Hundreds.	Tens.	UNITS.	Tenths of Units.	Hundredths.	THOUSANDTHS.	Ten-Thousandths.	Hundred-Thousandths.	MILLIONTHS.	Ten-Millionths.	Hundred-Millionths.	BILLIONTHS.
1	3	6,	3	8	4,	9	4	7,	8	6	7,	2	2	3,	4	2	0,	5	7	8,	0	9	3.
Period of Trillions.			Period of Billions.			Period of Millions.			Period of Thousands.			Period of Units.			Period of Thousandths.			Period of Millionths.			Period of Billionths.		

As is easily seen, decimals, with regard to their order, follow inversely the system of numeration of whole numbers; the *tenth* is ten times smaller than the unit, whereas *ten* is the unit repeated ten times; the *hundredth* expresses the hundredth part of the unit, and a hundred, the unit repeated hundred times, &c.

---

30. *What are* decimals?— 31. *What names are given to decimal parts?*— 32. *What is a* mixed number?

33. The formation of decimal parts is made obvious by the following example:

If an apple be divided into ten equal parts, each piece will represent the tenth part of the unit or the whole apple. Dividing again each tenth into ten equal parts, we obtain hundredths. The result would be the same for a *line*, a *dollar*, &c.

34. From the foregoing illustrations, we deduce the following rules:

I. *Write first the whole number, after which place the decimal point; then from the left to the right, write successively the tenths, the hundredths, the thousandths, &c.*

Thus, the number 3 units 25 hundredths is written 3.25.

II. *If some order of decimals be wanting, fill the vacant places with ciphers.*

Thus, the number 12 units 5 hundredths is written 12.05, in placing a cipher to represent the tenths; and 4 units 3 tenths 8 hundredths and 8 ten-thousandths is represented by 4.3808.

III. *If there are decimals only, a cipher is put in the place of the units, and the decimals follow in their regular order.*

Thus, one tenth is expressed by 0.1; 5 tenths 8 hundredths, 0.58 5 thousandths, 0.005.

There is always a figure less in decimals than in a corresponding whole number, because the figure of the unit which is included in the whole number is not included in decimals.

35. Annexing ciphers to decimals does not alter their value as long as the decimal point is not displaced; the parts are made ten, a hundred times more numerous, but they are ten, a hundred times smaller: there is then compensation.

Thus, 0.25 becomes 0.250 by the addition of one cipher, and 0.2500 by the addition of two, but the value of the decimal is always equivalent to 25 hundredths.

EXERCISES ON DECIMAL NOTATION AND NUMERATION.

WRITE IN FIGURES THE FOLLOWING MIXED NUMBERS.

1. Two hundred and sixteen, and three tenths.
2. Five hundred and seven, and twenty-five hundredths.
3. Twenty-seven, and four hundredths.
4. Three hundred and twenty-one, and nine millionths.
5. Forty-four, and twenty-three hundredths.
6. Three hundred, and forty-two ten-thousandths.
7. Twenty, and forty-eight thousandths.
8. Four hundred and ten, and five hundredths.

---

33. Explain by an example the formation of decimal parts?— 35. Is the value of decimals altered by placing ciphers at their right hand side?

9. Two hundred and twenty, and nine hundred-thousandths.
10. One thousand and six, and five ten-thousandths.
11. Four thousand and seven, and three hundred-thousandths.
12. Fifty-nine, and twenty-two millionths.
13. Eighty-two, and thirty-six hundred-millionths.
14. Eight hundred and fifteen, and sixteen thousandths.
15. Twenty-seven, and one hundred and two billionths.
16. Twenty thousand and ten, and thirty millionths.

EXPRESS ORALLY AND WRITE IN WORDS THE FOLLOWING MIXED NUMBERS AND SINGLE DECIMALS.

Mixed numbers.

1.	8.90	5.	354.0064	9	41.004064
2.	9.908	6.	352.06046	10	452.010778
3.	541.400	7.	76.26007	11	7657.008007
4.	703.2004	8.	375.500506	12	1898.04

Single decimals.

1.	0.004	5.	0.4072	9.	0.69804445
2.	0.000607	6.	0.401950	10.	0.736050210
3.	0.005	7.	0.9540626	11.	0.000500019
4.	0.0007007	8.	0.075003	12.	0.00000501

APPLICATION OF THE PRINCIPLES OF NUMERATION AS LAID DOWN IN NOS. 27 & 31.

36. According to the principles laid down in Nos. 27 & 31, it follows:

1st. That, to render a *whole* number, ten, a hundred, a thousand times greater, we must write at the right-hand side of the number, one, two, three naughts or ciphers (1).

Thus, the number 26 units, becomes 260 in adding a cipher after the 6, that is, ten times greater than the first, since the units become tens, and the tens, hundreds; or, in other words, the figure of the first order becomes a figure of the second order, and that of the second order, becomes a figure of the third order. If we add another cipher, we obtain 2600, which is a hundred times greater than the first number, since the 260 units have become 26 hundreds.

2nd. That, when the whole number has a decimal annexed, it suffices to remove the decimal point to one, two, three, &c., figures towards the right, to render the number ten, a hundred, a thousand, &c., times greater.

Thus, 26.35 becomes ten times greater if written 263.5, since the tenths become units, the units tens, &c.

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36. *How can a whole number be rendered* ten, *a* hundred, *a* thousand *times* greater?— 36. Do. *A whole number with a decimal annexed?*

(1) This means that the number obtained, equals ten, a hundred times, &c., the first; otherwise this reasoning would lead us to conclude that the unit is ten times greater than the unit, which would be absurd.

3rd. That, when the number of decimals is not sufficient to render the number as required, we must annex to its right-hand side as many ciphers as will answer the purpose.

Thus, to render the number 24.5 a thousand times greater, it would be necessary to remove the point three figures towards the right; but, as there is only one decimal, place two ciphers after the five, and the number becomes 24500, which number is evidently a thousand times greater than the first, since the units of the first order have been changed into units of thousands, or of the second order.

37. From the same principles, it follows also:

1st. That, to render a whole number ten, a hundred, a thousand times, &c., smaller, it suffices to cut off from the right-hand side one, two, three, &c., figures.

Thus, in the number 925; if we cut off two figures by the decimal point we obtain 9.25, which is a hundred times smaller than the first, since the hundreds have become units, the tens, tenths, &c.

2nd. That, if it be a mixed number, the decimal point must be removed one, two, three, &c., figures towards the left.

Thus, to render 26.35 **ten times smaller,** remove the decimal point one figure towards the left, and it becomes 2.635, that is, ten times smaller than the first, since the tens become units, &c.

3rd. That, if the number, either whole or decimal only, does not contain a sufficient number of figures at the left-hand side of the point, we must write as many ciphers as will answer the purpose of the question, taking care that one remain to take the place of the units.

Thus, to render the numbers 8 and 2.635 a thousand times smaller, place three ciphers on the left-hand side of each of those numbers; the first of these ciphers will hold the place of the units, and the others will reduce the primitive number to the required value; thus, the numbers become 0.008 and 0.002635, which are evidently a thousand times smaller than the first, since the units have been changed into thousandths, &c.

PRACTICAL EXERCISES

ON THE PROPERTIES OF DECIMAL NUMERATION.

1. Render the whole number 38

1°	10		*Ans.*	380.
2°	100		*Ans.*	
3°	1000	times greater.	*Ans.*	38000.
4°	10000		*Ans.*	
5°	100000		*Ans.*	3800000.
6°	1000000		*Ans.*	

37. *How can a whole number be rendered,* ten, a hundred, &c., *times* smaller?—37. Do *mixed number?*

2. Render the mixed number 42.1064231

1°	10		*Ans.*	
2°	100		*Ans.*	4210.64231
3°	1000	times greater.	*Ans.*	
4°	10000		*Ans.*	421064.231
5°	100000		*Ans.*	
6°	1000000		*Ans.*	42106423.1

3. Render the mixed number 4.20

1°	10		*Ans.*	42.
2°	100		*Ans.*	
3°	1000	times greater.	*Ans.*	
4°	10000		*Ans.*	42000.
5°	100000		*Ans.*	
6°	1000000		*Ans.*	

4. Render the decimal 0.05

1°	10		*Ans.*	0.5
2°	100		*Ans.*	
3°	1000	times greater.	*Ans.*	50.
4°	10000		*Ans.*	
5°	100000		*Ans.*	
6°	1000000		*Ans.*	

5. Render the whole number 6705415

1°	10		*Ans.*	
2°	100		*Ans.*	
3°	1000	times greater.	*Ans.*	
4°	10000		*Ans.*	67054150000.
5°	100000		*Ans.*	
6°	1000000		*Ans.*	

6. Render the mixed number 7610438.06

1°	10		*Ans.*	
2°	100		*Ans.*	76104.3806
3°	1000	times smaller.	*Ans.*	
4°	10000		*Ans.*	
5°	100000		*Ans.*	76.1043806
6°	1000000		*Ans.*	

7. Render the mixed number 5.45

1°	10		*Ans.*	
2°	100		*Ans.*	
3°	1000	times smaller.	*Ans.*	0.00545
4°	10000		*Ans.*	
5°	100000		*Ans.*	
6°	1000000		*Ans.*	0.00000545

8. Render the decimal 0.05

1°	10		*Ans.*	
2°	100		*Ans.*	
3°	1000		*Ans.*	
4°	10000	times smaller.	*Ans.*	0.000005
5°	100000		*Ans.*	
6°	1000000		*Ans.*	

9. Render the mixed number 206.007

1°	10		*Ans.*	
2°	100		*Ans.*	
3°	1000		*Ans.*	
4°	10000	times smaller.	*Ans.*	
5°	100000		*Ans.*	0.00206007
6°	1000000		*Ans.*	

10. Render the mixed number 1462.309.

1°	10		*Ans.*	146.2309
2°	100		*Ans.*	
3°	1000		*Ans.*	
4°	10000	times smaller.	*Ans.*	
5°	100000		*Ans.*	
6°	1000000		*Ans.*	

11.	Render	165.	10	times	greater.	*Ans.* 1650.
12.	"	3867.	100	"	smaller.	*Ans.* 38.67
13.	"	2064.15	1000	"	greater.	*Ans.*
14.	"	640.4	100	"	"	*Ans.* 64040.
15.	"	74.	1000	"	smaller.	*Ans.* 0.074
16.	"	746.	10000	"	greater.	*Ans.*
17.	"	9.35	100	"	smaller.	*Ans.*
18.	"	76874.	10000000	"	greater.	*Ans.*
19.	"	6.468	1000	"	"	*Ans.*
20.	"	0.45	1000	"	"	*Ans.* 450.
21.	"	9.10	1000	"	smaller.	*Ans.*
22.	"	0.05	1000	"	"	*Ans.* 0.00005
23.	"	9.6786	10000	"	greater.	*Ans.*
24.	"	4.0000007	100	"	smaller.	*Ans.*
25.	"	0.0007	100	"	greater.	*Ans.*
26.	"	14.666	10000	"	smaller.	*Ans.*
27.	"	0.7	10	"	"	*Ans.*
28.	"	674.867	10000000	"	greater.	*Ans.*
29.	"	40.6804	1000	"	"	*Ans.*
30.	"	60600867.	10000	"	smaller.	*Ans.* 6060.0867
31.	"	9.45	10000000	"	"	*Ans.*
32	"	46.520	1000000	"	greater.	*Ans.*
33.	"	76840.3	100000	"	smaller.	*Ans.*
34.	"	0.013	10000	"	greater.	*Ans.*

ADDITION.

38. Addition is the process of uniting together several numbers of the same kind, so as to form a single number called the **Sum** or **Amount.**

39. Numbers are of the same kind when they have the same denomination.

For instance, dollars can be added to dollars, pounds to pounds, and yards to yards, &c.; but dollars could not be added to yards, nor shillings to feet.

Example of an Addition with whole numbers.

What is the sum of the three following numbers: 428, 635, and 874?

OPERATION.

$$\begin{array}{r} 428 \\ 635 \\ 874 \\ \hline 1937 \end{array}$$

ANALYSIS.—Having arranged the numbers, so that all the units of the same order shall stand in the same column, we first add the column of *units;* thus, 8 and five are 13, and 4 are 17 units, = 1 ten and 7 units. We write the 7 units under the column of units and *carry* or add the 1 ten to the column of tens; thus, 1 added to 2 makes 3, and 3 are 6, and 7 are 13 tens = 1 hundred and 3 tens. We write the 3 tens under the column of tens, and carry the 1 hundred to the column of hundreds; thus, 1 added to 4 makes 5, and 6 are 11, and 8 are 19 hundreds = 1 thousand and 9 hundreds. We write the 9 hundreds under the column of hundreds; and there being no other column to be added, we set down the 1 thousand in the thousands' place, and find the amount of the three numbers to be 1937.

40. We begin the addition by the figures of the first column at the right-hand side, so that in whole numbers, we may carry the tens proceeding from the addition of the units to the column of the tens, the hundreds proceeding from the tens to the column of the hundreds, &c.; and also in decimals, carry the tenths proceeding from the hundredths to the column of the tenths, and the units proceeding from the addition of the tenths to the column of the units, and so on.

41. From the preceding illustrations we deduce the following:

RULE.—I. *Write the numbers to be added so that all the units of the same order shall stand in the same column; that is, units under units, tens under tens, etc.*

II. *Beginning at units, add downward, or upward, each column separately, and write the sum underneath, if it be less than ten.*

III. *If the sum of any column be ten, or more than ten, write the unit figure only, and add the ten or tens to the next column.*

IV. *Write the whole sum of the last column.*

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38. *What is* addition?— 39. *What is meant by numbers of the same kind?*— 40. *How is addition to be commenced?*— 41. *What is the general rule for addition?*

ADDITION OF DECIMALS.

42. RULE.—*In adding decimals, we follow the same process as for the addition of whole numbers; but we cut off from the right, by a point, as many decimals as there are in the number which contains the most of all the numbers added.*

Ex. Given the following numbers to be added: 3579 units 25 hundredths, 4682 units 15 hundredths, 573 units 75 hundredths, and 7856 units 80 hundredths.

OPERATION.

3579.25
4682.05
573.75
7856.80

Ans. 16691.85

which is read in the following manner: 16691 units 85 hundredths.

ANALYSIS.—We commence by the 1st. column at the right; thus, 5 and 5 are 10, and 5 are 15 hundredths = 1 tenth and 5 hundredths. We write the 5 hundredths under the column of hundredths, and carry the 1 tenth to the column of the tenths; thus, 1 and 2 are 3, and 7 are 10, and 8 are 18 tenths = 1 unit and 8 tenths. We write the 8 under the column of tenths and carry the 1 unit to the column of units; thus, 1 and 9 are 10, &c.

PROOF.

43. The Proof of an Arithmetical operation is another operation to prove the exactness of the first.

44. RULE.—*Separate the numbers to be added into two parts; add each of these two parts and set down their respective sums. Then add these two sums together; if their amount is equal to the first answer, the work is presumed to be correct.*

The numbers could be divided into a greater number of parts than two.

Example.

OPERATION.

123.24
349.00
56.25
149.34
967.32

Ans. 1645.15

PROOF.

1st. Part.	2nd. Part.
123.24	56.25
349.00	149.34
472.24	967.32
	1172.91

Addition of partial totals.

1172.91
472.24
1645.15

which is read 1645 units 15 hundredths.

USE OF ADDITION.—Addition serves to find the sum of several numbers: the whole cost when the buying price and other expenses are given. The selling price when the buying price and profit are given, &c.

We know that the *resolution* or solving of a problem requires an addition, when we must find a number equal to the sum or amount of several others.

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42. *How do you add decimals?*— 43. *What do you understand by an* arithmetical proof?— 44. *How is the proof of an addition made?*

PRACTICE IN ADDITION.

1. 600 + 850 + 501 + 49 + 904 + 759 + 215 + 555.
Ans. 4433 units.

2. 604 + 810 + 333 + 1226 + 3004 + 4004 + 5105.
Ans. 15086.

3. 19223 + 125979 + 189023 + 100610 + 3300. *Ans.* 438135.

4. 15879 + 15957 + 100101 + 810799 + 975020 + 100110.

5. 41 + 64 + 77 + 49 + 64 + 47 + 36 + 1012. *Ans.* 1390.

6. 110200 + 9104 + 4610 + 10110 + 95303 + 8888.

7. 100989 + 100001454 + 77777707 + 10110000 + 100000090.

8. 50319010 + 15015 + 132 + 20000020 + 109909 + 8888888 + 782704 + 189345. *Ans.* 80317134.

9. 49 + 97 + 68 + 45 + 54 + 68 + 38 + 97 + 75 + 63 + 49 + 98 + 57 + 95 + 59 + 87 + 65 + 43 + 11 + 10. *Ans.* 1238.

10. 49 + 468 + 429 + 47 + 64 + 46 + 36 + 49 + 94 + 39 + 93 + 29 + 92 + 87 + 78 + 57 + 86 + 39 + 47 + 74 + 98 + 57.

11. 56 + 48 + 64 + 46 + 57 + 86 + 54 + 36 + 95 + 34 + 66 + 44 + 33 + 99 + 65 + 67 + 66 + 77 + 59 + 96 + 69 + 49 + 95 + 67 + 27 + 45 + 36 + 97.

12. 52 + 34 + 42 + 29 + 423 + 695 + 987 + 429 + 678 + 542 + 249 + 75 + 99 + 88 + 89 + 98 + 36 + 674 + 99 + 89 + 69 + 429 + 98 + 103 + 138 + 274 + 391.

13. 94 + 569 + 439 + 590 + 694 + 678 + 534 + 864 + 684 + 468 + 94 + 95 + 649 + 946 + 495 + 789 + 647 + 963 + 769 + 956 + 875 + 708 + 1075 + 3548 + 739.

14. Express by figures and add up the following numbers: eighteen units, + ninety-five, + one hundred and one, + one hundred and twenty-three, + three hundred and ten, + six hundred. *Ans.* 1247.

15. Required the sum of six hundred units, + eight hundred and fifty, + five hundred and one, + forty-nine, + nine hundred and four, + seven hundred and fifty-nine, + two hundred and fifteen, and five hundred and fifty-five.

16. Express by figures one hundred and ninety-five, + two hundred and eleven, + one hundred and ten, + one hundred and ninety-nine, + eight hundred and one, + seven hundred and seventy-seven, + nine hundred and one. *Ans.* 3194.

17. Express by figures two thousand nine hundred and ninety-seven, + twenty-three thousand six hundred and fifteen, + twelve thousand six hundred and ten, + one thousand and fifteen, and make up the sum.

18. Required the sum of nineteen thousand two hundred and twenty-three units, + one hundred and twenty-five thousand nine hundred and seventy-nine, + one hundred and eighty-nine thousand and twenty three, + one hundred thousand six hundred and ten, + three thousand and three hundred.

19. Required the sum of fifteen thousand eight hundred and seventy-nine units, + fifteen thousand nine hundred and fifty-seven, + one hundred thousand one hundred and one, + eight hundred and ten thousand seven hundred and ninety-nine, + nine hundred and seventy-five thousand, + one hundred thousand and ten? *Ans.* 2017746.

20. Required the sum of fifty millions three hundred and nineteen thousand and ten units, + fifteen thousand and fifteen, + one hundred and thirty-two, + twenty millions and twenty, + one hundred and nine thousand nine hundred and nine, + eight millions eight hundred and eighty-eight thousand eight hundred and eighty-eight units, + eleven thousand, + eleven hundred and eleven?

21. 40.05 + 104.8 + 1003.025 + 7.38 + 2.15.

Ans. 1157.405 thousandths.

22. 0.4 + 0.20 + 0.0306 + 0.01 + 0.200 + 0.044 + 0.18.

Ans. 1.0646 ten-thousandths.

23. 0.05 + 0.00012 + 0.110 + 0.22 + 0.000015 + 0.014 + 0.0017.

24. 100. + 0.400 + 20.130 + 0.020 + 0.10012 + 0.0001005 + 0.1.

25. 6.96 + 3.99 + 6.78 + 4.39 + 4.79 + 2.98 + 4.67 + 7.69 + 4.42 + 6.81 + 7.59 + 9.76 + 4.36 + 7.95 + 5.35 + 7.77 + 3.79 + 9.99 + 7.889.

Ans. 117.929 thousandths.

26. 4.95 + 9.54 + 8.69 + 4.29 + 24.09 + 4.07 + 7.45 + 3.68 + 9.86 + 7.85 + 7.67 + 3.75 + 47.47 + 9.09 + 4.47 + 6.97.

27. 3.78 + 8.95 + 9.84 + 9.38 + 37.14 + 6.053 + 67 + 4.78 + 4.98 + 5.75 + 7.75 + 5.55 + 47 + 15 + 1.75 + 2.55 + 8.47.

28. 4.24 + 4.70 + 3.65 + 1.95 + 1640 + 49.65 + 3.45 + 2.90 + 9.80 + 1.40 + 3.55 + 7.40 + 4.65 + 9.09 + 7.60 + 55.45 + 2.95.

29. Express by figures forty units and five hundredths, + one hundred four units and eight tenths, + one thousand three units and twenty-five thousanths, + seven units and thirty hundredths, + two units and fifteen hundredths, and add them up. *Ans.* 1157.325.

30. Required the sum of four tenths, + twenty thousandths, + three hundred ten-thousandths, + one hundredth, + two hundred thousandths, + forty-four thousanths, + eighteen hundredths.

31. Required the sum of four hundredths, + twelve hundred-thousandths, + one hundred ten-thousandths, + eleven hundredths, + fifteen millionths, + fourteen thousandths. *Ans.* 0.174135.

32. Required the sum of three hundred-thousandths, + four thousandths, + seven tenths, + three hundred-thousandths, + eight billionths, + nineteen thousandths.

33. Add the following numbers: eight hundred-thousandths, + nine hundred ten-thousandths, + three hundred tenths, + one thousand hundredths, + thirteen ten-thousandths, + twenty millionths, + eight hundredths, + eleven hundred-thousandths, + three thousand and nineteen millionths. *Ans.* 40.174529.

34. Required the sum of one thousand tenths, + four hundred thousandths, + two thousand hundredths, + thirteen hundred tenths, + twenty thousand millionths, + ten thousand and twelve hundred-thousandths, + one thousand and five ten-millionths, + one hundred thousand millionths?

35. What is the sum of the following numbers: twenty-five, and seven millionths; one hundred forty-five, six hundred and forty-three thousandths; one hundred and seventy-five, and eighty-nine hundredths; seventeen, and three hundred and forty-eight hundred thousandths? *Ans.* 363.536487.

PRACTICAL PROBLEMS OR QUESTIONS IN ADDITION.

1. I bought a house for $25840, I paid $1565 for right of possession, and $238 for repairs; what did it cost me?

OPERATION.

$ 25840
1565
238
$ 27643 *Ans.*

ANALYSIS.—The whole cost of the house is equal to the amount of the sums expended, that is 25840 + 1565 + 238 = $27643 *Ans.*

NOTA.—The *whole cost* is the cost of an object when all expenses are paid.

2. I bought some merchandise for the sum of $245.65; how much must I sell them to gain $25.20?

OPERATION.

$ 245.65
25.20
$ 270.85 *Ans.*

ANALYSIS.—We must sell the merchandise at a price equal to what it cost, plus the sum we wish to gain; that is, 245.65 + 25.20 = $270.85, selling price.

3. A family spends on Monday, $4.75; on Tuesday, $1.15 more than on Monday, and $2.05 less than on Sunday: how much was spent during these three days?

ANALYSIS.—First, find the expenditure of Tuesday and Sunday. On Tuesday, they spent 4.75 + 1.15 = $5.90; on Sunday, 5.90 + 2.08 = $7.98. Then, 4.75 + 5.90 + 7.98 = $18.63, whole expense for the three days.

4. A family owes the baker $27; the butcher, $46; the shoe-maker, $69; the grocer, $108; and for house-rent, 145; how much does the family owe in all? *Ans.* $395.

5. Louis was born in 1847, in what year will he be 24 years old?

6. The population of Montreal is about 135000 souls, that of Quebec, 64150; Three Rivers, 8300; St. Hyacinth, 4102; Point-Levis, 5300; Sorel, 5250; Sherbrooke, 4300: what is the whole population of those seven towns? *Ans.* 226402.

7. A wholesale merchant sold during the year $9023 worth of cloth; $4500 of yellow cotton; $1592 of Irish linen; $1790 of calico; $856 of merino. For how much did he sell during the whole year?

8. A man owed a certain sum of money; he paid the 1st. time $240.50; the 2nd. $376.25; the 3rd. $109.40, after which he yet owes $92. How much did he owe at first? *Ans.* $818.15.

9. A company of soldiers have fired 29682 cartridges in an engagement and they have still 13403 remaining. How many had they before the engagement? *Ans.* 43085.

10. An army consists of three grand divisions; the 1st. contains 8610 men, the 2nd. 7960, and the 3rd. 8490. How many men are there in the army? *Ans.* 25090 men.

11. The hind-quarters of an ox weigh 390 pounds each; the fore-quarters 325 pounds each; the skin 97 pounds and the suet 95 pounds. What is the whole weight of the ox? *Ans.* 1622 pounds.

12. Andrew bought a horse and carriage for $310; and in selling both he gained $176. How much did he sell them for? *Ans.* $486.

13. A man bought three fields for $7680, he sells them at a profit of $750. For how much did he sell them? *Ans.* $8430.

14. John bought a new farm in a town-ship; the 1st. year it yielded 736 bush. of oats; the 2nd. year, 3697 bush.; the 3rd. year, 9982 bush.; the 4th. year, 10065 bush.; the 5th. year, 12760 bush.: how many bushels did it yield in the five years? *Ans.* 37240 bush.

15. How many years elapsed from the taking of Troy, which occurred 1184 years before Christ, till the year 1869 of the Christian era?

16. A person who was born in 1831, died at the age of 37. In what year did she die? *Ans.* 1868.

17. I have four bills to pay; the 1st., of $1405; the 2nd., $875.40; the 3rd., $96.15; and the 4th., $798. What sum do I require to pay them? *Ans.* $3174.55.

18. The area of the Dominion of Canada is computed as follows: the Province of Ontario, 180000 square miles; the Province af Quebec, 210000 square miles; the Province of Nova Scotia, 19650 square miles; and the Province of New Brunswick, 27710 square miles. What is the whole area? *Ans.* 437360 square miles.

19. A tanner bought 25 hides for the sum of $164.80; after having prepared them, he sold them for $112.60 more than he had paid. How much did he sell them for? *Ans.* $277.40.

20. A certain sum of money was divided among three persons: the 1st., received $65; the 2nd., $26.30 more than the first; the 3rd., $32.10 more than the second. How much did each receive and what was the sum divided? *Ans.* 1st. $65; 2nd. $91.30; 3rd. $123.40. Whole sum $279.70.

21. A merchant in selling cloth to the amount of $6218.50, lost $143.40 by the bargain; how much did he pay for it?

22. At the census of 1861, the population of Upper Canada was 1409430 inhabitants; that of Lower Canada, 1130800; Nova Scotia, 300000; New Brunswick, 250000. How many inhabitants were there in those four Provinces which compose the present Dominion of Canada? *Ans.* 3090230 inhabitants.

23. The battle of Marathon took place 490 before Christ. How many years since that period to 1868? *Ans.* 2358 years.

24. Eighteen tanned horse-hides weigh 486 pounds; they have lost 324 pounds in being tanned. What was their raw weight?

25. A number is such that if diminished by 6487 there remains but 5976. What is the number? *Ans.* 12463.

26. Raw wool is worth $0.75 per pound, when prepared it augments $2.45. What is the price of a pound of prepared wool? *Ans.* $3.20.

27. The population of Europe consists of 278694707 inhabitants; that of North America, 43879348; that of South America, 22007823; that of Asia, 588700000; that of Africa, 64035000; that of Oceanica, 20600000; that of Australia, 2025000; and that of Polynesia, 419000. What is the whole population of the globe?

Ans. 1020360878 inhabitants.

SUBTRACTION.

45. Subtraction is the process of finding the difference between two numbers of the same kind.

The larger number, or that which is to be diminished, is called the *Minuend;* and the smaller, or that which is to be subtracted, the *Subtrahend.*

46. The result of subtraction is called the **remainder, excess,** or **difference.**

CASE I.—*To subtract when each figure in the subtrahend is less than the figure above it in the minuend.*

Ex. From 547 take 324.

OPERATION.

Minuend	5 4 7
Subtrahend	3 2 4
Remainder	2 2 3

ANALYSIS.—We write the less number under the greater, so that units of the same order shall stand in the same column; then, we begin at the right and proceed as follows: 4 units from 7 units leave 3 units, which we write in units' place. Two tens from 4 tens leave two tens, which we write in tens' place. Three hundreds from 5 hundreds leave 2 hundreds, which we write in hundreds' place. Hence we have for the remainder, 2 hundreds, 2 tens, and 3 units, or 223.

EXAMPLES FOR PRACTICE.

	(1.)	(2.)	(3.)	(4.)	(5.)
Minuend	457	273	936	685	974
Subtrahend	325	132	714	423	631
Remainder	132	141	222	262	343

	(6.)	(7.)	(8.)	(9.)	(10.)
From	648	376	857	498	645
Take	234	164	522	175	542

11. 3692 — 1212 = *Ans.* 2480
12. 7634 — 3132 = *Ans.* 4502
13. 8742 — 5331 = *Ans.* 3411
14. 41763 — 11522 = *Ans.* 30241
15. 7839 — 5427 = *Ans.* 2412
16. 3724 — 2502 = *Ans.* 1222
17. 2945 — 832 = *Ans.* 2113
18. 69524 — 47321 = *Ans.* 22203
19. 56247 — 15123 = *Ans.* 41124
20. 72365 — 1243 = *Ans.* 71122
21. 1243 — 213 = *Ans.*...
22. 48673 — 16330 = *Ans.*...
23. 34272 — 13051 = *Ans.*...
24. 79832 — 57411 = *Ans.*...
25. 15475 — 4050 = *Ans.*...
26. 15768 — 4327 = *Ans.*...
27. 982876 — 120341 = *Ans.*...
28. 217951 — 5430 = *Ans.*...
29. 760142 — 570031 = *Ans.*...
30. 391657 — 141322 = *Ans.*...

CASE II.—*To subtract when any figure in the subtrahend is greater than the figure above it in the minuend.*

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45. *What is* subtraction?— *Define* minuend,— subtrahend.— 46. *How is the result called?*

Ex. Find the difference between 853029 and 360475.

METHOD BY BORROWING.

OPERATION.

	Hund. of Thousands.	Tens of Thousands.	Thousands.	Hundreds.	Tens.	Units.
Minuend	8	5	3	0	2	9
Subtrahend	3	6	0	4	7	5
Remainder	4	9	2	5	5	4

ANALYSIS.—Having placed the smaller number under the greater, with units under units, &c., as in Case I, we draw a line underneath. Then, beginning at the right-hand we say: 5 units from 9 units leave 4 units, which is the difference of the units, and which is written in the units' place below. We then proceed to take the 7 tens from the 2 tens above; but this cannot be done, since the 7 is greater than the 2. We cannot borrow from the next figure, as it is a cipher, we then borrow 1 from the 3 thousands, which equals 10 hundreds, leaving 9 above the cipher, and add the 1 hundred equal to 10 tens, to the 2 tens, making 12 tens; 7 tens from 12 leave 5 tens, which we write under; 4 from 9 leaves 5, which we write in hundreds' place below. As we have taken 1 thousand from the 3 thousands, 2 thousands remain; naught from 2 leaves 2, which we write under. We cannot take 6 ten-thousands from 5 ten-thousands; so from the 8 hundred-thousands we take 1 hundred-thousand, which equals 10 ten-thousands, and adding them to the 5 ten-thousands, make 15 ten-thousands; 6 ten-thousands from 15 ten-thousands leave 9 ten-thousands, which we write under. Having taken 1 hundred-thousand from the 8 hundred thousands, 7 hundred-thousands are left; 3 hundred-thousands from 7 hundred-thousands leave 4 hundred-thousands, which we write under; and thus find the difference, or remainder, to be 492554.

METHOD BY ADDING 10.

OPERATION.

8 5 3 0 2 9
3 6 0 4 7 5
4 9 2 5 5 4

ANALYSIS.—We first take the 5 units from the 9 units, and find the difference to be 4 units, which we write below. As we cannot take 7 tens from 2 tens, we add 10 tens to 2 tens, making 12 tens; 7 tens from 12 tens leave 5 tens. But having added 10 tens, or 1 hundred, to the minuend, we shall have a remainder 1 hundred too large, to compensate, we add 1 hundred to the 4 hundreds of the subtrahend, making 5 hundreds. We cannot take 5 hundreds from 0; so we add 10 hundreds to 0, making 10 hundreds; 5 hundreds from 10 hundreds leave 5 hundreds, which we write below. Now, as we have added 10 hundreds, or 1 thousand, to the minuend, we shall have a remainder 1 thousand too large, unless we add 1 thousand to the 0 of the thousands in the subtrahend, making 1 thousand; 1 thousand from 3 thousands leave 2 thousands. We then proceed to take the 6 ten-thousands from the 5 ten-thousands above it, as we cannot, we add 10 ten-thousands to the 5 ten-thousands, making 15 ten-thousands; 6 ten-thousands from 15 ten-thousands leave 9 ten-thousands. Then, to compensate for the 10 thus added to the 5 in the minuend, we add 1 to the 3 in the subtrahend, making 4 hundred-thousands, and subtract the 4 from the 8, which leave 4 hundred-thousands. Thus, we find the remainder to be 492554, the same as before.

This operation depends on the principle, *that, if any two numbers are equally increased, their difference remains the same.*

47. From the preceding illustrations we derive the following

RULE.—I. *Write the less number under the greater, so that units of the same order may stand under each other.*

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47. *What is the* rule *for subtraction?*

II. *Commencing at the right-hand, take each figure of the subtrahend from the figure above it, and write the result underneath.*

III. *If any figure in the subtrahend be greater than the corresponding figure above it, add* 10 *to that upper figure before subtracting, and then add one to the next left-hand figure of the subtrahend.*

PROOF OF SUBTRACTION.

48. We make the **Proof** of Subtraction in adding the remainder to the subtrahend, their sum will be equal to the minuend, if the work is correct.

Ex. From 35678 take 27899.

	3 5 6 7 8
	2 7 8 9 9
Rem.	7 7 7 9
Proof	3 5 6 7 8

ANALYSIS.—To prove this operation, we add the remainder 7779 to the subtrahend 27899 and obtain 35678, which sum is equal to the minuend, or greater number. Hence we conclude that the operation is correct.

This method of proof depends on the principle, *that the greater of any two numbers is equal to the less added to the difference.*

USE OF SUBTRACTION.—*Subtraction serves to find the gain or loss on goods; what we still owe on a sum of money of which we have already paid a part; in general to find the surplus of a number over another; the difference between two numbers, &c.*

We know that the solution *of a problem requires a subtraction, when we must find the difference between two numbers, or the excess of a number over another; and when it is required to find one of two numbers forming a total, that total or amount, and one of the numbers, being given.*

EXAMPLES FOR PRACTICE.

	(1.)	(2.)	(3.)	(4.)
Minuend	76518	57813	13042	250143
Subtrahend	49359	38675	9176	176158
Remainder	27159	19138	3866	73985
Proof	76518	Proof 57813	Proof 13042	Proof 250143

48. *How do you* prove *subtraction?*

5.	From	3900	take	351	*Ans.*	3549
6.	"	49469	"	15574	"	33895
7.	"	79906	"	16134	"	63772
8.	"	190540	"	30409	"	160131
9.	"	478754	"	97125	"	
10.	"	426542	"	179127	"	247415
11.	"	740070	"	471097	"	268973
12.	"	577405	"	198576	"	378829
13.	"	405907	"	55595	"	
14.	"	897452	"	508578	"	388874
15.	"	8950076	"	4137976	"	4812100
16.	"	14003325	"	988827	"	13014498
17.	"	15989700	"	154379	"	15835321
18.	"	21530600	"	737898	"	20792702
19.	"	97660054	"	14550045	"	83110009
20.	"	457652478	"	49876579	"	
21.	"	337008974	"	40073049	"	296935925
22.	"	154400000	"	91791994	"	62608006
23.	"	190054009	"	4590489	"	185463520
24.	"	7546748 5	"	64834795	"	689840100
25.	"	10007549	"	9068073	"	939476
26.	"	127321155	"	1300475	"	126020680
27.	"	418030450	"	27740761	"	390289689
28.	"	945000090	"	1500734	"	
29.	"	809005409	"	3740055	"	805265354
30.	"	490009076	"	5475904	"	484533172
31.	"	847653454	"	74375576	"	773277878
32.	"	850070452	"	97050654	"	753019798
33.	"	546807575	"	277451794	"	269355781
34.	"	101010101	"	9737350	"	91272751
35.	"	653405995	"	476294474	"	
36.	"	677454854	"	495647562	"	181807292
37.	"	764927074	"	676489572	"	88437502
38.	"	950076074	"	475207454	"	474868620
39.	"	456700750	"	45612495	"	411088255
40.	"	876007054	"	798435495	"	77571559
41.	"	753097507	"	194289778	"	558807729
42.	"	400075546	"	93457897	"	
43.	"	487054554	"	98047775	"	389006779
44.	"	432700769	"	71904257	"	360796512
45.	"	954875754	"	577469579	"	377406175
46.	"	453007527	"	276499619	"	176507908
47.	"	490700007	"	203405604	"	
48.	"	974500700	"	93235945	"	881264755
49.	"	839457354	"	745689835	"	93767519
50.	"	847654976	"	39787495	"	807867481
51.	"	905207246	"	746855472	"	158351774
52.	"	4184545945	"	178809709	"	4005736236
53.	"	9457385700	"	17073959	"	
54.	"	9707000591	"	19779883	"	9687220708

SUBTRACTION OF DECIMALS.

Ex. From 86.7 take 69.354.

OPERATION.

86.700
69.354
17.346

ANALYSIS.—Having placed the less number under the greater, so that figures of the same decimal place stand in the same column, we write two ciphers at the right of 7, in order that the minuend may have as many decimal figures as the subtrahend; then we subtract as in whole numbers, and finally place the decimal point in the remainder directly under that in the given number.

RULE.—I. *Write the less number under the greater, so that the decimal points shall stand directly under each other.*

II. *Subtract as in whole numbers, and place the decimal point in the result directly under the points in the given numbers.*

EXAMPLES FOR PRACTICE.

	(1.)	(2.)	(3.)	(4.)
From	12.067	8.11	36.105	1.0062
Take	9.71	6.7519	7.11892	0.43
Ans.	2.357	1.3581	28.98608	0.5762

5.	From	90.49	take	39.59	*Ans.* 50.90
6.	"	109.191	"	49.073	" 60.118
7.	"	5409.055	"	4045.997	" 1363.058
8.	"	764907.05	"	87929.795	" 676977.255
9.	"	897450.07	"	98776.095	"
10.	"	465742.5	"	76908.075	" 388834.425
11.	"	870079.04	"	1987-9.958	" 671289.082
12.	"	400048.2136	"	9372.016	" 390676.1976
13.	"	409004.9099	"	100.137	" 408904.7729
14.	"	570075.9004	"	4053.509	" 566022.3914
15.	"	49.1019	"	35.708	"
16.	"	610011.050	"	31971.9999	" 578039.0501
17.	"	71079.0013	"	7482.1736	" 63596.8277
18.	"	79073.07	"	7398.1204	" 71674.9496
19.	"	126001.0001	"	98996.9088	" 27004.0913
20.	"	191279.9709	"	50056.0099	" 141223.9610
21.	"	401645.1005	"	498.6709	" 401146.4296
22.	"	700007.0236	"	79797.0098	"
23.	"	411978.10359	"	36730.09671	" 375248.00688
24.	"	960945.00005	"	600979.00007	" 359965.99998
25.	"	0.0707	"	0.000607	" 0.070093
26.	"	0.0006	"	0.0000075	" 0.0005925
27.	"	0.90019	"	0.7300007	" 0.1701893
28.	"	0.0089	"	0.0070675	" 0.0018325
29.	"	0.0904	"	0.00289709	" 0.08750291
30.	"	0.7009	"	0.190007	" 0.510893
31.	"	0.0991	"	0.004500008	" 0.094599992

32.	From	0.0779	take	0.01011001	*Ans.*	0.06778999
33.	"	0.900	"	0.0019904	"	0.8980096
34.	"	0.19100	"	0.09900035	"	0.09199965
35.	"	0.4500	"	0.00550045	"	0.44449955
36.	"	0.09839	"	0.09500959	"	0.00338041

PRACTICAL PROBLEMS IN SUBTRACTION.

1. A field which had cost $2360 was sold for $2628. What is the gain?

OPERATION.

$2628
$2360
$ 268

ANALYSIS.—The gain is equal to the difference between the cost and selling prices; therefore, in subtracting the cost price $2360 from the selling price $2628, we obtain the gain. *Ans.* $268 gain.

2. How much does a merchant lose in selling for $4825.75 goods which cost him $5174.10?

OPERATION.

$5174.10
$4825.75
$ 348.35

ANALYSIS.—The loss is equal to the difference between the buying and selling prices; therefore, in subtracting the selling price $4825.75 from the buying price $5174.10, we obtain the loss. *Ans.* $345.35 loss.

3. A merchant bought flour for $5626, and sold the whole of it for $6853; how much did he gain? *Ans.* $1227.

4. Find the difference between 70401 and 6942? *Ans.* 63459.

5. What is the difference between 85450 and 54498? *Ans.* 30952.

6. I owed $1628; I paid $971; how much do I owe yet?

7. The greater of two numbers is 1302, and their difference 981; what is the smaller? *Ans.* 321.

8. A merchant sold in one day $2517.40 worth of dry goods, and thereby cleared a profit of $630.95. How much did the goods cost him? *Ans.* $1886.45.

9. To what number must we add 76 to increase it to 740?

10. The city of Quebec was founded by Champlain in 1608; how many years from that period to 1870? *Ans.* 262 years.

11. The area of the Province of Quebec is 210000 square miles; that of the Province of Ontario 180000 sq. m.: by how many square miles does the former exceed the latter? *Ans.* 30000 sq. m.

12. A father was 28 years old at the birth of his son, what will be the age of the son when the father will be 85 years old? *Ans.* 57 yr.

13. What number must be added to 357.75 to have 8000?

14. What will be the age in 1871 of a person who was born in 1792? *Ans.* 79 years.

15. What number must be added to 4 units 5 hundredths to have 10 units? *Ans.* 5 units 95 hundredths.

16. In 1857, Canada exported to the United States for a value of $13206436.10, and imported for $20224650.96. How much did the importations exceed the exportations? *Ans.* $7018214.86.

17. Napoleon I. died in 1821 at the age of 52. In what year was he born? *Ans.* 1769.

18. An army consisting of 41500 men, lost during a campaign 14704 men. How many remain? *Ans.* 26796 men.

19. What number must be added to 3 thousandths, to have 12 hundredths? *Ans.* 0.117 thousandths.

20. The population of Paris is 1953262 inhabitants and that of London 2863141; how much does the population of London exceed that of Paris? *Ans.* 909879 inhabitants.

21. Alfred the Great died in 901 at the age of 52, after a reign of 24 years: in what year was he born? *Ans.* 849.

22. Charlemagne was born in 742; he was crowned king of France in 768, emperor of the West in 800, and died in 814. How old was he, 1st. at his coronation as king; 2nd. as emperor; 3rd. at what age did he die; and 4th., how many years elapsed from his death until 1869? *Ans.* 1st. 26 as king, 2nd. 58 as emperor, 3rd. at the age of 72, and 4th. 1055 years.

23. Murillo's picture of the Immaculate Conception, being auctioned, the first bidding was $30000, but it was finally knocked down at $117000 and adjudged to the French Government who placed it in the museum of the Louvre. Required the difference between the 1st. and the last bidding? *Ans.* $87000.

24. The population of Montreal, in 1765, consisted of 7000 inhabitants; in 1851, it was 57715; in 1856, 75000; in 1860, 90000; and in 1868, about 135000. What was the increase of the population from 1851 to 1868? *Ans.* 77285 inhabitants.

25. A farmer reaped 1689 bushels of wheat, and 965 bushels of oats. He sold his neighbor John 890 bushels of wheat and 478 bushels oats, and the remainder to Joseph. How many bushels of each sort did he sell to Joseph? *Ans.* 799 bush. wheat and 487 bush. oats.

26. Two merchants, in commencing business, invested a capital of $18500; the 1st. invested $6590.40; how much must he add to his investment to equal that of the second? *Ans.* $5319.20.

27. Had I $508.50 more, I could pay a debt of $1015.80, and would have $75 left; how much have I? *Ans.* $582.30.

28. A merchant sold $11630 worth of cloth, which was $876 more than cost price; how much did it cost him? *Ans.* $10754.

29. A house which was sold for $14360, would have given a profit of $840 to its owner if he had paid it $300 less. How much dit it cost? *Ans.* $13820.

30. Gunpowder was invented in the year 1330; how long was this before the invention of printing, which was in 1441? *Ans.* 111 years.

PRACTICAL PROBLEMS COMBINING ADDITION AND SUBTRACTION.

1. A retail merchant places $45.25 in his drawer for change; on Monday he sells for $75.85; on Tuesday, for $68.40; on Wednesday, for $85; on Thursday, for $128.60; on Friday, for $54.85; and on Saturday, for $72.15; after which he pays a Bill of $95.60, another of $43.25, and takes $240.75 for his own expenses, and then there remains to him in cash a sum of $150. Are his accounts right?

ANALYSIS.—First find out what sum he would have had if he had not paid anything, and then what he has paid out.

He had, 45.25 + 75.85 + 68.40 + 85 + 128.60 + 54.85 + 72.15 = $530.10. He has taken out, 95.60 + 43.25 + 240.75 = $379.60. There should be left $530.10 — $379.60 = $150.50; difference 150.50 — 150 = *Ans.* $0.50 against him.

2. A market woman having 152 eggs, sold to one person 14 of them, to another 27, to another 73, to another 24, and to another 5: how many remain? *Ans.* 9.

3. A gentleman having $1128, lost $638, and spent $172: how much had he remaining? *Ans.* $318.

4. The waters of the St. Lawrence cover an area of 565000 square miles; two of its tributaries, the Saguenay and St. Maurice, cover, the one an area of 27000 square miles, and the other 21000 square miles. How much does the area of the St. Lawrence exceed those of its two tributaries? *Ans.* 517000 square miles.

5. A man has bought four building lots for the sum of $16860. For the 1st. he paid $2070.30; for the 2nd., $3674.50; for the 3rd., $4175: how much has he paid for the 4th.? *Ans.* $6940.20.

6. I deposited in a Savings Bank $8752.70; the first time I drew from it a sum of $4286; the second, $1650.50; the third, $972.75. How much have I left in the bank? *Ans.* $1843.45.

7. Moses was born about 1571 years before Christ, he left Egypt with the Hebrews the year 1491 before Christ, and died on Mount Nebo, in the year 1451 before Christ. What age was he, 1st. when he left Egypt; 2nd. at his death; and 3rd. how long from the period of his death to the year 1871 of the Christian era?

Ans. 1st. 80 years; 2nd. 120 years; 3rd. 3222 years.

8. A speculator gains $6570, and then loses $3762.40; at another time he gains $4545.72, and loses again $5632.10. Tell how much his gains exceed his losses? *Ans.* $1721.22.

9. A man deals in grains since 6 years; the 1st. year he lost $356; the 2nd., he gained $780.20; the 3rd., he gained $685.30; the 4th., he lost $2600; the 5th., he gained $4320.95; and the 6th., he lost again $3000. Did he gain or lose, and how much? *Ans.* $169.55 loss.

10. A owes a sum of $690, plus $55.20 for interest. He reimbursed at different times $87.50, $210.00, $318.45; how much does he still owe? *Ans.* $129.25.

11. A family owing its grocer $508.75, takes again effects to the amount of $240.32 and then gives in payment $704.65; what is yet the balance of its account? *Ans.* $44.42.

12. My brother owed a certain sum of money; he paid on account $284, $570.20, $210.08 and $345.30. Finally, in settling, he gave a Bank note of $1000, on which they returned him $454 change. What sum did he owe? *Ans.* $1955.58.

13. Peter has 360 sheep, Maurice 145 more than Peter, and Charles as many as Maurice and Peter together lacking 117. How many sheep has Charles? *Ans.* 748 sheep.

14. A merchant bought a whole cargo of Porto-Rico sugar worth $12347; he paid $311.70 for freight expenses, and $291.30 for commission and storage; after which, he sold his sugar for $12511.30. Required his gain or loss? *Ans.* $438.70 loss.

15. If I had sold $20 more a piece of linen which cost me $350, I would have gained $30; how much did I sell it? *Ans.* $360.

16. A speculator bought 217 cords of wood for $1085. He gave in payment 1800 pounds of salmon valued at $144.00; 700 bushels of potatoes worth $210, and 1200 pounds sugar equal to $72. How much does he owe yet? *Ans.* $659.

17. I have three creditors; I owe the 1st. $2500, the 2nd. $840, and the 3rd. $754. On the other hand, I have 2 debtors, the one owes me $1800, and the other, $2544. Besides I have $3768 in cash. Required what sum remains in hand after paying my debts? *Ans.* $4018.

18. How many pounds of bread will 200 pounds of flour give, knowing that it takes 114 pounds of water to knead them and that 44 pounds evaporate in baking? *Ans.*

19. Three boxes containing 1435 oranges have cost $17.15, and $3 each box for drayage: the first contains 240 oranges, the second 80 more; how many does the third contain? *Ans.* 875.

20. In adding $5.08, the price of an ox hide, to the sum expended by a tanner for 4 calf and 6 horse hides we obtain a sum of $22.98. Required the price of the 6 horse hides, knowing that the calf hides have cost $4.40? *Ans.* $13.50.

21. A cloth merchant bought 80 yards more than he had at first and then sold 140 yards; after which he has left half what he had in his shop before his last purchase. How many yards had he at first?

22. A dyer bought at three different times 109 pounds of dye for the sum of $3.84. The first time he bought 47 pounds and this quantity exceeded by 15 pounds his third purchase. How many pounds did he buy in his second purchase? *Ans.* 30 pounds.

23. A general starting for an expedition with 18000 men, left 600 of them to garrison a small town; at the same time he received a reinforcement of 800 more, 450 of whom he was obliged to leave in hospitals. Having asked 3500 more, he received only 2730; of these he left 1750 at different posts. Required the number of men he had on reaching his destination? *Ans.* 18730 men.

MULTIPLICATION.

49. Multiplication is the process of taking one number as many times as there are units in another.

50. The *terms* in Multiplication are:

1st. The **Multiplicand,** or number to be taken;

2nd. The **Multiplier,** or number by which we multiply, or which shows how many times the multiplicand is to be taken;

3rd. The **Product,** or the result obtained.

51. The multiplicand and multiplier are called **Factors,** because they *produce* or *make* the product.

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49. *What is* Multiplication?— *Define* Multiplicand.— Multiplier.— Product. — 51. What are the multiplicand and multiplier called?

MULTIPLICATION TABLE.

1 × 1 = 1	2 × 1 = 2	3 × 1 = 3	4 × 1 = 4
1 × 2 = 2	2 × 2 = 4	3 × 2 = 6	4 × 2 = 8
1 × 3 = 3	2 × 3 = 6	3 × 3 = 9	4 × 3 = 12
1 × 4 = 4	2 × 4 = 8	3 × 4 = 12	4 × 4 = 16
1 × 5 = 5	2 × 5 = 10	3 × 5 = 15	4 × 5 = 20
1 × 6 = 6	2 × 6 = 12	3 × 6 = 18	4 × 6 = 24
1 × 7 = 7	2 × 7 = 14	3 × 7 = 21	4 × 7 = 28
1 × 8 = 8	2 × 8 = 16	3 × 8 = 24	4 × 8 = 32
1 × 9 = 9	2 × 9 = 18	3 × 9 = 27	4 × 9 = 36
1 × 10 = 10	2 × 10 = 20	3 × 10 = 30	4 × 10 = 40
1 × 11 = 11	2 × 11 = 22	3 × 11 = 33	4 × 11 = 44
1 × 12 = 12	2 × 12 = 24	3 × 12 = 36	4 × 12 = 48
5 × 1 = 5	6 × 1 = 6	7 × 1 = 7	8 × 1 = 8
5 × 2 = 10	6 × 2 = 12	7 × 2 = 14	8 × 2 = 16
5 × 3 = 15	6 × 3 = 18	7 × 3 = 21	8 × 3 = 24
5 × 4 = 20	6 × 4 = 24	7 × 4 = 28	8 × 4 = 32
5 × 5 = 25	6 × 5 = 30	7 × 5 = 35	8 × 5 = 40
5 × 6 = 30	6 × 6 = 36	7 × 6 = 42	8 × 6 = 48
5 × 7 = 35	6 × 7 = 42	7 × 7 = 49	8 × 7 = 56
5 × 8 = 40	6 × 8 = 48	7 × 8 = 56	8 × 8 = 64
5 × 9 = 45	6 × 9 = 54	7 × 9 = 63	8 × 9 = 72
5 × 10 = 50	6 × 10 = 60	7 × 10 = 70	8 × 10 = 80
5 × 11 = 55	6 × 11 = 66	7 × 11 = 77	8 × 11 = 88
5 × 12 = 60	6 × 12 = 72	7 × 12 = 84	8 × 12 = 96
9 × 1 = 9	10 × 1 = 10	11 × 1 = 11	12 × 1 = 12
9 × 2 = 18	10 × 2 = 20	11 × 2 = 22	12 × 2 = 24
9 × 3 = 27	10 × 3 = 30	11 × 3 = 33	12 × 3 = 36
9 × 4 = 36	10 × 4 = 40	11 × 4 = 44	12 × 4 = 48
9 × 5 = 45	10 × 5 = 50	11 × 5 = 55	12 × 5 = 60
9 × 6 = 54	10 × 6 = 60	11 × 6 = 66	12 × 6 = 72
9 × 7 = 63	10 × 7 = 70	11 × 7 = 77	12 × 7 = 84
9 × 8 = 72	10 × 8 = 80	11 × 8 = 88	12 × 8 = 96
9 × 9 = 81	10 × 9 = 90	11 × 9 = 99	12 × 9 = 108
9 × 10 = 90	10 × 10 = 100	11 × 10 = 110	12 × 10 = 120
9 × 11 = 99	10 × 11 = 110	11 × 11 = 121	12 × 11 = 132
9 × 12 = 108	10 × 12 = 120	11 × 12 = 132	12 × 12 = 144

NOTE.—To repeat the Table by using the second columns as multipliers. Thus, 1 time 2 is 2, 2 times 2 are 4, 3 times 2 are 6, 4 times 2 are 8, etc.

CASE I.—*To effect a multiplication when the multiplier does not exceed* 12.

Ex. Multiply 542 by 7.

OPERATION.

Multiplicand	542
Multiplier	7
Product	3794

ANALYSIS.—In this example, it is required to take 542 seven times. If we take the units of each order 7 times, we shall take the entire number 7 times. Therefore, writing the multiplier under the unit figure of the multiplicand, we proceed thus: 7 times 2 units are 14 units = 1 ten and 4 units; we write the 4 units in the units' place, and reserve the 1 ten to add to the next product. Seven times 4 tens are 28 tens, and the 1 ten in reserve, added, are 29 tens = 2 hundreds and 9 tens; we write the 9 tens in the tens' place, and reserve the 2 hundreds to add to the product of hundreds. Seven times 5 hundreds are 35 hundreds, and the 2 hundreds reserved in the last product added, are 37 hundreds, which we write down in full; and the product is 3794.

EXAMPLES FOR PRACTICE.

	(1.)	(2.)	(3.)	(4.)
Multiplicand	3462	4276	5793	8634
Multiplier	4	5	3	6
Product	13848	21380	17379	51804

(5.)	(6.)	(7.)	(8.)	(9.)
2893	16812	48739	58607	76598
3	5	7	8	9

10.	873 × 3 = *Ans.*	2619	21.	76394 × 4 = *Ans.*...
11.	946 × 4 = *Ans.*	3784	22.	97631 × 5 = *Ans.*...
12.	4731 × 4 = *Ans.*	16924	23.	266532 × 7 = *Ans.*...
13.	5607 × 5 = *Ans.*	28035	24.	835456 × 6 = *Ans.*...
14.	6924 × 6 = *Ans.*	41544	25.	541378 × 8 = *Ans.*...
15.	8657 × 8 = *Ans.*	69256	26.	367542 × 9 = *Ans.*...
16.	27693 × 7 = *Ans.*	193851	27.	426985 × 8 = *Ans.*...
17.	51786 × 9 = *Ans.*	466074	28.	576483 × 11 = *Ans.*...
18.	45678 × 11 = *Ans.*	502458	29.	6932574 × 9 = *Ans.*...
19.	36397 × 9 = *Ans.*	327573	30.	397465 × 12 = *Ans.*...
20.	634576 × 12 = *Ans.*	7614912	31.	3745178 × 11 = *Ans.*...

CASE II.—*To effect a multiplication when the multiplier exceeds* 12.

Ex. Multiply 478 by 64.

OPERATION.

Multiplicand	478
Multiplier	64
Partial products.	1912
	2868
Entire product	30592

ANALYSIS.—We write the multiplicand and multiplier as in Case 1, and proceed thus. Four times 8 units are 32 units = 3 tens and 2 units; we write the 2 units in the place of units, and add the 3 tens to the product of tens. Four times 7 tens are 28 tens, + 3 tens are 31 tens = 3 hundreds and 1 ten; we write the 1 ten in the place of tens, and add the 3 hundreds to the product of hundreds. Four times 4 hundreds are 16 hundreds, + 3 hundreds are

19 hundreds, which we write in its proper place. We then, in like manner, multiply by the 6 tens in the multiplier, taking care to write the first figure obtained by this multiplication, in tens' place directly under the 6 of the multiplier; and, adding the *partial* products obtained by the two multiplications, we find the whole product of 478 by 64 to be 30592.

NOTE.—When there are ciphers between the significant figures of the multiplier, pass over them in the operation, and multiply by the significant figures only, remembering to set the first figure of the product under the figure of the multiplier that produces it.

52. From the foregoing illustrations we deduce the following

RULE.—I. *Write the multiplier under the multiplicand, so that units of the same order shall stand under one another, and draw a line underneath.*

II. *Multiply each figure of the multiplicand by each figure of the multiplier successively, beginning with the unit figure, and write the first figure of each partial product under the figure of the multiplier used, writing down and carrying as in addition.*

III. *If there are partial products, add them, and their sum will be the product required.*

PROOF OF MULTIPLICATION.

53. The **Proof** of multiplication is generally made by another multiplication (1) in which one of the factors equals the half, the third, or the fourth, etc., of one of the factors of the operation, and the other equals twice, three times, four times, etc., the other factor of the operation. Or,

In multiplying the multiplicand by the multiplier *diminished* by 1, and to the product adding the multiplicand; if the *sum* be the same as the product by the whole of the multiplier, the work is correct.

USE OF MULTIPLICATION.—*Multiplication serves to render any number so many times greater; to take several parts of a number; to find the value of several units or parts of units, when one of them is known; to bring a number expressing units of a certain nature to another number expressing units which are subdivisions of the first, &c.*

Generally we know that the solution *of a problem requires a multiplication, when the value of the unity is mentioned and that the value of several is required, or that of some parts of the unity.*

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52. *What is the general* rule *for multiplication?*— 53. *How is the proof of multiplication made?*

(1). In multiplying the multiplier by the multiplicand, the same product must be found.

EXAMPLES FOR PRACTICE.

	(1.)		(2.)		(3.)
	Multiply 8621		37215		167034
	By 47		65		304
	60347		186075		668136
	34484		223290		5011020
	Ans. 405187		*Ans.* 2418975		*Ans.* 50778336
4.	976	×	27	*Ans.*	26352
5.	697	×	34	"	23698
6.	749	×	46	"	34454
7.	8386	×	57	"	478002
8.	753537	×	68	"	51240516
9.	134679	×	79	"	10639641
10.	821956	×	387	"	319257972
11.	984765	×	756	"	744482340
12.	6654	×	789	"	5250006
13.	97248	×	865	"	84119520
14.	689834	×	943	"	650513462
15.	867894	×	996	"	864422424
16.	807497875	×	965	"	779235449372
17.	84966	×	7649	"	649904934
18.	543956	×	9475	"	5153983100
19.	96824	×	4696	"	454685504
20.	43208	×	4962	"	214398096
21.	90480	×	9007	"	814953360
22.	43	×	89006	"	3827258
23.	76496	×	87969	"	6729276624
24.	7674	×	12478	"	95756172
25.	3696	×	819162	"	3027622752
26.	69421	×	21754	"	1510184434
27.	4321	×	987654	"	4267652934
28.	756849	×	74323	"	56251288227
29.	968708	×	70469	"	69109512052
30.	4916	×	69678	"	342537048
31.	7654208	×	20963	"	160455162304
32.	80097	×	74269	"	5948724093
33.	900007	×	700608	"	630552104256
34.	4300407	×	700608	"	3012899547456
35.	460004	×	99804	"	45910239216
36.	960076	×	90708	"	87086573808
37.	690800	×	456007	"	315009635600
38.	7006924	×	540086	"	3784341555464
39.	786530746	×	357894	"	281494634808924
40.	416342505	×	987405	"	411098671149525
41.	896302456	×	943765	"	845888887386840
42.	495307429	×	936704	"	463956449974016
43.	7 7489007	×	900076	"	681797675464532
44.	879407854	×	698765	"	614499429100310

45.	954907089	×	600789	*Ans.*	573697675093221
46.	457907842	×	796807	"	364864173860494
47.	856407809	×	305407	"	261552939723263
48.	674396856	×	285679	"	192661019425224
49.	1864321	×	609649	"	1136581433329
50.	2465783	×	3686407	"	9089879711681
51.	7240036	×	4029008	"	29170162964288
52.	908007004	×	500123	"	454115186861492

MULTIPLICATION OF DECIMALS.

Ex. 1. Find the product of 4.35 by 8.26.

OPERATION.

4.35
8.26
2610
870
3480
35.9310 *Ans.*

ANALYSIS.—We multiply as in whole numbers, and point off on the right-hand of the product as many figures for decimals as there are decimal places in the multiplicand and multiplier.

The reason for pointing off the decimals in the product is, that in multiplying 4.35 by 8.26, or by 826 hundredths, which is the same thing, we take 826 times the hundredth part of 4.35 but we obtain the hundredth part in removing the point two figures towards the left (No. 37, 2nd.) which will give 0.0435; there remains then but to repeat 826 times this hundredth part to obtain the product required. As the number repeated consists of ten-thousandths, the product will be composed of decimals of the same nature; to separate the units it is then necessary to take its ten-thousandth part, that is, cut off 4 figures by the insertion of a point at the right side (No. 37). The same reasoning is applicable when there are three, four, &c. decimals in the multiplier.

If the factors are decimals only, we multiply as usual and cut off as many decimals in the product as there are in both factors; but if the product does not contain a sufficient number of figures, we fill up the vacant places by ciphers, placing one also for the units.

Ex. 2. Multiply 0.054 by 0.056.

OPERATION.

54
56
324
270
0.003024

ANALYSIS.—Multiplying 54 by 56, we obtain 3024; but as there are 6 decimals in the two factors, we place two ciphers at the left side of the product and having put the decimal point, we place another cipher for the units, and thus we find the number 0.003024, which is read 3 thousandths 24 millionths.

54. Hence the following

RULE.—I. *Multiply as in whole numbers, and point off as many figures for decimals, in the product, as there are decimals in the multiplicand and multiplier.*

II. *If there are not as many figures in the product as there are decimal places in the multiplicand and multiplier, supply the deficiency by prefixing ciphers.*

NOTE.—To multiply decimals by 10, 100, 1000, etc., (No. 36).

PROOF.—The proof is the same as in multiplication of whole numbers.

EXAMPLES FOR PRACTICE.

3.	15.27	×	9.	*Ans.*	137.43
4.	6.35	×	98.	"	622.3
5.	7.41	×	675.	"	5001.75
6.	197.19	×	56.	"	11042.64
7.	97.85	×	975.	"	95403.75
8.	69.78	×	596.	"	41588.88
9.	947.	×	4.65	"	4403.55
10.	869.	×	6.96	"	6048.24
11.	345.	×	3.95	"	1362.75
12.	57.	×	9.475	"	540.075
13.	786.	×	7.789	"	6122.154
14.	374.	×	2.967	"	1109.658
15.	9.47	×	6.694	"	63.39218
16.	39.47	×	28.9005	"	1140.702735
17.	676.49	×	60.705	"	41066.32545
18.	401.04	×	13001.4	"	521913.456
19.	9617.09	×	4281.45	"	41175089.9805
20.	6789.06	×	13808.927	"	93749640.72768
21.	3807.45	×	5321.807	"	20262510.2547
22.	489.04	×	37.00845	"	18098.612388

PRACTICAL PROBLEMS IN MULTIPLICATION.

1. If a workman earn $15 per week: how much will he earn in 9 weeks?

ANALYSIS.—In one week he earns $15; in 9 weeks he will earn nine times more, because he works nine times longer; therefore in multiplying by 9 we obtain the sum required $= 15 \times 9 = 135$. *Ans.* In 9 weeks he earns $135.

2. How much will 125 yards of cloth cost at $3.25 a yard?

ANALYSIS.—If one yard cost $3.25, 125 yards will cost 125 times more; in multiplying $3.25 by 125, the required sum $= 3.25 \times 125 =$ *Ans.* $406.25.

3. When a yard of cloth is worth $2.40, how much will 75 hundredths of a yard cost?

ANALYSIS.—The yard being worth $2.40, the 75 hundredths of a yard will be worth 75 times the hundredth part of $2.40; therefore, multiplying $2.40 by 0.75, we find the sum required $= 2.40 \times 0.75 = \$1.80$. *Ans.* $1.80.

4. What will 1635 barrels of sugar cost, at $25 a piece? *A.* $40875.

5. What will 785 kegs of tobacco cost, at $36 a keg? *A.* $28260.

6. What will 5679 bushels of wheat cost, at 85 cents a bushel?

7. How many pounds of flour are there in 387 barrels, there being 198 pounds in each barrel? *Ans.* 76626.

8. How many letters are there in a volume of 719 pages, each page containing 1539 letters? *Ans.* 1106541 letters.

9. A house has 295 windows and each window contains 24 panes of glass, how many panes in the whole edifice? *Ans.* 7080 panes.

10. Required how many trees in a nursery composed of 95 rows, if each row contains 178 trees? *Ans.* 16910 trees.

11. The circumference of the earth is divided into 360 degrees and each degree into 69.5 English miles; required how many miles around the earth? *Ans.* 25020.

12. Required how many hours in a year of 365 days? *Ans.* 8760.

13. How many days in 1000 years? *Ans.* 365000.

14. A man deposits $15 every week in a Savings Bank; how much does he deposit in one year or 52 weeks? *Ans.* $780.

15. A ream of paper contains 20 quires; how many quires are there in 572 reams? *Ans.* 11440.

16. If a cask of wine contains 213 quarts; required how many quarts in 136 casks? *Ans.* 28968 quarts.

17. How many eggs are there in 37 dozen? *Ans.* 444.

18. How many days has a person aged 84 years lived, reckoning 365 days to the year? *Ans.* 30660 days.

19. How many pens are there in 200 boxes each containing a gross or 144 pens? *Ans.* 28800 pens.

20. How many days elapsed from the birth of J. C. till the 31st. Dec. 1869 inclusively? (Not counting leap years.) *Ans.* 682185.

21. Europe produces yearly 3466 pounds of gold; what is the value in dollars knowing that a pound of this precious metal is estimated at $1718.50? *Ans.* $5956321.

22. A library is composed of 75 shelves and each shelf contains 86 volumes; how many pages are there in all the volumes supposing each volume to contain on an average 420 pages? *Ans.* 2709000.

23. A speculator has purchased 268 horses and 274 times as many sheep: how many sheep has he purchased? *Ans.* 73432.

24. There are 12 bags of wheat on a truck, each bag containing 3 bushels; how many pounds are there in the whole load, if the bushel weighs 50 pounds? *Ans.* 1800 pounds.

25. A workman earns $8 a week: how much will he earn in 7 years? *Ans.* $2912.

26. How much will 240 pieces of cloth, each containing 44 yds. cost, at $5.40 per yard? *Ans.* $57024.

27. How many pair of shoes can be made in 265 days, in a factory in which 86 pair can be made in 1 day?

28. If, at one load, a span of horses can draw 2997 pounds; how many pounds can they draw in 327 loads?

29. A field of 7 acres of land yields 45 bushels oats per acre; what is the value of the crops of the 7 acres at $0.40 a bush.? *Ans.* $126.

30. Supposing a sheep gives 6 pounds of wool a year; how many pounds will 28 sheep give in 3 years and what sum would it bring at 24 cents per pound? *Ans.* $120.96.

31. What is the value of the crop of a field containing 4 acres, if an acre yields 62 bush. oats worth 45 cents per bus.? *Ans.* 11160 cts.

32. A laborer thrashes 45 sheaves of wheat per day, giving 15 pecks; how many sheaves could 14 laborers thrash in 9 days, and what would be the quantity of grain obtained?

Ans. 5670 sheaves and 1890 pecks grain.

CONTRACTIONS IN MULTIPLICATION,

OR MULTIPLICATION BY FACTORS.

55. In many instances, by the exercise of judgment, as it will be seen, the operation may be very much *abridged*.

56 Any number that may be produced by multiplying together two or more numbers, is called a **Composite Number.** Thus, 6, 15, 18, are composite numbers; for $6 = 3 \times 2$; $15 = 5 \times 3$; $18 = 3 \times 3 \times 2$.

57. The **Factors** of a number are the several numbers which, multiplied together, produce the given number. Thus, the factors of 24 are 12 and 2 ($12 \times 2 = 24$); or, 4 and 6 ($4 \times 6 = 24$); or 2 and 3 and 4 ($2 \times 3 \times 4 = 24$).

NOTE.—The *factors* must not be confounded with the *parts* of a number. Thus, the *factors* of which 10 is composed, are 5 and 2, ($5 \times 2 = 10$); while the *parts* of which 10 is composed are 6 and 4, ($6 + 4 = 10$). The *factors* are *multiplied*, while the *parts* are *added*, to produce the number.

CASE I.—*To effect multiplication when the multiplier is a composite number.*

Ex. 1. What will 45 acres of land cost, at $367 an acre?

OPERATION.

367
5
1835
9
$16515 *Ans.*

ANALYSIS.—The factors of 45 are 5 and 9. Now, if we multiply the cost of 1 acre by 5, we obtain the cost of 5 acres; and, by multiplying the cost of 5 acres by the factor 9, we evidently obtain the cost of 9 times 5 acres, or 45 acres, the number bought. Hence the following

58. RULE.—I. *Separate the multiplier into two or more factors.*

II. *Multiply the multiplicand by one of these factors, and that product by another; and so on, till all the factors have been used. The last product will be the one required.*

NOTE.—The product of any number of factors is the same in whatever order they are multiplied. Thus, $4 \times 5 = 20$; and $5 \times 4 = 20$.

EXAMPLES FOR PRACTICE.

2. Multiply 2745 by $28 = 4 \times 7$. *Ans.* 76860.
3. Multiply 65742 by $35 = 5 \times 7$.
4. Multiply 78036 by $72 = 3 \times 3 \times 8$. *Ans.* 5618592.
5. Multiply 36783 by 81. *Ans.* 2979423.
6. What will 56 horses cost at $178 each? *Ans.* $9968.
7. What will 435 bushels of potatoes cost, at 32 cents a bushel?
8. What will 64 yards of merino cost, at 75 cents a yard?
9. In 1 mile there are 63360 inches; how many inches, 1st. in 45 miles?— 2nd. in 54 miles? *Ans.* 1st. 2851200;

56. *What is a* composite number?— 57. *What are the* factors *of any number?*

10. There are 8766 hours in one year; how many hours, 1st. in 84 years?— 2nd. in 125 years? *Ans.* 1st. 736344;

11. A town consumes 12432 pounds of bread in one day; how many pounds will the same consume, 1st. in 72 days?— 2nd. in 96? *Ans.* 1st. 897804;

12. An acre of land costs $475: what will cost, 1st. 15 acres?— 2nd. 70 acres?— 3rd. 144 acres? *Ans.* 1st. 7125;

CASE II.—*To effect multiplication when the multiplier is* 10, 100, 1000, *etc.* (No. 36, 1st.).

59. RULE.—*Annex to the multiplicand as many ciphers as there are in the multiplier.*

EXAMPLES FOR PRACTICE (p. 19).

CASE III.—*To effect multiplication when there are ciphers at the right-hand of one or both of the factors.*

Ex. 1. Multiply 1400 by 80.

OPERATION.

1400
80
112000

ANALYSIS.—We resolve the multiplicand into the factors 14 and 100, and the multiplier into the factors 8 and 10. Now, it is evident, (No. 5), that, if these several factors be multiplied together, they will produce the same product as the given numbers, 1400 and 80. Thus, $14 \times 8 = 112$, and $112 \times 100 = 11200$, and $11200 \times 10 = 112000$, the same result as in the operation.

60. From the preceding illustration we derive the following

RULE.—*Write the significant figures of the multiplier under those of the multiplicand, and multiply them together. To their product, annex as many ciphers as there are on the right of both multiplicand and multiplier.*

EXAMPLES FOR PRACTICE.

	(2.)	(3.)
Multiply	3764580	1306950000
By	270000	600800
	2635206	1045560
	752916	784170
Ans.	1016436600000	785215560000000

4. Multiply 610430 by 700500. *Ans.* 427606215000.
5. Multiply 3070607 by 7007000. *Ans.* 21515743249000.
6. Multiply 2020370 by 40302000. *Ans.* 814249517400000.
7. Multiply twenty-eight millions and four thousand, by three hundred and five thousand. *Ans.* 8541220000000.
8. Multiply seventy millions seven thousand and six hundred, by eight millions seven hundred and sixty. *Ans.* 560114005776000.

60. *What is the* rule *to multiply when there are ciphers at the right-hand of the multiplier or multiplicand, or both?*

9. Multiply forty-nine millions and forty-nine, by four hundred and ninety thousand. *Ans.* 24010024010000.

10. Multiply one billion and twenty thousand, by one thousand and one hundred. *Ans.* 1100022000000.

11. Multiply ten billions ninety-six thousand and eight hundred, by thirty thousand and seven hundred. *Ans.* 309971760000.

12. Multiply thirty millions ninety-thousand and eight hundred, by six hundred thousand and eighty. *Ans.* 18056887264000.

CASE IV.—*To effect multiplication when a part of the multiplier is a factor of another part.*

Ex. 1. Multiply 7439 by 328.

OPERATION.

7439
328
59512 = Prod. by 8 units.
238048 = Prod. by 32 tens.
2439992 = Prod. by 328.

ANALYSIS.—We consider the multiplier as separated into two parts, 32 *tens* and 8 *units*, or $320 + 8$; of which the smaller part is evidently a factor of the larger, since the 32 tens, or 320, is equal to 4 *tens* $\times$ 8. We next multiply by the 8 *units*, obtaining the product for that part of the multiplier. Now, as this product is the same as that by the factor 8 of the other part of the multiplier, we multiply it by 4 *tens*, obtaining the product of the multiplicand by 8 $\times$ 4 *tens*, or 32 *tens*. These products of the parts, added together, give the true product by 328; and,

61. From this illustration we derive the following

RULE.—*Multiply first by the smaller part of the multiplier; and then that partial product by a factor, or factors, of a larger part; and so on with all the parts. The sum of the several partial products will be the product required.*

EXAMPLES FOR PRACTICE.

2. Multiply 6526 by 568. *Ans.* 3706768.
3. Multiply 3785 by 721. *Ans.* 2728985.
4. Multiply 85065 by 2432.
5. Multiply 236428 by 54918. *Ans.* 12984152904.
6. Multiply 397821 by 25125.
7. Multiply 1146084 by 24816. *Ans.* 28441220544.
8. Multiply 5723605 by 4249784.

CASE V.—*To effect the multiplication of decimals when the multiplier is* 10, 100, 1000, *etc.* (No. 36, 2nd.)

62. RULE.—*Remove the decimal point as many places to the right as there are ciphers in the multiplier, annexing ciphers if required.*

---

61. *What is the* rule *for multiplying when a part of the multiplier is a factor of another part?* — 62. *What is the* rule *for effecting the multiplication of decimals when the multiplier is* 10, 100, 1000, *etc.?*

EXAMPLES FOR PRACTICE (p. 20 and 21).

CASE VI.—*To effect the multiplication of decimals when it is not necessary that all the decimal places of the product should be retained.*

Ex. 1. Multiply 6.5628 by 5.786, retaining only three decimal places in the product.

OPERATION.

6.5628
687.5

32814 = 6.562 × 5
4594 = 6.56 × .7 + 2
525 = 6.5 × .08 + 5
39 = 6. × .006 + 3

37.972 *Product.*

ANALYSIS.—We reverse the order of the figures of the multiplier and write them under the multiplicand; and, since thousandths is the lowest decimal figure to be retained in the product, we place the units' figure of the multiplier under the thousandths' figure of the multiplicand. Then, the unit of the product of any figure of the multiplicand by the figure of the multiplier that falls under it will be thousandths. When there are figures in the multiplicand on the right of that immediately above the figure of the multiplier, their product by the latter figure being expressed in units of lower orders than thousandths, may be neglected, except for the purpose of finding what must be carried to the thousandths' figure from their product.

63. From this illustration we deduce the following

RULE.—I. *Write the multiplier, with the order of its figures reversed, and with the units' place under that figure of the multiplicand which is the lowest decimal to be retained in the product.*

II. *Find the product of each figure of the multiplier by the figures above and to the left of it in the multiplicand increasing each partial product by as many units as would have been carried from the rejected part of the multiplicand, and one more when the highest figure in the rejected part of any product is* 5, *or greater than* 5; *and write these partial products with the lowest figure of each in the same column.*

III. *Add the partial products, and from the right-hand of the result point off the required number of decimal figures.*

NOTE.—1. Should the number of decimal places in the multiplicand be less than the number required in the product, supply the deficiency by annexing ciphers.

2. To obtain the number to be carried to each contracted partial product, it is generally necessary to multiply (mentally) only one figure at the right of the figure above the multiplying figure; but when the figures are large, the multiplication should commence at least *two* places to the right.

3. When the number of units in the highest order of the rejected part of the product is between 5 and 15, we carry 1; if between 15 and 25, we carry 2; if between 25 and 35, we carry 3; and so on.

63. *What is the* rule *for effecting the multiplication of decimals, when it is not necessary that all the decimal places of the product should be retained?*

EXAMPLES FOR PRACTICE.

2. Multiply 472.35 by 64.3645, and 3.657389 by 0.0536423, retaining, in the first, 2 decimal places, and, in the second, 5 decimal places.

OPERATION.	OPERATION.
472.350	3.657389
5463.46	3246350.0
2834100	182869
188940	10972
14170	2194
2834	146
189	7
23	1
3040.256	1.96189

3. Multiply 751.2037 by 38.7136, retaining 3 decimal places in the product. *Ans.* 29081.801.

4. Multiply 36.275 by 4.3678, retaining 1 decimal place in the product.

5. Multiply 843.7527 by 8634.175, retaining only the whole numbers in the product. *Ans.* 7285109.

6. Multiply 4256.785 by 0.00564, retaining only 3 decimal places in the product.

7. Multiply 73.27593 by 0.075325, carrying out the product to the seventh decimal place. *Ans.* 5.5195095.

8. Multiply 1.7323152 by 3962.57302, retaining 8 decimal places in the product.

PRACTICAL PROBLEMS COMBINING ADDITION, SUBTRACTION, AND MULTIPLICATION.

1. The hide of an ox costs \$6.15; it requires \$2 worth of bark, 9 quarts of oil at \$0.18 a quart, and \$0.60 for labor to prepare it. Required the gain if it be sold afterwards for \$12.75?

ANALYSIS.—The whole cost of the hide = \$6.15 + \$2 + (\$0.18 × 9 = \$1.62) + \$0.60 = \$10.37; \$12.75 − \$10.37 = *Ans.* \$2.38 gain.

2. A muslin manufacturer sold in one year, 540 pieces of it, viz.: 170 pieces to Montreal merchants; 85, to Quebec merchants; 130, to Toronto merchants; and the remainder to Ottawa merchants: what is that remainder? *Ans.* 155 pieces.

3. A man bought 25 barrels of flour at \$5.50 a barrel, and 40 barrels of apples at \$3 a barrel; what was the cost of all? *Ans.* \$257.50.

4. I paid for building my house \$1889, for my farm 3 times as much less \$892, and for my furniture \$140 more than I paid for building my house; how much did I pay for all, and for each?
Ans. \$4775; \$2029; and \$8693.

5. A young man receives \$1000 salary, and pays \$180 for board,

$215 for clothing, $120 for books, and $165 for other expenses: how much can he save in 4 years? *Ans.* $1280.

6. A merchant sold 75 yards of cloth at $2.47 per yard: he received in payment 132.25 yards of linen at $0.92, and a bill of $63.58; how much will the merchant receive? *Ans.* Nothing.

7. A book-seller made an invoice of books as follows: 125 volumes at $1.20; 248 at $0.90; 136 at $0.67; and 275 at $0.50; what is the amount of his invoice? *Ans.* $601.82.

8. Leo has $127; Peter, 3 times as much minus $205: and John has as much as Leo and Peter together: how much have Peter and John respectively, and how much have they all?

Ans. Peter, $176; John, $303; and all, $600.

9. A merchant bought 15 pieces of blue cloth, each containing 37 yards, and 12 pieces of black cloth, each containing 34 yards; how many yards of cloth did he buy of the two kinds altogether?

10. If a cow cost $28, a horse 6 times as much, and a farm 9 times as much as the cow and horse together, minus $112; how much more will the farm cost than 5 horses and 12 cows, at the same rate?

11. A wholesale grocer bought 95 barrels of salmon at $10.50 a barrel; he sold 84 barrels of them at $12 a barrel, and the remainder at $9 a barrel: how much did he gain or lose? *Ans.* Gained $109.50.

12. If an acre of land produces yearly 362 pounds of flax and 11 bushels of seed; it is required to know how many pounds of flax and how many bush. of seed will 7 acres produce, and how much will the whole be worth, if the flax be sold at $0.18 a pound and the seed at $2.50 per bush.? *Ans.* 2534 pounds flax; 77 bush. seed; $648.62.

13. In a dairy, there are 27 milch cows which give each, on an average, 108 pounds of butter; what sum will the dairy-man make in selling his butter at $0.18 a pound? *Ans.* $524.88.

14. A farmer desires to manure a field of 12 acres of land with manure worth $4 the hundred weight, and pays $1.45 for cartage per hundred weight; how much will it cost him to manure his field, supposing he requires 2 hundred weight per acre? *Ans.* $130.80.

15. A cabinet maker earns daily $1.55; his wife, $120: and his three sons, $0.65 each; how much can he lay by every week, the daily expenses of the whole family being $2.68? *Ans.* $9.44.

16. A lends B $19560, B lets A have bank stock to the amount of $3892, a farm 4 times as much as the bank stock – $1998, and pays the remainder in cash; how much cash did B pay A? *Ans.* $2098.

17. A jeweller bought a certain quantity of ivory at the rate of of $0.78 per pound; had he bought 6 pounds more, the cost would have been increased one eighth; how much did he pay for his ivory?

18. The repairs and superintendence of a railroad tract cost yearly $993 per mile; the expenses for improvements come to $4342.60; besides the company pays $626.40 for administration purposes and other items; required the total yearly expenditure for a track 132 miles long? *Ans.* $136045.

19. A plumber furnishes three kinds of zinc pipes: the diameter of the first, is 2 inches at $0.32 per yard; the second, 5 inches at $0.64; and the third, 8 inches at $0.96 per yard. The first kind is

99 yards long which is 20 yards more than the second and 34 more than the third; what sum must be paid to the plumber for his pipes? *Ans.* $144.64.

20. A handkerchief manufacturer bought 78 packages of thread, of which 40 are warp, at $10.90 per package, and 38 weft, at $10.55. He pays $0.85 per dozen for weaving and $26.30 for selling expenses; what will be his gain, knowing that he has made 640 dozen of handkerchiefs, and sold them at the rate of $2.58 per dozen?

Ans. $244.

DIVISION.

64. Division is the process of finding how many times one number is contained in another; or the process of finding one of the factors, the product and the other factor being known. Thus,

To divide 12 by 3, is to seek a number, which, being multiplied by 3, gives 12 for product; or, to find by what number 3 must be multiplied, to obtain 12 in the product.

The product is called **Dividend,** the known factor, **Divisor,** and the factor sought, **Quotient.**

When the dividend does not contain the divisor an exact number of times, the part of the dividend left is called the **Remainder,** and must be less than the divisor.

CASE I.—*To divide when the divisor does not exceed* 12.

NOTE.—When the process of dividing is carried on in the mind, and the quotient only is set down, the operation is called *Short Division.*

Ex. 1. How many times is 7 contained in 994?

OPERATION.

Divisor 7) 994 Dividend.

142 Quotient.

ANALYSIS.—We write the divisor on the left of the dividend with a line between them and another line beneath the dividend; then, beginning at the left-hand, we say: 7 is contained in 9, 1 time, and 2 hundreds remaining; we write the 1 directly under the 7, its dividend, for the *hundreds'* figure of the quotient. To 9, the next figure of the dividend, which is tens, we unite the 2 hundreds remaining, which equal 20 tens, in which we find the divisor 7 to be contained 4 times, and 1 ten remaining; we write the 4 for the *tens'* figure in the quotient, and the 1 ten remaining, equals 10 units, which, united to 4, the last figure of the dividend, make 14 units; in 14 units, 7 is contained 2 times; writing the 2 for the *units'* figure of the quotient, we have 142 for the entire quotient.

65. RULE.—I. *Write the divisor at the left-hand of the dividend, with a line between them, and draw a horizontal line beneath the dividend.*

64. *What is* division?— *What is the* dividend?— *The* divisor?— *The* quotient?— *The* remainder?— 65. *What is the* rule *for* short division?

II. *Beginning at the left, divide each term of the dividend by the divisor, and write each quotient figure under its dividend.*

III. *If there be a remainder after dividing any figure, consider it as prefixed to the next term of the dividend, and divide as before.*

IV. *Should any partial dividend be less than the divisor, write a cipher in the quotient, and prefix the number to the figure of the next lower order in the dividend, and divide as before.*

V. *If there be a remainder after dividing the last figure, place it after the quotient, and write the divisor under it.*

NOTE.—Besides the usual sign ÷, division is also indicated by writing the dividend *above*, and the divisor *below* a short horizontal line; thus $\frac{8}{4} = 2$.

PROOF.—Multiply the divisor and quotient together, and to the product add the remainder, if any; if the result obtained be equal to the dividend, the work is correct.

NOTE.—This method of proof follows from division being the reverse of multiplication. (64).

EXAMPLES FOR PRACTICE.

2. Divide 8154 by 6.

OPERATION.

Divisor 6) 8154 Dividend.

1359 Quotient.

PROOF.

1359 Quotient.
6 Divisor.

8154 Dividend.

(3.)	(4.)	(5.)	(6.)
5) 714325	3) 893763	7) 949112	4) 562845
142865		$135587\frac{3}{7}$	

	Quotients.
7. Divide 6375 by 5.	1275.
8. Divide 5592 by 6.	932.
9. Divide 98776 by 8.	12347.
10. Divide 174321 by 9.	19369.
11. Divide 1643784 by 12.	136982.
12. Divide 46215796 by 11.	4201436.
13. Divide 63412632 by 12.	

	Rem.
14. Divide 2271582 by 7.	5.
15. Divide 11357912 by 5.	2.
16. Divide 4056360 by 9.	6.
17. Divide 12980400 by 8.	
18. Divide 42084795 by 6.	3.
19. Divide 4507060 by 12.	4.
20. Divide 15023520 by 11.	6.

PRACTICAL PROBLEMS.

1. Nine yards of silk velvet cost $72; how much did it cost a yard?

ANALYSIS.—If the price of a yard were known, in multiplying it by 9, we would obtain $72; therefore, 72 is a product having for factors 9 and the price of a yard. Then, in dividing the product 72 by the factor 9, we obtain the price of a yard; $72 \div 9 =$ *Ans.* $8. *Or again*, as 9 yards cost $72, 1 yard will cost 9 times less, because there are 9 times less yards; then, in dividing 72 by 9, we obtain the price of a yard.

2. If 5 shillings make a dollar; how many dollars in 8890 shillings? *Ans.* 1778 dollars.

3. A gentleman divided $89622 equally among his 9 children; how much did each receive? *Ans.* $9958.

4. How many barrels of flour, at $8 a barrel, can be bought for $680? *Ans.* 85 barrels.

5. If 12 inches make one foot; how many feet in 7501464 inches?

6. Eleven horses were sold for $2531; what was the average sum received for each? *Ans.* $231.

7. A boy spent in one month 260 cents for oranges, giving 4 cents for each; how many oranges did he buy? *Ans.* 65.

8. A carpenter worked 11 months for $572; how much did he receive a month? *Ans.* $52.

9. If maple is worth $6 a cord; how many cords will be had for $1152? *Ans.* 192 cords.

10. A person wishes to distribute 168 apples equally among 4 boys and 3 girls; how many will each of them receive? *Ans.* 24.

CASE II.— *To divide when the divisor exceeds* 12.

NOTE.—When the whole process of division is written, the operation is termed *Long division.*

Ex. Divide 4738 by 34.

OPERATION.

Divisor.	Divd'd.	Quotient.
34)	4738	(139$\frac{12}{34}$.
	34	
2nd. partial dividend	133	
	102	
3rd. partial dividend	318	
	306	
	12 Remainder.	

ANALYSIS.—Taking 47 hundreds for the first partial dividend, we say: 35 is contained in 47, 1 time. The 1 we write in the quotient; $34 \times 1 = 34$, which we write under the 47; $47 - 34 = 13$, to which bringing down the next figure of the dividend, which is 3, we form 133; 34 in 133, 3 times. The 3 we write in the quotient; $34 \times 3 = 102$, which we write under the 133; $133 - 102 = 31$, to which bringing down the next figure of the dividend, we form 318; 34 in 318, 9 times. The 9 we write in the quotient; $34 \times 9 = 306$, which we write under the 318; $318 - 306 = 12$, a remainder, or a part of the dividend left undivided, which we write in the quotient with the divisor below it, thus completing the division.

66. RULE.—I. *Write the divisor and dividend as in short division, and draw a curved line at the right-hand of the dividend.*

66. *What is the* rule *to divide when the divisor exceeds* 12?

II. *Take for the first partial dividend, the least number of figures on the left that will contain the divisor, and place the quotient on the right.*

III. *Multiply the divisor by this quotient figure, place the product under the partial dividend, subtract, and to the remainder, annex the next term of the dividend, for the second partial dividend.*

annex the next term of the dividend, for the second partial dividend.

IV. *Divide as before, until all the figures of the dividend have been brought down and divided.*

V. *If any partial dividend will not contain the divisor, place a cipher in the quotient, and bring down the next figure of the dividend, and divide as before.*

VI. *If there be a remainder after dividing all the figures of the dividend, it must be written in the quotient, with the divisor underneath.*

NOTES.—1 If any remainder be *equal to*, or *greater* than the divisor, the corresponding figure in the quotient is too *small*.

2. If the product of the divisor by the quotient figure be *greater* than the partial dividend, the quotient figure is too *large*.

PROOF.—It is the same as in short division.

DIVISION ACCORDING TO THE FRENCH METHOD.

Ex. Divide 11812 by 72.

OPERATION.

Dividend 11812	(72	Divisor.
72	$164\frac{4}{72}$	Quotient.
461		
432		
292		
288		
4	Remainder.	

OBSERVATION.—We see by the example in the margin, that the divisor is placed on the right of the dividend, and the quotient below it. This mode gives the work a more compact and neat appearance, and possesses the advantage of having the figures of the quotient near the divisor, by which means, the practical difficulty of multiplying the divisor by a figure placed at a distance from it, is removed.

ABBREVIATION OF LONG DIVISION.

67. By the following method, we avoid writing the products in long division, as in the example of Case II, above.

Ex. 1. Divide 8764 by 365.

OPERATION.

365) 876.4	(24
146 4	
... 4	remainder.

ANALYSIS.—In this operation, we say : 3 is contained 2 times in 8; we write 2 at the quotient and multiply the divisor saying: 2 times 5 are 10, which subtracted from 16 (because we increase the 6 by 10), leaves 6 and carry one; 2 times 6 are 12 and 1 is 13, which subtracted from 17 leaves 4 and carry 1; again 2 times 3 are 6 and 1 is

7, which, subtracted from 8, leave 1; we bring down the 4 to form the second partial dividend. Then 3 in 14 is contained 4 times, which we write at the quotient, and multiply 365 by it; we subtract the product from the second partial dividend in a similar manner; there remains 4 which is to be added to the proof. Hence the following

RULE.—I. *Obtain the first figure of the quotient in the usual manner.*

II. *Multiply each figure of the divisor by this quotient figure, subtract from the first partial dividend, and write underneath the remainder.*

III. *At the right of the first remainder, annex the next figure of the dividend, for a new partial dividend, and proceed as with the former, till the work is finished.*

OBSERVATION 1.—When, after having employed all the figures of the dividend, there is still a remainder, we may reduce this remainder, firstly in tenths by adding a cipher at the right of it, and continue the division; but then, as we cannot have any more units, we place a point at the quotient. When we continue the division, the second remainder is reduced into hundredths by the addition of another cipher: but place no more points at the quotient, the units being indicated by the order they occupy. (Nos. 27 and 31.)

Ex. Divide 679 by 28.

OPERATION.

28) 679 (24.25
119
.70
140
..

ANALYSIS.—After the division, there remains 7; we reduce this remainder to tenths by writing a cipher at the right-hand of it, and we place a point at the quotient, and then proceed as before. But as there remains yet 14 tenths, we reduce this number to hundredths by the addition of another cipher. Multiplying and subtracting as before we find that nothing remains. Hence we conclude that 24.25 is the correct quotient of 679 by 28, as shown by the proof.

Had there been another remainder, we would have added one more cipher. Thus, we can carry the approximation to any order of decimal unity.

OBSERVATION 2.—When the dividend is smaller than the divisor, we first place a cipher and a point at the quotient to signify that there are no integers or whole numbers; then we reduce the dividend to tenths, hundredths, &c. (No. 36.), and proceed as before.

Ex. Given 6 to be divided by 25; what will be the operation?

OPERATION.

25) 6.0 (0.24
1 00
0

ANALYSIS.—Having disposed the terms, we say: 25 in 6 is not contained, we write a cipher and a point at the quotient. Then we reduce the 6 units in tenths by placing a cipher at the right-hand of it, and say: 25 in 60 is contained 2 and 10 tenths remain. We reduce them into hundredths by the addition of a cipher, and say: 25 in 100 is contained 4 times, and nothing remain; therefore, 0.24 hundredths is the quotient of 6 divided by 25 units.

USE OF DIVISION.—*Division serves to divide a number into equal parts; to render it a certain number of times smaller; to find how many times a number is contained into another; to find by what number must a given number be multiplied to produce another given number. Division serves also to find the value of the unity when a certain given number of units or parts of units are known, as for instance, the buying, the selling price of a yard, the gain or loss of a yard, &c. &c.; to find the number of units knowing their whole value and that of the unit, as for instance, the number of days that a laborer must work to earn a certain sum; and lastly, to find how many units there are in a number which expresses subdivisions of this unit, such as to find how many hours there are in any given number of minutes, &c.*

We know generally that the solution *of a problem requires a division when the value of several units, or parts of units, are given, to find only one;* Or, *when the value of the unit, and several units, or parts of units being given, we seek the number of units or parts of units.*

EXAMPLES FOR PRACTICE.

1. Find how many times is 72 contained in 23596.

OPERATION.

Divid'd.
Divisor. 72) 23596 (327 Quotient.
216
199
144
556
504
52 Remainder.

PROOF BY MULTIPLICATION.

327 Quotient.
72 Divisor.
654
2289
23544
52 Remainder.
23596 Dividend.

				Quotients.	Rem.
2.	27939	÷	16	1746	3
3.	38582	÷	18	2143	8
4.	405683	÷	20	20284	3
5.	743241	÷	25		16
6.	954992	÷	30	31833	2
7.	173469	÷	36		21
8.	497699	÷	40	12442	19
9.	218579	÷	42		11
10.	611286	÷	47	13006	4
11.	41126	÷	49		15
12.	4326 5	÷	50	8652	5
13.	845002	÷	53		23
14.	867532	÷	59	14703	55
15.	876701	÷	60		41

				Quotients.	Rem.
16.	4968	÷	64	77	40
17.	940025	÷	68		61
18.	445124	÷	70	6358	64
19.	4728	÷	75		3
20.	39006	÷	79	493	59
21.	1679407	÷	80		47
22.	4306404	÷	85	50663	49
23.	167008	÷	87		55
24.	7456029	÷	90	82844	69
25.	6717890	÷	98		88

To calculate with two decimals in the quotient.

				Quotients.	Rem.
26.	67980	÷	96	708.12	48
27.	432101	÷	69	6262.33	23
28.	470896	÷	72		16
29.	680094	÷	67	10150.65	45
30.	666648	÷	441	1511.67	153
31.	767642	÷	386		380
32.	124674	÷	126	989.47	78
33.	964321	÷	216	4464.44	196
34.	7246579	÷	612		328
35.	7890645	÷	367	21500.39	187
36.	9120128	÷	637		153
37.	687621	÷	4691	146.58	1422
38.	3466604	÷	1279		240
39.	4268901	÷	1467	2909.95	435
40.	2486930	÷	7614		4532
41.	4107129	÷	7614	539.41	6126
42.	81267904	÷	6174		3592
43.	69267421	÷	7186	9639.21	5794
44.	89064010	÷	7908		184
45.	694735210	÷	9087	76453.74	7462
46.	468904008	÷	7064		6768
47.	389006753	÷	8004	48601.54	2684
48.	86742807	÷	8906		5914
49.	707070709	÷	4260	165979.03	4120
50.	654380316	÷	49060		37440
51.	987654321	÷	49066	20129.09	39106
52.	8606000041	÷	60041		49042
53.	61247680241	÷	74085	826721.74	13310
54.	74238961401	÷	48647		11893
55.	9649646664	÷	42867	225106.64	32712
56.	8674289646	÷	74551		48424
57.	4247698734	÷	94672	44867.52	88056
58.	63.2460086	÷	59866		46810
59.	45680108007	÷	300452	152037.95	185360
60.	37894216118	÷	987684		88784

DIVISION OF DECIMALS.

Ex. 1. Divide 3.456 by 2.4.

OPERATION.

2.4) 3.456 (1.44 *Ans.*
2 4
1 05
96
96
96

ANALYSIS.—We divide as in whole numbers; and, since the divisor and quotient are the two factors, which, being multiplied together, produce the dividend, we point off two decimal figures in the quotient, to make the number in the two factors equal to the number in the product or dividend.

Ex. 2. Divide 0.525 by 7.5.

OPERATION.

7500) 525.00 (0.07.
525 00

ANALYSIS.—As the decimal places in the dividend exceed those in the divisor, we make them equal by annexing two ciphers to the divisor; and, having proceeded in the division as in Obs. 2, p. 54, we find the quotient to be 0 07, or 0 unit 7 hundredths.

67. From the preceding illustrations we deduce the following

RULE I.—*Divide as in whole numbers, and point off as many decimals in the quotient as the decimals in the dividend exceed those of the divisor; but, if there are not as many, supply the deficiency by prefixing ciphers.*

Or,

RULE II.—*If the dividend and divisor have not the same number of decimals, annex ciphers at the right-side of the term which has the least, so that it may have as many decimal figures as the other; then divide, without any regard to the point, as in whole numbers.*

NOTE 1.—To divide decimals by 10, 100, 1000, etc. (No. 37).

PROOF.—The proof is the same as in division of whole numbers.

EXAMPLES FOR PRACTICE.

				Quotients.	Rem.
3.	79.1	÷	2.5	31.64	
4.	67.8632	÷	16.4	4.174	96
5.	2.3421	÷	42.2	0.055	211
6.	0.338	÷	0.15		8
7.	14.	÷	0.7852	17.	6516
8.	0.21318	÷	8.34		468
9.	10.85	÷	0.0775	140.	
10.	0.1728	÷	0.012		

67. *What is the* rule *for the division of* decimals?

To calculate with five decimals in the quotient.

				Quotients.	Rem.
11.	16.6	÷	10.2	1.62745	34
12.	40.72	÷	16.12		740
13.	46.634	÷	39.122	1.19201	18478
14.	79.683	÷	14.244		6084
15.	76.1234	÷	9.24	8.23846	296
16.	59.2687	÷	91.42		1998
17.	79.4	÷	9.04	8.78318	528
18.	70.8	÷	10.08		960
19.	29.40	÷	18.126	1.62197	17178
20.	16.74	÷	17.261		
21.	0.7	÷	3.7	0.18918	34
22.	0.2	÷	3.2		
23.	0.42	÷	3.07	0.13680	249
24.	0.009	÷	0.000014		4

PRACTICAL PROBLEMS.

1. If 45 yards of cloth cost $123.75; how much will 1 yard cost?

ANALYSIS.—If the price of a yard were known, in multiplying it by 45, we would obtain $123.75; therefore, $123.75 is a product having for factors 45 and the price of a yard. Dividing 123.75 by 45, we obtain the price of a yard = $123.75 ÷ 45 = *Ans.* $2.75.

2. A laborer earns $2.65 per day; in how many days will he earn $47.70? *Ans.* 18 days.

ANALYSIS.—As many times as $2.65, the price of a day's labor, are contained in $47.70, as many days will be required; therefore, in dividing 47.70 by 2.65, we obtain the number of days required = 47.70 ÷ 2.65 = *Ans.* 18 days.

3. The product of two numbers is 661045; one of the numbers is 85; what is the other? *Ans.* 7777.

4. What is the number that, being multiplied by 72 will give 70344?

5. One of two factors is 4.75 and their product 4222.18. Find the other factor? *Ans.* 888 88 hundredths.

6. I paid $806 for 196 reams of paper; how much is that per ream? *Ans.*

7. What number is that which is 25 times smaller than 3575?

8. At $0.30 per volume, how many volumes can be got for $69?

9. If a cord of maple wood cost $4.60; how many cords can be bought for $989? *Ans.* 215.

10. How many sheets of paper in a volume in-8° of 1280 pages? (The sheet in-8° contains 16 pages.) *Ans.* 80.

11. How many yards of carpet, at $4.60 per yard, can be bought for $676.20? *Ans.* 147 yards?

12. If 63 gallons make a hogshead; how many hogsheads will 2016 gallons make? *Ans.* 32.

13. In how many days could 35 men accomplish as much work, as one man in 805 days? *Ans.* 23.

14. During a cruise of 64 days, a ship sailed 11648 miles; how far did she sail each day? *Ans.* 182 miles.

15. Having multiplied 6.55 by a certain number, we obtained 57.3125; what is that number? *Ans.* 8.75.

16. A train on the Grand Trunk Railway runs 62 miles an hour; at the same rate, how long would it take to go round the world, the distance being about 25000 miles? *Ans.* 403 + hours.

17. The large wheels of a coach are 15 feet in circumference, the small ones 6 feet; how many turns will each make in a distance of 140182 feet? *Ans.* Large, 9345 + ^{7}; small, 23363 + ^{4}.

18. Find a number whose product by 0.005 would be 0.00025.

19. I bought a farm containing 175 acres for $4375; how many dollars did it cost per acre? *Ans.* $25.

20. A butcher gave $66 for sheep, at the rate of $3.30 each; how many sheep did he buy? *Ans.* 20 sheep.

21. How many pair of slippers must be made by a shoemaker, to earn $1.35 per day, if he be paid $0.15 for every pair he makes?

22. The annual receipts on a railroad 500 miles long amount to $3600000. Required the average daily receipts, and how much is received per mile annually? *Ans.* Average daily receipts 9863.01\frac{1}{365}$.
Do do per mile $7200.

23. The air contained in a puncheon weighs 9.75 drachms; the water it would contain would weigh 7507.5 drachms; how many times is the weight of the water greater than that of the air? *A.* 770 times.

24. A charcoal maker places 127 cords of wood in a kiln which cost him $580; he consumes 13 cords of fuel for the operation, and the value of the charcoal obtained is estimated at $231.14, at the rate of $0.28 per bushel. Required how many bushels of coal have been produced by a cord of wood? *Ans.* 6.5 bushels.

25. The population of the globe is about 1300860000 inhabitants; supposed that it is renewed every 33 years (in Canada, the mean length is about 37 years). It is required to know how many persons die yearly, daily, every hour and every minute? *Ans.* 39420060 persons yearly; 10800 daily; 4500 every hour; 75 every minute (or 5 every 4 seconds.)

CONTRACTIONS IN DIVISION,

OR DIVISION BY FACTORS.

CASE I.—*To divide when the divisor is a composite number.*

Ex. 1. Divide $1596 equally among 28 persons.

OPERATION.

4) 1596
7) 399
57 *Ans.*

ANALYSIS.—The factors of 28 are 4 and 7. We divide $1596 by 4, and the resulting quotient by 7, and obtain for the final result, 57, which must be the same as the quotient of 1596 divided by 4 times 7, or 28. We might have obtained the same result by dividing first by 7, and then by 4. Hence the following

GS. RULE.—*Divide the dividend by one of the factors, and*

the quotient thus obtained, by a second factor, and so on, till every factor of the divisor has been used. The last quotient will be the required quotient.

EXAMPLES FOR PRACTICE.

2. Divide 4536 by $14 = 2 \times 7$.	*Ans.* 324.
3. Divide 9774 by $18 = 3 \times 6$.	*Ans.* 543.
4. Divide 14560 by $35 = 5 \times 7$.	*Ans.* 416.
5. Divide 126375 by $75 = 3 \times 5 \times 5$.	*Ans.* 1685.
6. Divide 69384 by 42, using its factors.	*Ans.* 1652.
7. Divide 57456 by 72, using its factors.	*Ans.* 798.
8. Divide 246792 by 84, using its factors.	*Ans.* 2938.
9. Divide 2962875 by 125, using its factors.	*Ans.* 23703.

To find the true remainder when there are several in the operation.

Ex. 1. Divide 10183 by 105, using the factors 3, 5, and 7, and find the true remainder.

OPERATION.

3) 10183
5) 3394 ..... 1 rem.
7) 678 .....$4 \times 3 = 12$
96 ... $6 \times 5 \times 3 = 90$
103 true rem.

ANALYSIS.—Dividing 10183 by 3, we have a quotient of 3394, and a remainder of 1 undivided, which, being a part of the given dividend, must also be a part of the true remainder. The 3394 being a quotient arising from dividing by 3, its units are 3 times as great in value as the units of the given dividend, 10183. Dividing the 3394 by 5, we have a quotient of 678, and a remainder of 4. As this 4 is a part of the 3394, it must be multiplied by 3 to change it to the same kind of units as the 1. This makes a true remainder of 12 arising from dividing by 5. Dividing the 678 by 7, we have a quotient of 96 and a remainder of 6. This 6 is a part of the 678, the units of which are 5 times as great in value as those of the given dividend, 10183; therefore, to change this last remainder 6, to units of the same value as the dividend, we multiply it by 5 and 3, and obtain a true remainder of 90, arising from dividing by 7. Adding the three partial remainders, we obtain 103 for the true remainder. Hence, the

69. RULE.—I. *Multiply each partial remainder by all the divisors preceding the one that produces it.*

II. *Add the several products with the first remainder; the sum will be the true remainder.*

EXAMPLES FOR PRACTICE.

2. Divide 3026 by 15, using the factors 3 and 5, and find the true remainder. *Ans.* 11.

3. Divide 34709 by 42, using the factors 6 and 7, and find the true remainder. *Ans.* 17.

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69. *What is the* rule *for finding the* true remainder?

4. Divide 5858 by 84, using the factors 3, 4, and 7, and find the true remainder. *Ans.* 62.
5. Divide 9078147 by 90, using the factors 3, 5, and 6, and find the true remainder. *Ans.* 27.
6. Divide 7360481 by 96, using the factors 2, 6, and 8, and find the true remainder. *Ans.* 65.
7. Divide 10165 by 120, using the factors 2, 3, 4, and 5, and find the true remainder. *Ans.* 85.
8. Divide 63724 by 135, using the factors 3, 5, and 9, and find the true remainder. *Ans.* 4.

CASE II.—*To divide a whole number by* 10, 100, 1000, *etc.* (No. 37, 1st)

70. RULE.—*From the right-hand of the dividend, cut off as many figures as there are ciphers in the divisor. Under the figures so cut off, place the divisor, and the whole will form the quotient.*

EXAMPLES FOR PRACTICE.

1. Divide 87 by 10. *Ans.* $8\frac{7}{10}$.
2. Divide 5813 by 100.
3. Divide 7009 by 1000. *Ans.* $7\frac{9}{1000}$.
4. Divide 510040 by 10000.
5. Divide 200371 by 100. *Ans.* $2003\frac{71}{100}$.

CASE III.—*To divide when there are ciphers on the right-hand of the divisor.*

Ex. 1. Divide 85726 by 4500.

OPERATION.

45|00) 857|26 ($19\frac{226}{4500}$.
45
407
405
226 Remainder.

ANALYSIS.—The factors of 4500 are 100 and 45. First, dividing by 100, (70), we obtain for a quotient 857, and for a remainder 26. Dividing this quotient by the remaining factor, 45, we obtain for a quotient 19, and for a remainder 2, to which annex 26, the first remainder, and underneath write the divisor, and we have for the entire quotient $19\frac{226}{4500}$.

71. RULE.—I. *Cut off the ciphers from the divisor, and as many figures from the right of the dividend.*

II. *Divide the remaining figures of the dividend by the remaining figures of the divisor.*

III. *Place the entire divisor under the true remainder, and annex it to the integral part of the quotient, for the entire quotient.*

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70. *What is the* rule *to divide by* 10, 100, *etc.*?— 71. *What is the* rule *for dividing when there are ciphers on the right-hand of the divisor.*

EXAMPLES FOR PRACTICE.

2. Divide 33100 by 6000. *Ans.* $5\frac{3100}{6000}$.
3. Divide 1047628 by 2400. *Ans.* $436\frac{1228}{2400}$.
4. Divide 72002 by 1200. *Ans.* . . . $\frac{2}{1200}$.
5. Divide 96031425 by 92000.
6. Divide 1247701 by 47000. *Ans.* $26\frac{25701}{47000}$.
7. Divide 1703945642 by 4160000.
8. Divide 460352000 by 8100.

CASE IV.—*To divide a decimal by* 10, 100, 1000, *etc.* (No. 37).

72. RULE.—*Remove the decimal point as many places to the left as there are ciphers in the divisor, and if there be not figures enough in the number, prefix ciphers.*

EXAMPLES FOR PRACTICE (p. 20 and 21).

CASE V.—*To abridge the division of decimals, when the divisor contains a large number of them.*

Ex. 1. Divide 675.4563 by 23.54738, extending the quotient to three decimal places.

CONTRACTED OPERATION.		COMMON OPERATION.	
23.54738) 675.4563 (28.684		23.54738) 675.4563 (28.684	
470 95	= product by 2, + 1.	470 94	76
204 50		204 50	870
188 38	= product by 8, + 6.	188 37	904
16 12		16 12	9660
14 13	= product by 6, + 3.	14 12	8428
1 99		2 00	12320
1 88	= product by 8, + 4.	1 88	37904
11		11	744160
9	= product by 4, + 1.	9	418952
2		2	325208

ANALYSIS.—In the contracted method we first ascertain how many places of figures the above example will have in the quotient. Comparing the entire part of the divisor with the entire part of the dividend, it is evident that the first quotient figure will be of the order of tens, and therefore the quotient will contain *two* places of whole numbers; and as there are to be *three* places of decimals, it must contain *five* figures. Hence, we divide at first by five figures of the given divisor, counting them from the left towards the right, thus, using the 23.547, and rejecting the figures 38, on the right. In multiplying each contracted divisor by its quotient figures, we increase the product by having regard to rejected figures, as in contracted multiplication of decimals (Case VI, p. 57).

By comparing the contracted with the common method, we see the extent of the abbreviation, and the agreement of the corresponding intermediate results. Hence, he

72. *What is the* rule *to divide a decimal by* 10, 100, 1000, *etc.?*

73. RULE.—I. *Compare the significant figure on the left of the divisor with those on the left of the dividend, and determine how many figures will be required in the quotient.*

II. *For the first contracted divisor, take as many significant figures from the left of the given divisor as there are places required in the quotient; and, at each subsequent division reject one place from the right of the last preceding divisor.*

III. *In multiplying by the several quotient figures, carry from the rejected figures of the divisor as in contracted multiplication.*

NOTE.—Annex ciphers to either dividend or divisor, when necessary, before commencing the work.

EXAMPLES FOR PRACTICE.

2. Divide 487.24 by 1.003675, extending the quotient to 2 decimal places. *Ans.* 485.46.

3. Divide 2.3748 by 1.4736, extending the quotient to the third decimal place. *Ans.* 1.611.

4. Divide 3.2682 by 2.4736, and carry the quotient to four places of decimals. *Ans.* 1.3212.

5. Divide 0.079085 by 0.83497, and carry the quotient to the fifth decimal place. *Ans.* 0.09471.

6. Divide 8972.436 by 756.3452, extending the quotient to 4 decimal places. *Ans.* 11.8629.

7. Divide 0.4879357 by 0.002963, extending the quotient to the second decimal place. *Ans.* 164.69.

8. Divide 12193263.1112635269 by 1234.56789, extending the quotient to as many decimal places, plus one, as there will be whole numbers in it. *Ans.* 9876.54321.

DECIMAL CURRENCY.

74. Decimal Currency is the currency whose denominations increase and decrease in a tenfold ratio.

75. Currency is coin, bank bills, treasury notes, etc., in circulation as a medium of trade.

76. Coin is money stamped, and has a given value established by law.

NOTES.—1. The currency of the present Dominion of Canada is decimal currency; it had been adopted by each of the Provinces before their *Federation*.

2. Decimal currency is also the currency of the United States, and is sometimes called *Federal Money*; it was established by Congress in 1792.

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73. *What is the* rule *for abridging the division of decimals?*— 74. *What is* decimal currency?— 75. Currency?— 76. Coin?

77. The present **Coins** of the Dominion of Canada are of silver and copper.

The *silver* coins are the fifty-cent piece, the twenty-five-cent piece, the ten-cent piece, and the five-cent piece.

NOTE.—The shilling or twenty-cent piece, though still in circulation, is no longer to be coined.

The *copper* coins are the two-cent piece and the cent.

100 cents (*cts.*) make 1 dollar, marked $1.

78. The **Coins** of the United States are of gold, silver, and nickel.

The *gold* coins are the double-eagle, eagle, half-eagle, quarter-eagle, three-dollars, and dollar.

The *silver* coins are the dollar, half-dollar, quarter-dollar, dime, and half-dime.

The *nickel* coins are the 5-cent, 3-cent, 2-cent, and 1-cent pieces.

NOTES.—1. The mill is not coined; it is used only in computation.

2. To make the metal of coins more serviceable, *gold* coins contain 9 parts by weight of gold and 1 part of an alloy consisting of silver and copper. *Silver* coins contain 9 parts of silver and 1 part of copper.

TABLE OF THE UNITED STATES CURRENCY.

10 mills (*m.*)	make	1 cent,	marked	1 *ct.* or *c.*
10 cents	"	1 dime,	"	1 *d.*
10 dimes	"	1 dollar,	"	$1.
10 dollars	"	1 eagle,	"	1 *E.*

79. The **Dollar** is the *unit* of currency in the Dominion of Canada and the United States. Accounts are kept in *dollars, cents,* and *mills.*

Dimes, cents, and mills, being fractions of a dollar, are separated from the dollar by the decimal point; thus, four dollars two dimes three cents five mills, or four dollars two hundred thirty-five mills, are written $4.235.

To express any number of cents less than 10, a cipher must be placed between the figure expressing that number and the decimal point; thus, 8 cents is written .08, or 0.08.

NOTES.—1. Business men frequently write cents as common fractions of a dollar; thus, $3 14 is also written 3\frac{14}{100}$, read 3 and $\frac{14}{100}$ dollars.

2. In business transactions, when the *final* result of a computation contains 5 mills or *more,* they are called one cent, and when *less* than 5, they are rejected.

EXAMPLES FOR PRACTICE.

1. Write fifteen dollars twenty-three cents. *Ans.* $15.23.
2. Write seven dollars six cents. *Ans.* $7.06.
3. Write ten dollars nine cents.
4. Write forty-two cents. *Ans.* $0.42.

77. *What are the* coins *of the Dominion of Canada?*—78. *Of the United States?* — 79. *What is the* unit *of currency in the D. C. and U. S?*

5. Write five dollars eight mills. *Ans.* $5.008.
6. Write thirty cents.
7. Write one hundred cents.
8. Write one thousand mills.
9. Write one cent five mills.
10. Write seventeen dollars four mills.
11. Write $6 and 7 cents.
12. Write 3 eagles 4 dollars 3 dimes 3 mills.

REDUCTION OF DECIMAL CURRENCY.

80. Reduction is the process of changing a number of one denomination to another number of a different denomination which shall have the same value.

We see, p. 64, that $1 = 100 cents = 1000 mills; hence the

81. RULE.—I. *To change dollars to cents, multiply by* 100; *that is, annex two ciphers.*

II. *To change dollars to mills, annex three ciphers.*

III. *To change cents to mills, annex one cipher.*

NOTE.—Dollars, cents, and mills, expressed by a single number, are changed to mills by merely removing the decimal point to the right; and dollars and cents, by annexing one cipher and removing the decimal point to the right.

Conversely,

RULE.—I. *To change cents to dollars, divide by* 100; *that is, point off two figures from the right.*

II. *To change mills to dollars, point off three figures.*

III. *To change mills to cents, point off one figure.*

EXAMPLES FOR PRACTICE.

1. In $7 how many mills?

ANALYSIS.—In $1 there are 1000 mills, and in $7 there are 7 times 1000 mills = 7 00 mills.

2. In 356 cents how many dollars?

ANALYSIS.—In $1 there are 100 cents, therefore, $\frac{1}{100}$ of the number of cents equals the number of dollars; $\frac{1}{100}$ of 356 = $3.56.

3. Change $464 to cents. *Ans.* 46400 cts.
4. Change 612 cents to dollars. *Ans.* $6.12.
5. Reduce $3.10 to mills.
6. Reduce 35 cents to mills.
7. Reduce 7045 mills to dollars.
8. Change 10426 cents to dollars.
9. Reduce $4605 to mills.
10. In 2064 mills how many cents?

NOTE.—We give no separate rules for Decimal currency, since it may be added, subtracted, multiplied and divided in the same manner as decimals.

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80. *What is* reduction?— *What is the* rule *for changing dollars to cents and mills?— Cents to mills?—* 81. *What is the* rule *for changing cents to dollars?— Mills to dollars?— Mills to cents?*

PRACTICAL PROBLEMS COMBINING THE FUNDAMENTAL RULES.

1. A broker bought stocks for $3729.90, and sold them for $4168.135; how much did he gain? *Ans.* $438.235.

2. If 1 month's wages amount to $35.50, what will 12 months' wages amount to? *Ans.* $426.

3. At $7.40 a barrel, how much flour can be bought for $111?

4. If 15 quarts of strawberries cost $0.9375, what will 1 quart cost? *Ans.* $0.0625.

5. A farmer sold an equal number of chickens, ducks, and geese, for 2130 cents; the chickens at 14, the ducks at 46, and the geese at 82 cents each; how many of each kind did he sell? *Ans.* 15.

6. Bought 144 acres of land for $5040, and sold 95 acres of it at $48 an acre, and the remainder for what it cost; how much did I gain by the bargain? *Ans.* $1235.

7. A man sold 135 bushels of wheat at $0.62 a bushel, and received in payment 86 pounds of sugar at $0.09 a pound, 25 pounds of coffee at $0.375 a pound, and the remainder in cash; how much cash did he receive? *Ans.* $66.585.

8. If a gentleman's income be $3000 a year, and his expenses $4.20 a day, what will he save at the end of a year, or 365 days?

9. A man divided $360 among three persons; to the first he gave $130; and to the second, $20 less than to the first: how much did the third receive? *Ans.* $120.

10. A lumber merchant bought 680 logs for the sum of $3644.80; what is the price of each log? *Ans.* $5.36.

11. With a Bank note of $1000, I paid my tailor's bill of $348; my shoemaker's of $75 and my house-rent of $375; how many dollars have I left? *Ans.* $202.

12. If a hat cost $4.25, how much will five dozen of similar hats cost? *Ans.* $255.

13. An army composed of 62100 men on the eve of a battle, has 13708 men less after the engagement; how many men are there yet in the army? *Ans.* 48392.

14. What is the price of a silver cover, if 15 cost $117? *A.* $7.80.

15. How much must I sell goods which cost me $286 to gain $76 in giving $18 commission? *Ans.* $380.

16. Joseph bought 73 casks of syrup at $39 the cask, and sold them again for $52; what is his profit? *Ans.* $949.

17. A Banker is to receive $13950 in three payments; the first amounting to $5800, and the second, to $4320; what will be the amount of the third? *Ans.* $3830.

18. A silver cover costs $19.20; how much will 2 dozen of similar ones cost? *Ans.* $460.80.

19. I bought 150 apples for $1.05; how many can I buy for $17.85? *Ans.* 2550.

20. A banker received during the first quarter, $15936; during the second, $31940; during the third, $27674; during the fourth, $42769. He paid out during the whole year $96843; required how much he has left supposing he had $24375 in his safe at the beginning of the year? *Ans.* $45851.

21. I sold at 65 cents a bushel, 58 bushels of barley for which I had paid 52 cts. per bushel; how much did I gain? *Ans.*

22. Frank was born in 1857, in what year will he be 21 years old? *Ans.* 1878.

23. A father was 34 years old at the birth of his son; what will be the age of the son when the father will be 75 years old? *Ans.* 41.

24. An omnibus able to seat 18 persons makes 12 trips per day; how many travellers will it carry in one year of 365 days supposing that there are always 18 persons at each trip? *Ans.* 78840.

25. If we can buy a yard of flannel for $1.76; how many yards of the same quality can be got for $626.56? *Ans.* 356.

26. A pedestrian sets out from Quebec to Montreal, the distance being 180 miles; he walks during 5 days at the rate of 27 miles per day; required what distance he has yet to go? *Ans.* 45 miles.

27. Of a certain sum, 82 persons have received each $24 and there remains yet $36.40; what is that sum? *Ans.* $2004.40.

28. I bought 15 yards of linen at $0.25 a yard, 37 gallons of oil at $1.30 a gallon, 40 pounds of tea at $0.80 a pound, and 108 pounds of coffee at $0.37 a pound; required the amount of my Bill? *A.* $123.81.

29. A bookbinder has 720 volumes to bind at the rate of $0.18 a volume; if it takes him 45 days to do the work: how much will he earn per day? *Ans.* $2.88.

30. Having bought a barrel of oil containing 28 gallons, at $0.75 a gallon, I lost 7 gallons by leakage and sold the remainder for $1.20 per gallon; did I lose or gain and how much? *Ans.* Gained $4.20.

31. Having bought a house for $3740 and making repairs for the sum of $1438, I wish to sell it so as to gain $600: for how much must I sell it? *Ans.* $5778.

32. What sum of money is required to pay 34 workmen each of whom has worked during 28 days, at $0.80 per day? *Ans.* $761.60.

33. I bought 97 barrels of codfish at $5 a barrel, I gave 17 barrels to the poor and sold the remainder at $8 a barrel. Have I lost or gained and how much? *Ans.* Gained $155.

34. Louis bought 500 acres of land for the sum of $17876. He afterwards sold it in lots as follows: 127 acres, at $47; 212 acres at $96; and the remainder, at $37; how much did he gain by his bargain? *Ans.* $14402.

35. Henry receives 45 cents to buy 6 pounds bread at 5 cents a pound, and 2 copies at 3 cents a piece; what is his change?

36. The overcoat of Wilfrid costs 3 times as much as the hat of Julius, which is worth $2.70; what is the cost of Wilfrid's overcoat?

37. A fruit dealer has received 15 dozen oranges in two boxes, one of which contains 30 oranges more than the other; how many oranges are there in each box? *Ans.* 105 and 75.

38. A milliner bought silk in a shop for 36 cents, thread for 20 cents, needles for 9 cents, and cotton for 18 cents; after paying her bill she has 72 cents left; how much money had she? *Ans.* $1.55.

39. What is the dividend when the divisor is 3061 and the quotient 0.065? *Ans.* 198.965.

40. A butcher sells a pound of meat for 9 cents which cost him 6 cents; what profit does he make on 175 pounds? *Ans.* $5.25.

41. A person having an income of $3285, wishes to lay by $3 a day. Required how much that person can spend daily, the year being of 365 days. *Ans.* $6.00.

42. A merchant sold 75 yards of cloth at $2.70 a yard, and has received in payment 132 yards of linen at 85 cents a yard and a note of $52.40; how much has he yet to receive? *Ans.* $37.90.

43. What is that number which, being augmented by 85 and divided by 9, gives 25 for quotient? *Ans.* 140.

44. A millionnaire owes a sum of $6549 which he agrees to liquidate in ten equal payments one every year for ten years. His annual income is $5925; how much can he spend daily after paying the tenth agreed upon? *Ans.* $14.44.

45. What number must be divided by 37 so that the quotient may be 13.25 and the remainder 0.35? *Ans.* 490.60.

46. At 39 cents a pound, how much must be paid for 9 bales of wool, each bale containing 317 pounds? *Ans.* $1112.67.

47. If a pair of boots be sold for $3.16; how much must be paid for 20 boxes, each containing 60 pairs? *Ans.* $3792.

48. How much will 3550 laths cost at 22 cents per hundred?

49. How many barrels of apples containing 3 bushels each at 50 cents a bushel can I buy for $40.50? *Ans.* 27 barrels.

50. A literary work consists of 6 volumes; in each volume there are 560 pages, in each page, 42 lines, and in each line, 40 letters. How many letters are there in the work, if it is divided into 60 chapters, and if 5 blank lines are left between each chapter?

51. How many cords of wood at $3.25 a cord did I buy for $136.50? *Ans.* 42.

52. Sold 20 pounds butter for $3.80, how much will 59 pounds come to at the same price? *Ans.* $11.21.

53. A cabinet-maker has earned $45 in a certain number of days by working; had he worked 9 days more, he would have earned $67.50; how much did he earn per day? *Ans.* $2.50.

54. The sum of two numbers is 2458, and their difference, 154; what are the two numbers? *Ans.* 1306 and 1152.

55. When a son, who is now 30 years old, was born, his father was 35 and his mother 19; what are the actual ages of the father and mother? *Ans.* 65 and 49.

56. Having some money at my disposition, I bought two farms at the rate of $1750 each, and 19 shares of Bank Stock at $103 per share, and I have $113 left; how much money had I at my command? *Ans.* $5579.

57. In selling cloth for $610, a merchant gained as much as the cloth cost him, less $5 0; what was the cost? *Ans.* $555.

58. Although I was robbed of $25, yet after having paid $546 which I owed, I have $17 left; how much money had I?

BILLS AND ACCOUNTS.

82. A **Bill**, in business transactions, is a written statement of articles bought or sold, together with the prices of each, and the whole cost.

NOTES.—1. The party who buys, or who receives money, goods, or services, etc., from another, is a *Debtor*; and the party who sells, or who parts with money, goods, etc., is a *Creditor*.

2. A bill of goods bought or sold, or of services received or rendered at a single transaction, and containing only one date, is often called a *Bill of Parcels*.

83. An **Account** is a registry of debts and credits.

NOTES.—1. An account should always contain the names of both parties in the transaction, the price or value of each item or article, and the date of the transaction.

2. Accounts may have only one side, which may be either debit or credit; or it may have two sides, debit and credit.

84. The **Balance of an Account** is the difference between the amount of the debit and credit sides.

85. An **Account Current** is a full copy of an account, giving each item of both debit and credit sides to date.

NOTE.—An account current having only one side is sometimes called a *Bill of Items*.

86. An **Invoice** is a full statement in detail of goods sent to a purchaser or agent at the time the goods are forwarded, giving the marks and contents of each package, the charges paid, and how sent.

87. The **Footing of a Bill** is the total amount or cost of all the items.

NOTES.—1. When a creditor receives the amount of a bill or an account current, he acknowledges it to be paid by writing at the bottom of the bill or account "Received Payment," and signing his name. If the payment be made to a person authorized by the creditor to receive it, he should receipt the bill or account by writing the creditor's name first and his own name under it, as in Form I.

2. Bills and accounts are sometimes paid by the debtor giving to the creditor a promissory note for the amount.

In the following bills and accounts the abbreviations are:

Dr. for debit or debtor.	bbl. for barrel.
Cr. for credit or creditor.	bush. for bushel.
yd. for yard.	lb. for pound.
doz. for dozen.	cwt. for hundred weight.

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82. *What is a* Bill?— *What is meant by* debtor *and* creditor?— *By a* Bill of Parcels?— 83. *What is an* Account?— 84. *The* Balance of an Account?— 85. *An* Account current?— *A* Bill of Items?— 86. *An* Invoice?— 87. *The* Footing of a Bill?

(FORM 1.)

KINGSTON, Sept. 8, 1870.

MR. G. MURRAY,

Bought of E. P. HEALEY & Co.

32	yards Cassimere,	@	$1.70	$ 54	40	
15	" Blue Cloth,	@	3.25	48	75	
24	" Flannel,	@	.67	16	08	
15	" Drilling,	@	.12			
34	" Fine Muslin,	@	.18			
8	" Gingham,	@	.30			
4	doz. Shirt Bosoms,	@	5.60			
2	" Wool Hose,	@	3.25			
						$158 45

Received Payment,

E. P. HEALEY & Co.,
per N. RYAN.

(FORM 2.)

MONTREAL, Sept. 17, 1870.

MR A. SEYMOUR,

Bought of T. MCGREEVY & Co.

May	6	4	boxes Oranges,	@	$ 3.55	$ 14	20	
June	10	15	" Raisins,	@	2.90	43	50	
July	21	3	chests Black Tea,	@	25.00	75	00	
"	"	4	" Green Tea,	@	28.50			
"	24	7	" Imperial Tea,	@	45.10			
"	"	15	bbls. Coffee Sugar,	@	27.20			
Aug.	3	10	sacks Coffee,	@	18.60			
"	12	150	bushels Corn Meal,	@	.85			
								$1283 90
Sept.	2		Credited by Cash,					80 00
								$1203 90

Received Payment,

T. MCGREEVY & Co.

(FORM 3.)

QUEBEC, June 2, 1870.

MR. D. JOHNSON,

Bought of BYRNE, O'BRIEN & CO.

No.							
2	40	pair Gaiters, ...	@ $2.30	$ 92	00		
7	75	" Rubbers, ...	@ .72	54	00		
14	108	" Calf Boots, ...	@ 3.80	410	40		
10	67	" Thick "	@ 2.65	177	55		
		Cooperage and Cartage, ...		4	37		
		Insurance, ...		1	30		
						$739	62

By "Canadian Express Line."

(FORM 4.)

TORONTO, Oct. 5, 1870.

L. JACKSON & CO.,

To W. PRICE & SON. *Dr.*

1870.							
July	3	To 140 bbls. Flour, ...	@ $ 7.60	$1064	00		
"	12	" 95 " Fish, ...	@ 18.50	1757	50		
Aug.	9	" 36 chests Green Tea, . . .	@ 31.80	1144	80		
						$3966	30
1870.		*Cr.*					
July	20	By 200 yards Broadcloth, ...	@ $5.10	$1020	00		
"	27	" 75 " Black Cloth. . . .	@ 4.67	350	25		
Aug.	4	" 280 " Red Flannel, . . .	@ .72	201	60		
Sept.	2	" 24 gross Silk Buttons, . . .	@ .43	10	32		
						$1582	17
		Balance due W. P. & Son				$2384	13

Received Payment,

W. PRICE & SON.

(FORM 5.)

S. R. TAYLOR & Co. in % current with C. LYONS & SON.

DR.						CR.					
1870.						1870.					
Jan.	2	To 986 lbs. Cheese, ...	$ .12	118	32	May	3	By 85 bush. Potatoes, ...	$ .37	31	45
Feb.	27	" 780 " Butter, ...	.21	163	80	"	24	" 71 bbls. Apples, ...	2.40	170	40
"	28	" 592 " Lard, ...	.14	82	88	June	2	" 23 " Plums, ...	4.50	103	50
May	4	" 609 " Tallow, ...	.13	69	17	"	28	" 70 lbs. Almonds, ...	.09	6	30
"	18	" 17 bbls. Salt, ...	1.70	28	90	July	29	" Note at 3 months to Bal.		282	82
"	30	" 96 dozen Eggs, ...	.15	14	40						
July	16	" 130 bush. Oats, ...	.90	117	00						
				$ 594	47					$ 594	47

HALIFAX, July 25, 1870.

C. LYONS & SON.

EXAMPLES TO BE MADE OUT, AS INDICATED.

On Form 1.

1. Sold in Montreal, Feb. 2. 1870, by John Hogan, to Mr. A. Larue, viz.: 7 lbs. chocolate, at 25 cts.; 15 lbs. candles, at 22 cts.; 12 lbs. sugar, at 15 cts.; 18 lbs. flour, at 24 cts. Footing of the bill, $11.17.

On Form 1.

2. Edmond O'Shea of Kingston sold to T. Lee, Feb. 10, 1870, and L. Norris, his agent, collected the amount of the bill: 15 lbs. butter, at 17 cts.; 25 lbs. cheese, at 20 cts.; 750 lbs. maple sugar, at 9 cts.; 278 lbs. coffee, at 36 cts. Footing of the bill, $175.13.

On Form 2.

3. James Owen of Toronto, sold Jan. 8, 1870, to W. C. Maher, 37 yds. sheeting, at 26 cts.; 43 yds. lace, at 82 cts.; Feb. 3, 75 yds. Irish linen, at 45 cts.; 209 yds. muslin, at 14 cts.; 330 yds. dowlas, at 16 cts. Footing of the bill, $160.69.

On Form 4.

4. Messrs. B. Sharples & Co., Ottawa, sold to D. Hall; Feb. 12, 1870, 110 pair thick boots, at $3.75; 28 pair buskins, at 86 cts.; Feb. 20, 40 pair slippers, at 85 cts.; March 2, 67 pair gaiters, at $1.15; 120 pair boys' brogans, at $1.25. On this are the following credits: Feb. 27, by cash, $280; March 15, 110 boxes lemons, at $3.20. What balance was due B. S. & Co., March 23, when the account was settled? *Ans.* $65.63.

On Form 5.

5. L. A. Conroy & Co., Ottawa, sold to G. Morin & Bro., Jan. 2, 1870, 17 yds. broadcloth, at $5.25; Jan. 15, 29 yds. cassimere, at $1.62; Feb. 3, 60 yds. bleached shirting at 17 cts.; Feb. 7, 49 yds. ticking, at 27 cts.; Feb. 15, 18 yds. blue cloth at $3.19; June 17, 27 yds. gray cloth, at $2.75; Aug. 3, 75 yds. flannel, at 61 cts. Remitted by G. Morin & Co. in part payment, as follows: Jan. 28, 1870, cash, $83; June 25, 14 bbls. Ontario Flour, at $7.20; and settled by Note, at 60 days, Aug. 4, the bal. then due L. A. C. & Co. What was the amount of the note? *Ans.* $153.28.

On Form 2.

6. W. Duffy, Montreal, sold to P. Maurice: March 1, 1870, 18 lbs. tobacco, at 32 cts.; 25 lbs. snuff, at 40 cts.; 72 lbs. tobacco leaf, at 18 cts.; 54 lbs. sugar, at 12 cts; 20 lbs. soap, at 14 cts.; April 2, 45 gallons molasses, at 37 cts. April 6, credited by cash, $18. What balance was due W. D., April 8? *Ans.* $36.65.

On Form 3.

7. Sold, May 2, 1870, by L. T. Nolan, dealer in fruits to R. S. Lemoine, Toronto: 32 bbls. Montreal apples, marked 4, at $2.95; 56 bbls. Greenings, marked 5, at $2.25; 16 bbls Harveys, marked 6, at $1.80; 40 bbls. Russets, marked 8, at $2.75; paid $7.50 for packing and $13.40 for transportation. Find the amount of bill. Forwarded by the "Western Line." *Ans.* $380.10.

On Form 1.

8. G. A. Parker of Quebec, sold to S. Montigny, May 5, 1870: 20 lbs. Rio coffee, at 24 cts.: 50 lbs. W. I. sugar, at 7 cts.; 75 lbs. Pearl starch, at 13 cts.; 12 gallons syrup, at 65 cts.; 90 lbs. butter crackers, at 9 cts.; 54 lbs. picnic crackers, at 11 cts.—Footing of the bill, $39.89.

On Form 5.

9. Philip Doyle, grocer, Toronto, sold to W. Morris & Co.: June 11, 1870, 473 gallons alcohol at 95 cts.; 308 gal. old rum, at $1.90; 610 gal. Holland gin, at $1.05; Aug. 5, 207 gal. rum, at $1.75; 119 gal. cognac, at $2.10; Sept. 22, 401 gal. Scotch gin, at $1.15. On this the following payments were made by W. Morris & Co.: Oct. 4, 30 bbls. salmon, at $8.75; Nov. 6, cash, $520: Nov. 22, draft on London, at 30 days, for the balance due P. D. What was the amount of the draft? *Ans.* $1965.85.

On Form 4.

10. Mr. P. I. Gordon, Kingston, sold to J. Kelly: June 15, 1870, 23 yds. silk, at 95 cts.; 15 yd. ribbon, at 45 cts.; 12 yds. muslin, at 18 cts.; July 10, 4 yds. blue cloth, at $3.60; 3 yd. broadcloth, at $4.50; 9 yds. deskin, at $1.25; 1 cravat, at $1.30; Aug. 15, 5 pair boots, at $6.50; 3 doz. hose, at $2.40; 1 doz. sleeve buttons, 50 cts. On this are the following credits: July 20, by 3 bbls. green apples, at $3.20; 15 bushels potatoes, at 22 cts.; Aug. 20, by cash, $7.30. What balance was due P. I. G., Aug. 24, when the account was settled? *Ans.* $91.21.

On Form 2.

11. O. J. Larkin bought of R. Hamilton & Co., Montreal: May 12, 1870, 18 plows, at $11; 23 handsaws, at $3.50; 90 spades, at 86 cts.; May, 30, 86 shovels, at 50 cts.; 46 cwt. iron, at $12; June 7, 14 hammers, at 62 cts.; 12 mitresaws, at $12.12. June 7, credited by cash, $140; June 15, credited by cash, $375. What balance was due R. H. & Co., June 16? *Ans.* $590.02.

On Form 3.

12. Invoiced by L. Casey & Son of Halifax, to A. C. Samson, May 4, 1870: 12 crates Antwerp ware, marked 6, at $175; 43 casks Rouse

illon wine, marked 8, at $99; 19 bbls. superfine flour, marked 10, at $7; 23 bbls. peas, marked 3, at $1.52; 42 chests black tea, marked 5, at $17.50; 37 chests green tea, marked 1, at $23.75; cooperage, $15; cartage, $6.80; Insurance, $32.50. Forwarded by the "Maine Express Line." Amount of Invoice, $8193.01.

On Form 4.

13. Messrs. Hall & Brothers, St. John, N. B., sold, June 1, 1870, to P. N. Walsh, 15260 lbs. pork, at 5½ cts.; 7265 lbs. cheese, at 8½ cts.; July 3, 11521 bushels corn, at 50 cts.; July 10, 1560 bbls. flour, at $6.12½. On the above are the following credits: June 25, by 1150 lbs. cotton, at 6¼ cts; June 30, by cash, $750; July 12, 8256 lbs. maple sugar, at 7 cts.; 6450 gallons molasses, at 37½ cts. What is the amount of cash requisite to balance the account on July 13? *Ans.* $12953.78.

On Form 2.

14. B. C. Willis, bought of A. Murphy & Co., publishers, Montreal: Aug. 4, 1870, 75 Juneau's Mental Arithmetic, at 15 cts.; 50 Smith's Practical Arithmetic, at 37 cts.; 2 doz. Miller's Reader, at $4.50; Aug. 12, 60 Henry's Grammar, at 7 cts.; 36 Kerney's Compendium of History, at 72 cts.; Sept. 1, 30 Walkingame's Primary Algebra, at 18 cts.; Sept. 1, credited by 50 Commercial Arithmetic of the Christian Brothers, at 40 cts. What balance was due A. M. & Co, Sept. 2? *Ans.* $54. 27.

On Form 5.

15. S. N. Kelly bought of H. Hamel & Co., Quebec, Feb. 3, 1870, 18 yds. cambric, at 14 cts.; 60 yds calico, at 42 cts.; 39 yds. cassimere, at $3.75; March 10, 37 yds. cotton, at 35 cts.; 6 yds. velvet, at $4.70; May 2, 30 yds. linen, at $2.65; May 4, 24 yds. merino, at 75 cts. S. N. Kelly's credits are: April 1, 50 lbs. coffee, at 25 cts.; April 9, 7 cords of maple, at $3.50: May 20, draft on Halifax, $78; June 25, 1 gal. oil, $1.50. What balance was due Hamel & Co., June 26, 1870? *Ans.* $196.12.

Let the pupils make out Bills or Accounts, as the case may be, in proper form, from the following.

16. Andrew Whelan of Three Rivers, sold to John Gosselin, July 5, 1870, and I. Kane, his clerk, collected the amount of the bill: 36 lbs. maple sugar, at 13 cts.; 16 lbs. coffee, at 15 cts.; 13 lbs. tea, at 98 cts.; 13 lbs. chocolate, at 61 cts.; 7 lbs. ginger, at 17 cts.; 47 lbs. cheese, at 9 cts.; 12 lbs. pepper, at 19 cts.; 20 lbs. butter, at 16 cts.; 2 gal. vinegar, at 68 cts. Footing of the bill, $40.01.

17. Forwarded per the Eastern Line, June 3, 1870, by B. Ellis & Co., Ottawa, to S. T. Garneau, Quebec: 18 pair worsted stockings, No. 6, at 90 cts.; 15 doz. napkins, No. 10, at 47 cts.; 24 pair men's

kid gloves, No. 7, at 85 cts.; 20 doz. women's kid gloves, No. 2, at 75 cts.; 12 pair silk stockings, No. 16, at $2.85; 6 pair thread stockings, No. 11, at $1.12½. Paid for cartage, 75 cts.; charges for packing, $1.60. Amount $101.95.

18. Sold by J. M. O'Reilly, Montreal, April 10, 1870, to A. Gauthier: 278 lbs. coffe , at 36 cts.; 1270 lbs. lard, at 13 cts.; 800 lbs. ham, at 11 cts.; 1540 lbs. corned beef, at 8 cts.; 750 lbs. butter, at 17 cts.; 217 lbs. maple sugar, at 7 cts.; 126 doz. eggs, at 12 cts.; 150 bushels oats, at 65 cts. Footing of the bill, $731.69.

19. Sold in Toronto, April 20, 1870, by Isaac Chambers, to Mrs. Julia Mere lith, and the bill paid: 3 doz. silver table forks, at $43.75 a doz.; 2 doz. silver table spoons, at $35 a doz.; 2½ doz. silver teaspoons, at $18.50 a doz.; 1½ doz. ivory handle knives, at $7.50 a doz.; 1 gold guard chain, at $136. Footing of the bill, $394.75.

20. P. Barry & Son, Kingston, sold to H. Miller, March 6, 1870, as follows: 2 loaves white sugar, 52 lbs., at 15 cts.; 4 bbls. extra flour, at $7.80; 9½ lbs. cheese, at 16 cts.; 15 lbs. raisins, at 15 cts.; 7 lbs. black pepper, at 42 cts.; 20 lbs butter, at 23 cts; 3 bushels peas, at 70 cts.; 5 bush. beans, at $1.10; 14½ lbs. bacon, at 16 cts.; 1 gal. molasses, 60 cts. Footing of the bill, $60.83.

21. M. Peter Nelson owes D. I. Hogan, Toronto, as follows: June 5, 1870, 3 gross shirt-studs, at 85 cts.; June 17, 15 doz. woolen stockings, at $3.18½; 3 doz. shirt fronts, at $5.05; Aug. 2, 12½ yds. ribbon, at 25 cts.; 30 pair silk gloves, at $1.37½; 4 doz. linen towels, at $2.85; 22½ yds. ticking, at 45 cts. Footing of the bill, $131. 37½.

22. G. Turner & Son, Quebec, sold to A. I. Green, March 6, 1870, 17 pair boots, at $3.00; March 18, 19 pair shoes, at 1.08; April 9, 80 pair hose, at $1.20; 23 pair gloves, at 75 cts. They received of A. I. Green, the following as credits: April 5, 27 Second Readers, at 20 cts.; 10 Third Readers, at $3 90; May 11, 7 Brown's Dictionaries, at $4.75; 19 Golden Manuals, at $2.93; 20 Christian Duties, at 37 cts. The balance due G. T. & Son, which was paid, May 15, 1870, amounted to $44.05.

23. Sold by Smith & Watters, Kingston, July 24, 1870, to O. S. Peters: 275 bbls. Patapsco flour, at $7.16; 150 bbls. Ontario flour, at $6.25; 170 bbls. Chicago flour, at $5.87½; 214 bushels corn, at 82 cts.; 326 bush. wheat, at $1.62½; 300 bush. oats, at 91 cts.; 500 bush. rye, at $1.06. Footing of the bill, $5413.48.

24. Joseph R. Simon, bought of C. T. Adams, Montreal, April 20, 1870, as follows: 5 yds. black cloth, at $3.50; 1 satin waistcoat, $5.50; Trimmings, $3.75; 3 yds. yellow linen, at 19 cts.; 10 yds. gray fringe, at 68 cts.; 3 pieces of ribbon, at 31 cts.; 3 yds. black cassimere, at $2.25; 7½ yds. alpaca, at 55 cts.; 16 yds. cambric, at 10½ cts.; 3 skein silk thread, at 5½ cts.; 4 yds. wadding, at 6 cts.; 9 yds. white flannel, at 90 cts.; 2 cravats, at $1.12½; 4 yds. green baize, at 58 cts.; 6 cotton shirts, at 65½ cts.; 5 yds. merino, at 80 cts.; 10 yds. muslin, at 14 cts. Footing of the bill, $70.01.

25. Sold by P. Mayrand & Co., Halifax, to Ed. O'Neil, grocer, as follows: June 8, 1870, 4 pieces muslin, each 37 yds., at $3.45 a yd.; 8 pieces printed calico, each 47 yds., at 82 cts. a yard; June 27, 5

pieces Dutch linen, each 30 yds., at 70 cts. a yard; July 10, 11 pieces serge, each 19 yds., at 56 cts.; Aug. 6, 1750 yds. Low II cotton, at 20 cts.; 974 yds. Manchester stuffs, at 25 cts. July 30, E. O'Neil, paid in part $350. What balance was due P. M. & Co., Aug. 2, when the account was made out? *Ans.* $1284.46.

26. Messrs. Fraser, O'Donnell & Co., wholesale dealers, Montreal, sold to Dugal & Lane: Aug. 4, 1870, 18 fine dress coats, No. 52, at $27.50; 46 cashmere vests, No. 20, at $4.30 each; Sept. 9, 3 doz. men's black wool hats, No. 22, at $12.50 per doz.; ½ doz. men's Pearl hats, No. 54 at $27 per doz.; 5 umbrellas 28-in., at $1.75; Oct. 12, 5 doz. men's white cotton hose, No. 7, at $2.60 per doz.; 3 black leather valises, No. 72, at $9.50. On this are the following credits: Sept. 10, by cash, $400; Sept. 30, by cash, $150; Oct 7, by 50 bushels corn, at 65 cts. What balance was due F. O'D. & Co., Oct. 18, when the account was settled? *Ans.* $211.55.

27. Bought of L. R. Williams, Quebec, by H. S. Connolly: June 3, 1870, 75 lbs. maple sugar, at 6¼ cts.; 9 lbs. green tea, at 65 cts.; 21 gals. maple syrup, at 70 cts.; July 1, 12 lbs. pepper, at 25 cts.; 10 lbs. spice, at 20 cts.; 12 lbs. ginger, at 18 cts.; 15 lbs. coffee, at 12½ cts.; July 12, 20 lbs. dried apples, at 10 cts.; 18 lbs. dried peaches, at 12½ cts.; 2 bushels onions, at 80 cts.; Aug. 1, 13 lbs. mackerel, at 8 cts.; 9 lbs. smoked herrings, at 20 cts.; Aug. 10, 25 lbs. rice, at 5 cts.; 12 lbs. dried beef, at 12½ cts.; Sept. 4, 5 bush. corn meal, at 80 cts.; 5 sacks table salt, at 20 cts.; 17 lbs. soda crackers, at 9 cts. Amount of the bill, $52.24, which was paid to L. R. Williams, Sept. 7.

28. Sold by L. Trudel, Montreal, to J. B. Poston, as follows: Oct. 20, 1870, 48 pair tongs, at 37½ cts.; 2 doz. pewter-polished bits, at 85 cts. per doz.; 96 doz. hinges, at 18 cts. per doz.; Nov. 3, 32 doz. curry-combs, at 45½ cts. a doz. 20 packets shoemakers' awls, at 58 cts. per packet; Nov. 12, 75 packets 3½ in. screws, at 95 cts. per packet. L. Trudel received of J. B. Poston on account: Nov. 8, 2 casks Medoc wine, each 45 gal., at 80 cts. per gallon; Dec. 5, cash, $50. What balance was due L. T., Dec. 6? *Ans.* $32.79

29. Invoiced by D. Molson, Quebec, to V. R. Lewis, Ottawa, Feb. 1, 1870: 2 cases calf boots, No. 3, each 67 pairs, at $3.75; 4 cases thick boots, No. 4, each 54 pairs, at $2.62; 2 cases gaiters, No. 7, each 75 pairs, at $1.12; 2 cases buskins, No. 10, each 27 pairs, at 86 cts.; 2 cases slippers, No. 14, each 35 pairs, at 70 cts.; 2 cases rubbers, No. 13, each 50 pairs, at $1.04; charged for packing, cartage, etc., $3.90. Footing of the bill, $1439.76.

30. N. P. Morris & Co., Halifax, sold to U. S. Brown, Sept. 7, 1870, 50 yds. print, at 12½ cts.; 15 yds. cambric, at 9 cts.; 6 yds. cassimere, at $ .60; Sept. 25, 33 yds. sheeting, at 11 cts.; 3 yards velvet, at $3.00; 6½ yds. broadcloth, at $4.37½; Oct. 29, 20 yards French print, at 17 cts.; 15 yds. merino, at 70 cts. On this bill are the following credits: Nov. 1, by 22 lbs. butter at 20 cts.; 6 cords cherry wood, at $3.00; Dec. 4, by cash, $16.00; Dec. 10, by 8 days' labor, at $1.50. What balance was due N. P. M. & Co., Dec. 30, when the account was settled? *Ans.* $21.76¾

PROPERTIES OF NUMBERS.

EXACT DIVISORS AND PRIME NUMBERS.

88. An **Exact Divisor** of a number is one that divides it without a remainder, or which gives an integer for the quotient.

89. All numbers are either *even* or *odd*.

90. An **Even Number** is a number of which 2 is an exact divisor; as 2, 6, 8, 24.

91. An **Odd Number** is a number of which 2 is not an exact divisor; as 1, 3, 7, 15.

Every number must be either *prime* or *composite*.

92. A **Prime Number** is one which can not be resolved or separated into two or more integral factors; as 1, 3, 5, 7.

NOTES.—1. All prime numbers except 2 are odd numbers.

2. Numbers are prime to each other, when they have no common divisor; thus, 7 and 13 are prime to each other, as are also 4, 11, and 15.

93. A **Composite Number** is one that has other exact divisors besides 1 and itself; as 6, 9, 14.

94. The **Prime Factors** of a number are its exacts divisors; thus, 1 3, and 7, are factors of 21.

95. The **Power** of a number is the product obtained by taking the number a certain number of times as a factor; thus, 16 is a power of 4.

NOTE.—When the number is taken once, it is called its first power; when taken twice, as a factor, the product is called its second power; and so on.

96. The **Exponent** of a power is a figure written at the right of a number, and a little above it, to show how many times it is taken as a factor; thus, in the expression 5^2, the exponent is 2, and the whole is read 5 second power.

From these principles,

1st. *Any number which will exactly divide one of two numbers will divide their product.*

2nd. *Any number which will exactly divide each of two numbers will divide their sum.*

3rd. *Any number which will exactly divide each of two numbers will divide their difference.*

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88. *What is an exact divisor?*— 89. *What are all numbers?*— 90. *What is an* even number?— 91. *An* odd number?— 92. *A* prime number?— *When are numbers prime to each other?*— 93. *What is a* composite number?— 94. *What are* prime factors?— 95. *What is the* power *of a number?*— 96. *What is an* exponent?

We derive the following properties:

I. *Two* is an exact divisor of all even numbers.

II. *Three* is an exact divisor of every number the sum of whose digits it will exactly divide.

III. *Four* is an exact divisor when it will exactly divide the tens and units of a number.

IV. *Five* is an exact divisor of every number whose unit figure is 0 or 5.

V. *Six* is an exact divisor of every even number, the sum of whose digits it will exactly divide, or that 3 will exactly divide.

VI. *Eight* is an exact divisor when it will exactly divide the hundreds, tens, and units of a number.

VII. *Nine* is an exact divisor when it will exactly divide the sum of the digits of a number.

VIII. *Ten* is an exact divisor when 0 occupies the units' place.

IX. *Eleven* is an exact divisor of every number whose sum of the digits, standing in the *even* places is equal to the sum of the digits standing in the *odd* places.

TABLE OF PRIME NUMBERS FROM 1 TO 1109.

1	59	139	233	337	439	557	653	769	883	1013
2	61	149	239	347	443	563	659	773	887	1019
3	67	151	241	349	449	569	661	787	907	1021
5	71	157	251	353	457	571	673	797	911	1031
7	73	163	257	359	461	577	677	809	919	1033
11	79	167	263	367	463	587	683	811	929	1039
13	83	173	269	373	467	593	691	821	937	1049
17	89	179	271	379	479	599	701	823	941	1051
19	97	181	277	383	487	601	709	827	947	1061
23	101	191	281	389	491	607	719	829	953	1063
29	103	193	283	397	499	613	727	839	967	1069
31	107	197	293	401	503	617	733	853	971	1087
37	109	199	307	409	509	619	739	857	977	1091
41	113	211	311	419	521	631	743	859	983	1093
43	127	223	313	421	523	641	751	863	991	1097
47	131	227	317	431	541	643	757	877	997	1103
53	137	229	331	433	547	647	761	881	1009	1109

FACTORING.

97. CASE I.— *To resolve a number into its prime factors.*

NOTE.—The process of factoring numbers depends upon the following principles:

When is 2 *an exact divisor?*— 3?— 4?— 5?— 6?— 8?— 9?— 10?— 11?

I. Every prime factor of a number is an exact divisor of that number.

II. The only exact divisors of a number are its prime factors, or some combinations of its prime factors.

Ex. What are the prime factors of 1596?

OPERATION.

2	1596
2	798
3	399
7	133
19	19
	1

ANALYSIS.—We divide by 2, the least prime factor, and the result by 2; this gives an odd number, 399, for a quotient. We then divide by the prime numbers 3, 7, and 19, successively, and the last quotient is 1. The divisors, 2, 2, 3, 7, and 19, are the prime factors required. Hence, the

98 RULE.—*Divide the given number by the smallest prime factor; divide the quotient in the same manner, and so continue the division until the quotient is a prime number. The several divisors and the last quotient will be the prime factors required.*

PROOF. The product of all the prime factors will be the given number.

EXAMPLES FOR PRACTICE.

Required the prime factors of

1. 28.	*Ans.* 2, 2, 7.	6. 1140.	*Ans.*	11. 12673.	*Ans.*
2. 36.	*Ans.*	7. 3420.	*Ans.*	12. 12196.	*Ans.*
3. 86.	*Ans.*	8. 2445.	*Ans.*	13. 21504.	*Ans.*
4. 144.	*Ans.*	9. 2431.	*Ans.*	14. 13981.	*Ans.*
5. 360.	*Ans.*	10. 2205.	*Ans.*	15. 17199.	*Ans.*

99. CASE II.—*To find the prime factors common to two or more numbers.*

Ex. What are the prime factors common to 84, 126, and 210?

OPERATION.

2	84,	126,	210.
3	42,	63,	105.
7	14,	21,	35.
	2,	3,	5.

ANALYSIS.—We find 2 to be an exact divisor of all the numbers, and 3, therefore, a common factor; 3 is an exact divisor of the first set of quotients, and 7 of the second set of quotients, therefore, 3 and 7 are also common factors of the numbers. There is no exact divisor of the third set of quotients. Hence, 2, 3, and 7 are the only prime factors common to 84, 126, and 210.

100. RULE.—I. *Divide each of the numbers by the smallest prime number which is an exact divisor of each.*

II. *Divide each set of quotients in the same manner, until they become prime to each other. The divisors will be the common prime factors.*

98. *What is the* rule *to resolve a number into its prime factors?*— 100. *What is the* rule *to find the prime factors common to two or more numbers?*

EXAMPLES FOR PRACTICE.

Required the prime factors common to

1. 12, and 24. *Ans.* 2, 2, and 3.
2. 48, 96, and 120.
3. 42, 63, and 105. *Ans.* 3 and 7.
4. 225, 435, and 540.
5. 48, 72, and 96.
6. 140, 210, and 280. *Ans.* 2, 5, and 7.
7. 252, 336, and 420.
8. 960, 1568, and 5824.
9. 330, 495, and 165.
10. 2340, 11934, 12987, and 14859.

CANCELLATION.

101. Cancellation is the process of rejecting equal factors from numbers sustaining to each other the relation of dividend and divisor.

Ex. 1. Divide 112 by 56.

OPERATION.

$$\frac{112}{56} = \frac{\cancel{7} \times \cancel{2} \times \cancel{2} \times \cancel{2} \times 2}{\cancel{7} \times \cancel{2} \times \cancel{2} \times \cancel{2}} = \frac{2}{1} = 2.$$

ANALYSIS.—The factors of the dividend are 7, 2, 2, 2, and 2. The factors of the divisor are 7, 2, 2, and 2. Rejecting the common factors 7, 2, 2, and 2, we obtain 2 for the quotient.

NOTES.—1. When a dividend contains a divisor an exact number of times, there is a factor in the dividend equal to the divisor.

2. When a factor is cancelled, 1 is supposed to take its place.

Ex. 2. Divide the product of $7 \times 10 \times 12 \times 5$ by the product of $14 \times 18 \times 6$.

OPERATION.

$$\text{Dividend, } \frac{\cancel{7} \times \overset{5}{\cancel{10}} \times \overset{2}{\cancel{12}} \times 5}{\underset{2}{\cancel{14}} \times \underset{9}{\cancel{18}} \times \cancel{6}} = \frac{25}{9} = 2\tfrac{7}{9}.$$

$$\text{Divisor,}$$

OBS.—We have performed this division without factoring the dividend and divisor, by rejecting the factors that are common to both dividend and divisor, and writing the remaining factors in their proper places.

102. RULE.—I. *Write the dividend above and the divisor below a horizontal line.*

II. *Cancel all the factors common to both dividend and divisor.*

III. *Divide the product of the remaining factors of the dividend by the product of the remaining factors of the divisor, and the result will be the quotient.*

101. *What is* cancellation?— 102. *What is the* rule *for cancellation?*

EXAMPLES FOR PRACTICE.

3. $16 \times 24 \times 48 \div 32 \times 36 \times 38 =$ *Ans.* $\frac{8}{19}$.
4. $12 \times 7 \times 5 \div 2 \times 4 \times 3.$ *Ans.* $17\frac{1}{2}$.
5. $16 \times 5 \times 10 \times 18 \div 8 \times 6 \times 2 \times 12.$ *Ans.* $12\frac{1}{2}$.
6. $84 \times 12 \times 18 \div 21 \times 24 \times 9.$ *Ans.* 4.
7. $72 \times 18 \times 16 \div 24 \times 16 \times 9.$ *Ans.* 6.
8. $22 \times 9 \times 12 \times 5 \div 3 \times 11 \times 6 \times 4.$ *Ans.* 15.
9. $76 \times 34 \times 96 \div 17 \times 51 \times 32.$
10. $25 \times 7 \times 14 \times 36 \div 4 \times 10 \times 21 \times 54.$
11. $184 \times 145 \times 80 \div 23 \times 29 \times 60.$
12. $28 \times 27 \times 21 \times 15 \times 18 \div 7 \times 54 \times 7 \times 3 \times 9.$
13. $12 \times 5 \times 183 \times 18 \times 70 \div 3 \times 14 \times 9 \times 5 \times 20 \times 6.$
14. $213 \times 84 \times 190 \times 264 \div 30 \times 56 \times 36.$

DIVISORS OF NUMBERS.

103. A **Common Divisor** or **Measure** of two or more numbers is any number that will exactly divide each of them.

104. The **Greatest Common Divisor** of two or more numbers is the greatest exact divisor of each of them.

105. GENERAL PRINCIPLES.—I. *One is a divisor of all integers.*

II. *Every number is a divisor of itself.*

III. *Every prime factor of a number is a divisor of that number.*

IV. *Every product of any two or more prime factors of a number is a divisor of that number.*

V. *Every number equals the product of its prime factors.*

VI. *A number has no divisors except its prime factors and the product of every two or more of them. Hence, the product of all the prime factors common to two or more numbers is their greatest common divisor.*

COMMON DIVISOR.

106. *To find a common divisor of two or more numbers.*

Ex. Required a common divisor of 9, 15, and 21.

OPERATION.

$9 = 3 \times 3$
$15 = 3 \times 5$
$21 = 3 \times 7$

ANALYSIS.—We resolve each of the given numbers into two factors, one of which is common to all of them. In the operation 3 is the common factor, and is therefore a common divisor of the numbers.

107. RULE.— *Resolve the given numbers into their prime factors, then if any factor be common to all, it will be a common divisor.*

---

103. *What is a* common divisor ?— 104. *What is the* greatest common divisor ?

EXAMPLES FOR PRACTICE.

Find the common divisors of the following numbers:

1. 10, 15, and 25. *Ans.* 5.
2. 15, 18, 24, and 36. *Ans.* 3.
3. 3, 9, 18, and 24.
4. 21, 77, 35, and 42.
5. 28, 14, 42, and 35. *Ans.* 7.
6. 10, 35, 50, and 75. *Ans.* 5.
7. 4, 12, 16, and 28.
8. 82, 118, 48, and 146.

GREATEST COMMON DIVISOR.

108. *To find the greatest common divisor of two or more numbers.*

Ex. What is the greatest common divisor of 168, 210, and 252?

FIRST METHOD.

OPERATION.

2	168	210	252
3	84	105	126
7	28	35	42
	4	5	6

ANALYSIS.—First find the prime factors common to the numbers, (99), which are 2, 3, and 7. Therefore the greatest common divisor is $2 \times 3 \times 7 = 42$. (105, VI).

109. RULE.—*Find the prime factors common to all the numbers* (99), *and their product will be the greatest common divisor.*

SECOND METHOD.

OPERATION.

The prime factors of
$168 = 2 \times 2 \times 2 \times 3 \times 7$
$210 = 2 \times 3 \times 5 \times 7$
$252 = 2 \times 2 \times 3 \times 3 \times 7$

ANALYSIS.—The prime factors common to the three numbers are 2, 3, and 7. Therefore the greatest common divisor is $2 \times 3 \times 7 = 42$. (105, VI.)

110. RULE.—*Resolve the numbers into their prime factors, and find the product of the common prime factors.*

THIRD METHOD.

111. PRINCIPLES.—I. *If the less of two numbers is a divisor of the greater, it is their greatest common divisor.*

II. *A divisor of a number is a divisor of any number of times that number.*

III. *A common divisor of two numbers is a divisor of their sum, and also of their difference.*

IV. *The greatest common divisor of the difference of two numbers and one of them, is the greatest common divisor of the two numbers.*

---

109. *What is the rule to find the greatest common divisor, first method?— Second method?— Third method?*

Ex. Required the greatest common divisor of 117 and 1365.

OPERATION.

```
117) 1365 (11
     117
     ---
     195
     117
     ---
      78) 117 (1
           78
           --
           39) 78 (2
               78
               --
                0
```

ANALYSIS.—Since 117 is the greatest divisor of 117, if it be a divisor of 1365, it will be their greatest common divisor. By trial 117 is found not to be a divisor of 1365, since there is a remainder, 78.

If 78, the greatest divisor of itself, is a divisor of 117, it is the greatest common divisor of 78 and 117, and also, of 117 and 1365. (111, IV.) By trial, 78 is found not to be a divisor of 117, since there is a remainder, 39.

If 39, the greatest divisor of itself, is a divisor of 78, it is the greatest common divisor of 39 and 78, also of 78 and 117, and of 117 and 1365. By trial, 39 is found to be a divisor of 78, and is, therefore, the greatest common divisor of 117 and 1365.

OBS.—A knowledge of the Principles (111), will render the above analysis plain, since 39 is an exact divisor of 78, it is a divisor of $117 = 39 + 78$, and $1365 = 11 \times 117 + 78$.

112. RULE.—*Divide the greater number by the less, and the divisor by the remainder, and so on, till there is no remainder. The last divisor will be the greatest common divisor sought.*

NOTE.—The greatest common divisor of three or more numbers can be found by finding the greatest common divisor of two of the numbers, then the greatest common divisor of this greatest common divisor and a third number, and so on. The last common divisor will be the greatest common divisor of all the numbers.

EXAMPLES FOR PRACTICE.

Find the greatest common divisors of the following numbers:

1. 72 and 168. *Ans.* 24.
2. 175 and 455. *Ans.* 35.
3. 169 and 866. *Ans.* 1.
4. 84, 126, and 210. *Ans.* 42.
5. 12, 18, 24, and 30. *Ans.* 6
6. 385, 462, and 154.
7. 12, 15, and 18.
8. 210, 350, and 770.
9. 70, 105, and 245.
10. 16, 20, and 24. *Ans.* 4.
11. 78, 234, and 468.
12. 2041 and 8476.
13. 286, 429, and 715.
14. 1649 and 5423.
15. 92, 116, and 124.
16. 252, 630, 1134, and 1386.
17. 49373 and 147731.
18. 3013, 2231, and 2047.

LEAST COMMON MULTIPLE.

113. A **Multiple** is a number exactly divisible by a given number; thus, 15 is a multiple of 3.

114. A **Common Multiple** is a number exactly divisible by two or more given numbers; thus, 24 is a common multiple of 2, 3, 4, 6, 8, and 12.

---

113. *What is a* multiple?— 114. *A* common multiple?

NOTE.—It is evident that the product of two or more numbers, or any number of times their product, must be a common multiple of the numbers. Hence, *A common multiple of two or more numbers may be found by multiplying the given numbers together.*

115 The **Least Common Multiple** is the least number exactly divisible by two or more given numbers; thus, 30 is the least common multiple of 10 and 15.

NOTE.—The least common multiple of two or more numbers contains all the prime factors of each of those numbers, and no other factors.

116. *To find the least common multiple.*

FIRST METHOD.

Ex. What is the least common multiple of 9, 12, 16, and 20?

OPERATION.

$9 = 3 \times 3$
$12 = 2 \times 2 \times 3$
$16 = 2 \times 2 \times 2 \times 2$
$20 = 2 \times 2 \times 5$
$2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5 = 720$ *Ans.*

ANALYSIS.—Resolving the numbers into their prime factors, we find these to be 2, 3, and 5. The greatest number of times the 2 occurs as a factor in any of the given numbers is 4 times; the greatest number of times 3 occurs in any of the given numbers is twice; and the greatest number of times the 5 occurs in any of the given numbers is once. Hence, 2, 2, 2, 2, 3, 3, and 5, must be all the prime factors necessary to produce in composing 9, 12, 16, and 20. Therefore 720, the product of these factors, is the least common multiple required.

117. RULE.—I. *Resolve the given numbers into their prime factors.*

II. *Take all the prime factors of the largest number, and such prime factors of the other numbers as are not found in the largest number, and their product will be the least common multiple.*

SECOND METHOD.

Ex. What is the least common multiple of 10, 16, 24, and 32?

OPERATION.

2)	10,	16,	24,	32.
2)	5,	8,	12,	16.
2)	5,	4,	6,	8.
2)	5,	2,	3,	4.
	5,	1,	3,	2.

ANALYSIS.—We first write the given numbers on a horizontal line, then we divide by 2, a prime number that will divide all of them without a remainder, and write the quotients in a line underneath. Now, since some of the numbers in the second line contain the factor 2, we again divide by 2, and write the quotients, and the undivided number, 5, in a line underneath as before. We continue to divide by a prime number, till the divisor and remainders are all prime to each other; the product of both divisors and remainders gives 480 as the least common multiple.

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115. *What is the* least common multiple?— 117. *What is the* rule *for finding the least common multiple* first method?

118. RULE.—I. *Divide by the smallest prime number that is an exact divisor of two or more of the numbers, and write the quotients and the undivided numbers underneath.*

II. *Proceed with the resulting numbers in like manner, until there is no exact divisor of any two of them.*

III. *The product of the divisors and the resulting numbers will be the least common multiple sought.*

NOTES.—1. When numbers are prime to each other, their product is their last common multiple.

2. When any of the given numbers is a factor of any of the others it may be canceled.

EXAMPLES FOR PRACTICE.

Required the least common multiples of the following numbers:

1. 24, 36, and 20. *Ans.* 360.
2. 7, 14, 21, and 15. *Ans.* 210.
3. 14, 19, 38, and 57. *Ans.* 798.
4. 8, 12, 16, and 20.
5. 32, 34, and 36.
6. 20, 36, 48, and 50.
7. 9, 18, 27, and 54.
8. 12, 15, 42, and 60.
9. 10, 45, 75, and 90. *Ans.* 450.
10. 12, 15, 18, and 35. *Ans.* 1260.
11. 25, 60, 100, and 125.
12. 22, 12, 44, and 11.
13. 18, 27, 36, and 40.
14. 270, 189, 297, and 243.
15. 64, 84, 96, and 216.
16. 84, 100, 224, and 300.

FRACTIONS.

119. A **Fraction** is one or more of the equal parts of a unit.

120. Two integers are required to write a fraction, one to express the number of parts into which the whole number is divided, and the other to express the number of these parts taken. Thus,

If an apple be divided into 2 equal parts, one of the parts is called *one half;* if divided into 3 equal parts, one of the parts is called *one third*, two of the parts *two thirds;* if divided into 4 equal parts, one of the parts is called *one fourth*, etc.; if divided into 5 equal parts, one of the parts is called *one fifth*, etc.

The parts are expressed by figures; thus,

One half is written	$\frac{1}{2}$	Three fourths is written	$\frac{3}{4}$
One third "	$\frac{1}{3}$	One fifth "	$\frac{1}{5}$
Two thirds "	$\frac{2}{3}$	Four fifths "	$\frac{4}{5}$
One fourth "	$\frac{1}{4}$	Five sevenths "	$\frac{5}{7}$

118. *What is the* rule *for finding the least common multiple* second method? — 119. *What is a* fraction? — 120. *How many numbers are required to write a fraction?*

121. The two integers of a fraction are its **Terms**; the one *below* the line, the *Denominator;* and the one *above*, the *Numerator*.

122. The **Denominator** names the parts, and shows how many of them are equal to a unit.

123. The **Numerator** numbers the parts, and shows how many of them are taken or expressed by the fraction.

124. From the foregoing definitions, it follows,

I. That the value of a fraction in units, is the quotient of the numerator divided by the denominator.

II. That fractions indicate division, the numerator being a dividend and the denominator a divisor.

125. To *Analyze* a fraction is to name the unit or quantity divided, the value of one of its equal parts and the number of parts expressed.

Ex. Analyze $\frac{5}{6}$ of a yard.

In $\frac{5}{6}$ of a yard, the unit of the fraction is 1 yard; the part or fractional unit, $\frac{1}{6}$ of a yard; and the number of fractional units expressed or numbered is 5. Six is the denominator, and shows that the yard is considered as 6 equal parts. Five is the numerator, and shows that 5 of these equal parts are enumerated. 5 is the dividend, and 6, the divisor. Hence, $\frac{5}{6}$ of a yard expresses 5 equal parts of such value that 6 of them equal 1 yard, the unit of the fraction.

126. Fractions are classified as *Simple, Compound,* and *Complex*.

127. The *Simple fraction* is distinguished as *Proper* and *Improper*.

128. A **Simple Fraction** is one whose terms are integral; as $\frac{2}{3}$, $\frac{3}{4}$, $\frac{7}{5}$.

129. A **Proper Fraction** is one whose numerator is less than its denominator; as $\frac{1}{4}$, $\frac{5}{6}$, $\frac{12}{13}$.

130. An **Improper Fraction** is one whose numerator equals or exceeds its denominator; as $\frac{3}{3}$, $\frac{5}{4}$, $\frac{7}{5}$.

131. A **Compound Fraction** is a fraction of a fraction; as $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{7}{8}$. $\frac{2}{5} \times \frac{3}{4} \times \frac{8}{9}$.

132. A **Complex Fraction** is one having a fraction or a mixed number in either or both of its terms; as, $\frac{\frac{2}{3}}{\frac{5}{9}}$, $\frac{6}{7\frac{3}{4}}$, $\frac{9\frac{1}{2}}{13}$, $\frac{5\frac{1}{3}}{9\frac{2}{5}}$.

133. A **Mixed Number** is an integer and a fraction united in the same expression; as $5\frac{2}{7}$.

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121. *What are the* terms *of a fraction?*— 122. *Define the* denominator?— 123. *The* numerator?— 126 *How are fractions* classified?— 128. *What is a* simple fraction?— 129. *What is a* proper fraction?— 130. *An* improper fraction?— 131. *A* compound fraction?— 132. *A* complex fraction?— 133. *A* mixed number?

134. Since fractions are expressions indicating the division of one number by another, it follows,

1st. *That, if the numerator be multiplied, or the denominator be divided, by any number, the fraction is multiplied by the same number.*

2nd. *That, if the numerator be divided, or the denominator multiplied, by any number, the fraction is divided by the same number.*

3rd. *That, if the numerator and denominator be both multiplied, or both divided, by the same number, the fraction will not be changed in value.*

REDUCTION OF FRACTIONS.

135. The **Reduction** of a fraction is the process of changing its terms, or its form, without altering its value.

136. CASE I.—*To reduce a whole or mixed number to an equivalent improper fraction.*

Ex. 1. Reduce 12 yards to fifths.

OPERATION.

$5 \times 12 = \frac{60}{5}$, *Ans.*

ANALYSIS.—In 1 yard there are 5 fifths, and in 12 yards there are 12 times 5 fifths $= \frac{60}{5}$.

137. RULE.—*Multiply the whole number by the given denominator; take the product for a numerator, under which write the given denominator.*

Ex. 2. To reduce $15\frac{3}{4}$ to fourths.

OPERATION.

$15\frac{3}{4}$

4

$\frac{63}{4}$, *Ans.*

ANALYSIS.—In 1 there are 4 fourths; therefore, 4 times the number of whole ones equals the number of fourths; therefore, $15 = \frac{60}{4}$, to which add $\frac{3}{4}$ and we have $15\frac{3}{4} = \frac{63}{4}$.

138. RULE.—*Multiply the whole number by the denominator of the fraction; to the product add the numerator, and under the sum write the denominator.*

EXAMPLES FOR PRACTICE.

1. Reduce 9 to thirds. *Ans.* $\frac{27}{3}$.
2. Reduce 12 to eighths. *Ans.* $\frac{96}{8}$.
3. Reduce 25 to fourths.
4. Reduce 36 to fifths.
5. Reduce 40 to thirteenths.
6. Reduce 16 to ninths. *Ans.* $\frac{144}{9}$.
7. Reduce 70 to tenths.
8. Reduce 52 to fifteenths.
9. Reduce 35 to sevenths.
10. Reduce 81 to elevenths.

135. *What is* reduction *of a fraction?*— 137. *What is the* rule *for reducing a whole number to an equivalent improper fraction?*— 138. *For reducing a mixed number to an equivalent improper fraction?*

Reduce the following mixed numbers to improper fractions.

11. $37\frac{3}{5}$.	*Ans.* $\frac{188}{5}$.	19. $125\frac{9}{13}$.	*Ans.* $\frac{1634}{13}$.
12. $45\frac{7}{8}$.	*Ans.* $\frac{367}{8}$.	20. $172\frac{3}{20}$.	*Ans.* $\frac{3443}{20}$.
13. $92\frac{9}{16}$.		21. $260\frac{2}{15}$.	
14. $23\frac{7}{60}$.		22. $171\frac{2}{7}$.	*Ans.* $\frac{1199}{7}$.
15. $13\frac{6}{7}$.		23. $167\frac{40}{120}$.	
16. $134\frac{8}{9}$.	*Ans.* $\frac{1214}{9}$.	24. $209\frac{12}{17}$.	
17. $96\frac{7}{12}$.		25. $331\frac{1}{19}$.	
18. $44\frac{5}{18}$.		26. $116\frac{3}{8}$.	

139. CASE II.—*To reduce an improper fraction to an equivalent whole or mixed number.*

Ex. In $\frac{37}{8}$ of a yard, how many yards?

OPERATION.

$\frac{37}{8} = 37 \div 8 = 4\frac{5}{8}$, *Ans.*

ANALYSIS.—Since 8 eighths make 1 yard, there will be as many yards in 37 eighths of a yard as 37 contains times 8, or $4\frac{5}{8}$ yards.

140. RULE.—*Divide the numerator by the denominator, and the quotient will be the number required.*

EXAMPLES FOR PRACTICE.

Reduce the following improper fractions to whole or mixed numbers:

1. $\frac{18}{6}$.	*Ans.* 3.	10. $\frac{1631}{116}$.	*Ans.* $14\frac{7}{116}$.
2. $\frac{27}{4}$.	*Ans.* $6\frac{3}{4}$.	11. $\frac{978}{56}$.	*Ans.* $17\frac{13}{28}$.
3. $\frac{124}{5}$.	*Ans.* $24\frac{4}{5}$.	12. $\frac{192}{16}$.	*Ans.* 12.
4. $\frac{205}{9}$.		13. $\frac{2070}{7}$.	
5. $\frac{828}{15}$.		14. $\frac{227}{15}$.	
6. $\frac{170}{34}$.		15. $\frac{4107}{8}$.	
7. $\frac{1000}{7}$.		16. $\frac{3802}{28}$.	
8. $\frac{1738}{13}$.		17. $\frac{4442}{11}$.	
9. $\frac{2431}{83}$.		18. $\frac{232261}{105}$.	

141. CASE III.—*To reduce fractions to their lowest terms.*

NOTE.—A fraction is in its *lowest terms*, when its numerator and denominator are prime to each other.

Ex. Reduce $\frac{36}{84}$ to its lowest terms.

OPERATION.

2) $\frac{36}{84} = \frac{18}{42}$

2) $\frac{18}{42} = \frac{9}{21}$

3) $\frac{9}{21} = \frac{3}{7}$ *Ans.*

Or,

12) $\frac{36}{84} = \frac{3}{7}$

ANALYSIS.—Dividing both terms of the fraction by the same number does not alter the value of the fraction (134, 3rd.); hence, we divide both terms of $\frac{36}{84}$ by 2, both terms of the result, $\frac{18}{42}$, by 2, both terms of this result by 3, and obtain $\frac{3}{7}$ for the final result. As 3 and 7 are prime to each other, the lowest terms of $\frac{36}{84}$ are $\frac{3}{7}$.

Instead of dividing by the factors 2, 2, and

140. *What is the rule for reducing an improper fraction to a whole or mixed number?*

3, successively, we may divide by the greatest common divisor of the given terms, and reduce the fraction to its lowest terms in a single operation. Hence, the

142. RULE.—*Cancel or reject all factors common to both numerator and denominator.* Or,

Divide both terms by their greatest common divisor.

EXAMPLES FOR PRACTICE.

Reduce the following fractions to their lowest terms:

1. $\frac{5}{35}$.	*Ans.* $\frac{1}{7}$.	9. $\frac{16}{84}$.	*Ans.* $\frac{4}{21}$.	17. $\frac{81}{567}$.	*Ans.* $\frac{1}{7}$.
2. $\frac{27}{36}$.	*Ans.* $\frac{3}{4}$.	10. $\frac{156}{208}$.	*Ans.* $\frac{3}{4}$.	18. $\frac{124}{456}$.	*Ans.* $\frac{31}{114}$.
3. $\frac{64}{72}$.	*Ans.* $\frac{8}{9}$.	11. $\frac{120}{180}$.	*Ans.* $\frac{2}{3}$.	19. $\frac{288}{360}$.	*Ans.* $\frac{4}{5}$.
4. $\frac{72}{126}$.	*Ans.* $\frac{2}{7}$.	12. $\frac{14}{196}$.	*Ans.* $\frac{7}{54}$.	20. $\frac{851}{1215}$.	
5. $\frac{96}{111}$.	*Ans.* $\frac{32}{37}$.	13. $\frac{107}{214}$.		21. $\frac{5643}{5940}$.	
6. $\frac{144}{432}$.		14. $\frac{216}{232}$.		22. $\frac{3127}{3233}$.	
7. $\frac{336}{384}$.		15. $\frac{615}{915}$.		23. $\frac{3060}{5940}$.	
8. $\frac{960}{1536}$.		16. $\frac{4563}{1057}$.		24. $\frac{7891}{8116}$.	

143. CASE IV.—*To reduce a fraction to a decimal.*

Ex. Reduce $\frac{7}{8}$ to its equivalent decimal.

FIRST OPERATION.

$\frac{7}{8} = \frac{7000}{8000} = \frac{875}{1000} = 0.875$, *Ans.*

SECOND OPERATION.

$$8\,)\,\underline{7.000}$$
$$0.875$$

ANALYSIS.—We first annex the same number of ciphers to both terms of the fraction; this does not alter its value, (134. 3rd.); we then divide both resulting terms by 8, the significant figure of the denominator, to obtain the decimal denominator, 1000. Omitting the denominator, and prefixing the sign, we have the equivalent decimal 0.875.

In the second operation, we omit the intermediate steps, and obtain the result, practically, by annexing the three ciphers to the numerator, 7, and dividing the result by the denominator, 8.

144. RULE.—I. *Annex ciphers to the numerator, and divide by the denominator.*

II. *Point off as many decimal places in the result as there are ciphers annexed.*

NOTE.—If the division is not exact when a sufficient number of decimal figures have been obtained, the sign, +, may be annexed to the decimal to indicate that there is still a remainder.

EXAMPLES FOR PRACTICE.

Reduce the following fractions to equivalent decimals.

1. $\frac{1}{5}$.	*Ans.* 0.5.	6. $\frac{1}{25}$.	*Ans.* 0.04.	11. $\frac{1}{3}$.	*Ans.* 0.333+
2. $\frac{3}{4}$.	*Ans.* 0.75.	7. $\frac{15}{16}$.		12. $\frac{13}{16}$.	
3. $\frac{4}{5}$.	*Ans.* 0.8.	8. $\frac{7}{5}$.		13. $\frac{5}{64}$.	
4. $\frac{5}{8}$.		9. $\frac{17}{20}$.	*Ans.* 0.85.	14. $\frac{17}{250}$.	
5. $\frac{5}{7}$.	*Ans.* 0.714+	10. $\frac{3}{400}$.		15. $\frac{19}{320}$.	

142. *What is the* rule *for reducing fractions to their lowest terms?*— 144. *What is the* rule *for reducing a fraction to a decimal?*

145. CASE V.—*To reduce a decimal to a fraction.*

Ex. 1. Reduce 0.875 to an equivalent fraction.

OPERATION.

$0.875 = \frac{875}{1000} = \frac{7}{8}$.

ANALYSIS.—Writing the decimal figures, .875, over the common denominator, 1000, we have $\frac{875}{1000} = \frac{7}{8}$. Hence, the

146. RULE.—*Omit the decimal point, and supply the proper denominator.*

Ex. 2. Reduce $0.5\frac{1}{3}$ to a fraction.

OPERATION.

$$.5\frac{1}{3} = \frac{5\frac{1}{3}}{10} = \frac{\frac{16}{3}}{10} = \frac{16}{30} = \frac{8}{15}.$$

147. RULE.—*Omitting the decimal point, write the denominator under the decimal, and reduce the fraction to its lowest terms* (142).

EXAMPLES FOR PRACTICE.

Reduce the following decimals to equivalent fractions:

1. 0.06.	*Ans.* $\frac{3}{50}$.	9. 0.000125.	
2. 0.75.	*Ans.* $\frac{3}{4}$.	10. $0.3\frac{3}{4}$.	*Ans.* $\frac{3}{8}$.
3. 0.12.		11. 4.00075.	*Ans.* $4\frac{3}{4000}$.
4. 0.125.		12. $0.66\frac{2}{3}$.	*Ans.* $\frac{2}{3}$.
5. 0.024.		13. $0.57\frac{1}{7}$.	*Ans.* $\frac{4}{7}$.
6. 0.655.	*Ans.* $\frac{131}{200}$.	14. $0.16\frac{2}{3}$.	
7. 0.0008.	*Ans.* $\frac{1}{1250}$.	15. $5.625\frac{1}{6}$.	
8. 0.68.			

148. CASE VI.—*To reduce a compound fraction to a simple one.*

Ex. 1. Reduce $\frac{2}{3}$ of $\frac{5}{7}$ to a simple fraction.

OPERATION.

$\frac{2}{3} \times \frac{5}{7} = \frac{10}{21}$, *Ans.*

ANALYSIS.—By multiplying the denominator of $\frac{5}{7}$ by 3, the denominator of $\frac{3}{3}$, it is evident we we obtain $\frac{1}{3}$ of $\frac{5}{7} = \frac{5}{21}$, since the parts into which the number is divided are 3 times as many, and consequently only $\frac{1}{3}$ as large as before; and, since $\frac{1}{3}$ of $\frac{5}{7} = \frac{5}{21}$, $\frac{2}{3}$ of $\frac{5}{7}$ will be twice $\frac{5}{21} = \frac{10}{21}$.

Ex. 2. Reduce $\frac{3}{4}$ of $\frac{4}{7}$ of $\frac{5}{9}$ of $\frac{9}{22}$ of $\frac{4}{5}$ of $3\frac{2}{3}$ to a simple fraction.

OPERATION.

$$\frac{\cancel{3}}{\cancel{4}} \times \frac{\cancel{4}}{7} \times \frac{\cancel{5}}{\cancel{9}} \times \frac{\cancel{9}}{\cancel{22}} \times \frac{\overset{2}{\cancel{4}}}{\cancel{5}} \times \frac{11}{\cancel{3}} = \frac{2}{7}, \text{ Ans.}$$

146, 147. *What is the* rule *for reducing a decimal to a fraction?*

149. RULE.—I. *Cancel the factors common to the numerators and denominators, if any.*

II. *Multiply the remaining numerators together for a new numerator, and the remaining denominators for a new denominator.*

NOTE.—All whole and mixed numbers that occur in compound fractions must be reduced to improper fractions, before the required reduction is performed.

EXAMPLES FOR PRACTICE.

1. What is $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$? *Ans.* $\frac{2}{5}$.
2. What is $\frac{1}{4}$ of $\frac{4}{5}$ of $\frac{5}{6}$? *Ans.* $\frac{1}{6}$.
3. What is $\frac{3}{8}$ of $\frac{9}{11}$ of $\frac{7}{8}$ of $\frac{4}{7}$? *Ans.* $\frac{27}{176}$.
4. Required the value of $\frac{6}{7}$ of $\frac{1}{4}$ of $\frac{6}{11}$ of 21. *Ans.* $2\frac{5}{11}$.
5. Reduce $\frac{3}{7}$ of $\frac{5}{8}$ of $\frac{1}{5}$ of $\frac{8}{9}$ of $\frac{9}{10}$ to a simple fraction.
6. What is the value of $\frac{2}{5}$ of $\frac{5}{6}$ of $\frac{3}{9}$ of $\frac{9}{12}$ of $\frac{24}{13}$?
7. Reduce $\frac{6}{14}$ of $\frac{7}{8}$ of $\frac{8}{12}$ to a simple fraction. *Ans.* $\frac{2}{3}$.
8. Reduce $\frac{22}{35}$ of $\frac{8}{11}$ of $\frac{15}{22}$ of $9\frac{5}{8}$ to a whole number.
9. What is the value of $\frac{1}{5}$ of $2\frac{3}{4}$ of $1\frac{3}{11}$? *Ans.* $\frac{7}{12}$.
10. What is the value of $\frac{3}{15}$ of $\frac{5}{7}$ of $\frac{4}{16}$ of $\frac{11}{5}$ of $8\frac{3}{4}$?
11. Reduce $\frac{9}{17}$ of $\frac{4}{9}$ of $\frac{17}{18}$ of $3\frac{3}{5}$ to a simple fraction.
12. Required the value of $\frac{7}{8}$ of $7\frac{1}{2}$ of $1\frac{9}{15}$ of $\frac{8}{21}$ of $3\frac{1}{3}$.

150. CASE VII.—*To reduce fractions to a common denominator.*

151. A **Common Denominator** is a denominator common to two or more fractions.

Ex. Reduce $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{4}{5}$ to other fractions of equal value, having a common denominator.

FIRST OPERATION.

$$\frac{2 \times 4 \times 5}{3 \times 4 \times 5} = \frac{40}{60}$$

$$\frac{3 \times 3 \times 5}{4 \times 3 \times 5} = \frac{45}{60}$$

$$\frac{4 \times 3 \times 4}{5 \times 3 \times 4} = \frac{48}{60}$$

SECOND OPERATION.

$\frac{2}{3}, \frac{3}{4}, \frac{4}{5}, = \frac{40}{60}, \frac{45}{60}, \frac{48}{60}$.

ANALYSIS.—The product of the denominators is evidently a common multiple of the denominators. Multiplying both terms of the fraction $\frac{2}{3}$ by 4 and 5, and of $\frac{3}{4}$ by 3 and 5, and of $\frac{4}{5}$ by 3 and 4, does not change the value of the fractions (134, 3rd.), and reduces them to equivalent fractions having a common denominator. Hence, the

153. RULE.—*Multiply the terms of each fraction by all the denominators but its own (for new numerators and a common denominator).*

NOTE.—Mixed numbers must first be reduced to improper fractions, and compound fractions, to simple ones.

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119. *What is the* rule *for reducing a compound fraction to a simple one.*—153. *What is the* rule *for finding a common denominator?*

EXAMPLES FOR PRACTICE.

Reduce the following fractions to their common denominator:—

1. $\frac{3}{4}$ and $\frac{5}{6}$. *Ans.* $\frac{18}{24}$, $\frac{20}{24}$.
2. $\frac{1}{2}$ and $\frac{3}{5}$. *Ans.* $\frac{5}{10}$, $\frac{6}{10}$.
3. $\frac{4}{7}$ and $\frac{3}{4}$. *Ans.* $\frac{16}{28}$, $\frac{21}{28}$.
4. $\frac{4}{5}$ and $\frac{5}{9}$. *Ans.* $\frac{36}{45}$, $\frac{25}{45}$.
5. $\frac{6}{7}$ and $\frac{7}{11}$. *Ans.* $\frac{66}{77}$, $\frac{49}{77}$.
6. $\frac{8}{17}$ and $\frac{9}{13}$. *Ans.* $\frac{153}{221}$, $\frac{104}{221}$.
7. $\frac{7}{9}$, $\frac{4}{5}$, and $\frac{1}{2}$. *Ans.* $\frac{70}{90}$, $\frac{72}{90}$, $\frac{45}{90}$.
8. $\frac{2}{3}$, $\frac{3}{5}$, and $\frac{1}{8}$.
9. $\frac{4}{7}$, $\frac{3}{8}$, and $\frac{6}{11}$. *Ans.* $\frac{352}{616}$, $\frac{231}{616}$, $\frac{330}{616}$.
10. $\frac{3}{7}$, $\frac{5}{8}$, $\frac{2}{3}$, and $\frac{1}{2}$.
11. $\frac{3}{4}$, $2\frac{1}{2}$, $\frac{1}{3}$, and $\frac{5}{6}$. *Ans.* $\frac{108}{144}$, $\frac{360}{144}$, $\frac{48}{144}$, $\frac{120}{144}$.
12. $1\frac{3}{5}$, $5\frac{2}{3}$, and $\frac{1}{2}$.
13. $\frac{1}{2}$, $\frac{2}{3}$, $\frac{4}{7}$, and 8.
14. $\frac{4}{9}$, $\frac{7}{11}$, and $\frac{2}{3}$ of $7\frac{5}{8}$. *Ans.* $\frac{528}{1188}$, $\frac{756}{1188}$, $\frac{6539}{1188}$.
15. $\frac{11}{12}$, $\frac{4}{5}$ of 6, and $21\frac{1}{3}$.

154. Case VIII.—*To reduce fractions to their least common denominator.*

155. The **Least Common Denominator** of two or more fractions is the least denominator to which they can all be reduced, and it must be the least common multiple of their denominators.

Ex. Reduce $\frac{5}{6}$, $\frac{3}{8}$, and $\frac{7}{12}$ to their least common denominator.

OPERATION.

$$\begin{array}{r|lll} & \frac{5}{6}, & \frac{3}{8}, & \frac{7}{12}. \\ \hline 2) & \frac{5}{3}, & \frac{3}{4}, & \frac{7}{6}. \\ \hline 2) & 3, & 4, & 6. \\ \hline 3) & 3, & 2. & 3. \\ \hline & 1, & 2, & 1. \end{array}$$

$2 \times 2 \times 3 \times 2 = 24$

$\frac{5}{6} = \frac{20}{24}$
$\frac{3}{8} = \frac{9}{24}$
$\frac{7}{12} = \frac{14}{24}$
} *Ans.*

Analysis.—We find the least common denominator, by (117), to be 24. We then take such a part of it as is expressed by each of the fractions separately for their respective new numerators. Thus, to get a new numerator for $\frac{5}{6}$, we take $\frac{5}{6}$ of 24, the least common denominator, by dividing it by 6, and multiplying the quotient by 5. We proceed in like manner with each of the fractions, and write the numerators thus obtained over the least common denominator. Hence, the

156. Rule.—I. *Find the least common multiple of the given denominators, for the least common denominator.*

II. *Divide this common denominator by each of the given denominators, and multiply each numerator by the corresponding quotient. The products will be the new numerators.*

EXAMPLES FOR PRACTICE.

Reduce the following fractions to their least common denominator.

1. $\frac{3}{4}$, $\frac{5}{9}$, $\frac{7}{8}$, and $\frac{5}{12}$. *Ans.* $\frac{54}{72}$, $\frac{40}{72}$, $\frac{63}{72}$, $\frac{30}{72}$.
2. $\frac{2}{9}$, $\frac{1}{21}$, 6, and $\frac{3}{4}$. *Ans.* $\frac{56}{252}$, $\frac{12}{252}$, $\frac{1512}{252}$, $\frac{189}{252}$.

156. *What is the* rule *for reducing fractions to their least common denominator?*

3. $\frac{3}{4}$, $\frac{2}{5}$, $\frac{4}{9}$, and $\frac{2}{11}$. *Ans.* $\frac{1485}{1980}$, $\frac{792}{1980}$, $\frac{880}{1980}$, $\frac{360}{1980}$.

4. $\frac{11}{14}$, $\frac{1}{2}$, $\frac{16}{21}$, and $\frac{4}{7}$. *Ans.* $\frac{66}{120}$, $\frac{25}{120}$, $\frac{74}{120}$, $\frac{112}{120}$.

5. $\frac{7}{15}$, $\frac{5}{24}$, $\frac{37}{60}$, and $2\frac{3}{5}$.

6. $\frac{2}{9}$, $\frac{3}{4}$, 6, and $\frac{1}{27}$. *Ans.* $\frac{210}{308}$, $\frac{84}{308}$, $\frac{176}{308}$, $\frac{231}{308}$.

7. $\frac{15}{22}$, $\frac{3}{11}$, $\frac{4}{7}$, and $\frac{3}{4}$.

8. $\frac{3}{7}$, $\frac{9}{14}$, $\frac{11}{28}$, and $5\frac{3}{7}$. *Ans.* $\frac{24}{66}$, $\frac{40}{66}$, $\frac{330}{66}$, $\frac{495}{66}$.

9. $\frac{4}{11}$, $\frac{20}{33}$, 5, and $7\frac{1}{2}$.

10. $5\frac{6}{11}$, 7, $7\frac{3}{4}$, and 8. *Ans.* $\frac{21}{30}$, $\frac{20}{30}$, $\frac{26}{30}$, $\frac{28}{30}$.

11. $\frac{7}{10}$, $\frac{2}{3}$, $3\frac{1}{5}$, and $\frac{14}{15}$.

12. $\frac{3}{4}$, 9, 7, 5, and 4.

13. $\frac{7}{8}$, $\frac{7}{12}$, $\frac{7}{16}$, and $\frac{7}{20}$. *Ans.* $\frac{210}{240}$, $\frac{140}{240}$, $\frac{105}{240}$, $\frac{84}{240}$.

14. $\frac{32}{56}$, $\frac{5}{7}$, $4\frac{1}{8}$, and $\frac{4}{15}$.

15. $\frac{8}{15}$, $\frac{6}{11}$, $\frac{5}{24}$, and $\frac{19}{20}$.

ADDITION OF FRACTIONS.

NOTES.—1. Fractions, to be added or subtracted, must be abstract or of like denomination, and must have a common denominator.

2. Only units of the same kind, whether fractional or integral can be added together.

Ex. 1. What is the sum of $\frac{3}{8}$, $\frac{5}{6}$, and $\frac{7}{12}$?

OPERATION.

$$\frac{3}{8} + \frac{5}{6} + \frac{7}{12} = \frac{9 + 20 + 14}{24} = \frac{43}{24} = 1\frac{19}{24}, \textit{Ans.}$$

ANALYSIS.—We first reduce the given fractions to a common denominator. And as the resulting fractions, $\frac{9}{24}$, $\frac{20}{24}$, and $\frac{14}{24}$ have the same fractional unit, we add them by uniting their numerators into one sum, making $\frac{43}{24} = 1\frac{19}{24}$, the answer.

Ex. 2. Add $7\frac{3}{4}$, $8\frac{5}{12}$, and $1\frac{2}{3}$.

OPERATION.

$$\begin{aligned} 7 + 8 + 1 &= 16 \\ \tfrac{3}{4} + \tfrac{5}{12} + \tfrac{2}{3} &= 1\tfrac{5}{6} \\ &\overline{17\tfrac{5}{6}}, \textit{Ans.} \end{aligned}$$

ANALYSIS.—The sum of the integers, 7, 8, and 1, is 16; the sum of the fractions, $\frac{3}{4}$, $\frac{5}{12}$, and $\frac{2}{3}$, is $1\frac{5}{6}$. Hence, the sum of both fractions and integers is $16 + 1\frac{5}{6} = 17\frac{5}{6}$. Hence the

157. RULE. I. To add fractions.—*When necessary, reduce the fractions to their least common denominator; then add the numerators and place the sum over the common denominator.*

II. To add mixed numbers.—*Add the integers and fractions separately, and then add their sums.*

NOTE.—All fractional results should be reduced to their lowest terms, and if improper fractions, to whole or mixed numbers.

157. *What is the general* rule *for adding fractions?*

EXAMPLES FOR PRACTICE.

1. What is the sum of $\frac{7}{8}$, $\frac{3}{6}$, and $\frac{2}{3}$? *Ans.* $2\frac{1}{24}$.
2. What is the sum of $\frac{4}{5}$, $\frac{7}{10}$ and $\frac{12}{15}$. *Ans.* $2\frac{11}{30}$.
3. What is the sum of $\frac{7}{11}$, $\frac{9}{22}$, $\frac{5}{8}$, and $\frac{1}{6}$? *Ans.* $2\frac{29}{33}$.
4. Add $\frac{4}{7}$, $\frac{7}{15}$, $\frac{3}{5}$, and $\frac{13}{21}$. *Ans.* $2\frac{9}{35}$.
5. Add $\frac{4}{5}$, $\frac{7}{10}$, and $\frac{13}{15}$.
6. Add $\frac{13}{15}$, $\frac{5}{12}$, and $\frac{7}{20}$.
7. Find the sum of $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, and $\frac{17}{18}$. *Ans.* $3\frac{7}{36}$.
8. Find the sum of $\frac{1}{3}$, $\frac{3}{4}$, $\frac{11}{12}$, and $\frac{29}{42}$.
9. Add $14\frac{4}{5}$, $3\frac{9}{10}$, $\frac{19}{20}$, and $1\frac{2}{3}$. *Ans.* $21\frac{19}{60}$.
10. Add $4\frac{7}{15}$, $8\frac{5}{21}$, and $2\frac{8}{35}$.
11. Add $4\frac{1}{4}$, $1\frac{1}{3}$, and $4\frac{7}{11}$. *Ans.* $10\frac{29}{132}$.
12. What is the sum of $\frac{7}{8}$, $1\frac{7}{12}$, 5, and $10\frac{5}{6}$.
13. Add $\frac{1}{2}$ of $\frac{2}{3}$, $\frac{4}{5}$ of $\frac{5}{7}$, and $\frac{3}{4}$. *Ans.* $1\frac{55}{84}$.
14. Add $\frac{3}{5}$ of $\frac{2}{3}$, $\frac{1}{2}$ of $\frac{2}{7}$ of $\frac{3}{4}$, and $\frac{1}{2}$.
15. Add $41\frac{1}{2}$, $105\frac{2}{9}$, $300\frac{3}{4}$, $241\frac{3}{5}$, and $472\frac{1}{4}$. *Ans.* $1161\frac{23}{90}$.
16. Add $125\frac{4}{7}$, $327\frac{5}{12}$, and $25\frac{1}{4}$.
17. Add $\frac{1}{4}$, $\frac{2}{9}$, $\frac{3}{11}$, $\frac{7}{18}$, and $\frac{17}{33}$. *Ans.* $1\frac{89}{99}$.
18. Find the sum of $17\frac{3}{4}$, $18\frac{5}{12}$, and $11\frac{2}{3}$.
19. Add $\frac{4}{5}$ of $18\frac{3}{11}$, and $\frac{11}{12}$ of $\frac{4}{9}$ of $6\frac{3}{11}$. *Ans.* $12\frac{891}{893}$.
20. Add $\frac{1}{3}$ of $\frac{3}{11}$ of $\frac{11}{12}$ to $\frac{1}{2}$ of $\frac{2}{5}$.
21. Add $16\frac{2}{3}$, $8\frac{7}{8}$, $9\frac{3}{5}$, $3\frac{1}{4}$, and $1\frac{7}{8}$. *Ans.* $40\frac{4}{15}$.
22. Add $3\frac{1}{2}$ of $5\frac{1}{4}$, $\frac{9}{22}$ of $7\frac{1}{9}$, and $\frac{2}{5}$ of $\frac{4}{5}$.

SUBTRACTION OF FRACTIONS.

Ex. 1. From $\frac{3}{4}$ take $\frac{2}{3}$.

OPERATION.

$$\frac{3}{4} - \frac{2}{3} = \frac{9 - 8}{12} = \frac{1}{12}.$$

ANALYSIS.—We reduce the given fractions to a common denominator, and have $\frac{9}{12}$ and $\frac{8}{12}$ which express fractional units of the same value. Then 9 twelfths less 8 twelfths equal 1 twelfth $= \frac{1}{12}$, the answer.

Ex. 2. From $24\frac{3}{7}$ take $16\frac{4}{5}$.

FIRST OPERATION.

$24\frac{3}{7} = 24\frac{15}{35}$
$16\frac{4}{5} = 16\frac{28}{35}$
$7\frac{22}{35}$ *Ans.*

ANALYSIS.—We first reduce the fractional parts $\frac{3}{7}$ and $\frac{4}{5}$, to a common denominator, 35. Since we cannot take $\frac{28}{35}$ from $\frac{15}{35}$, we add $1 = \frac{35}{35}$, to $\frac{15}{35}$, making $\frac{50}{35}$, and $\frac{28}{35}$ from $\frac{50}{35}$ leaves $\frac{22}{35}$. We next add 1 to the 6 in the subtrahend, and subtracting, we have $7\frac{22}{35}$ for the entire remainder.

SECOND OPERATION.

$24\frac{3}{7} = \frac{171}{7} = \frac{855}{35}$
$16\frac{4}{5} = \frac{84}{5} = \frac{588}{35}$
$\frac{267}{35} = 7\frac{22}{35}$.

ANALYSIS.—In this operation, we reduce the mixed numbers to improper fractions, and these fractions to a common denominator. We then subtract the less fraction from the greater, and, reducing the remainder to a mixed number, obtain $7\frac{22}{35}$, as before.

158. RULE. I. To subtract fractions.—*When necessary, reduce the fractions to their least common denominator. Subtract the numerator of the subtrahend from the numerator of the minuend, and place the difference of the new numerators over the common denominator.*

II. To subtract mixed numbers.—*Reduce the fractional parts to a common denominator, and then subtract the fractional and integral parts separately.* Or,—*Reduce the mixed numbers to improper fractions, then to a common denominator, and subtract the less fraction from the greater.*

EXAMPLES FOR PRACTICE.

1. $\frac{4}{5} - \frac{3}{5} =$ *Ans.* $\frac{1}{5}$.
2. $\frac{11}{12} - \frac{9}{12} =$
3. $\frac{2}{3} - \frac{4}{9} =$ *Ans.* $\frac{1}{9}$.
4. $\frac{9}{10} - \frac{12}{20}$.
5. $\frac{13}{20} - \frac{11}{10} =$ *Ans.* $\frac{21}{80}$.
6. $\frac{4}{17} - \frac{1}{3}$.
7. $\frac{11}{34} - \frac{1}{10} =$ *Ans.* $\frac{19}{85}$.
8. $\frac{18}{19} - \frac{3}{38}$.
9. $\frac{10}{27} - \frac{5}{18} =$ *Ans.* $\frac{5}{54}$.
10. $9\frac{3}{8} - 11\frac{11}{12} =$
11. $10\frac{5}{7} - 4\frac{3}{7}$.
12. $14\frac{1}{4} - 3\frac{7}{8} =$ *Ans.* $10\frac{3}{8}$.
13. $4\frac{1}{7} - \frac{8}{9}$.
14. $9\frac{1}{2} - 3\frac{7}{9} =$ *Ans.* $5\frac{13}{18}$.
15. $8\frac{1}{8} - 6\frac{7}{8}$.
16. $19\frac{1}{18} - 3\frac{18}{19} =$ *Ans.* $15\frac{32}{247}$.
17. $71\frac{1}{15} - 13\frac{7}{12} =$ *Ans.* $57\frac{28}{60}$.
18. $75 - 7\frac{1}{3}$.
19. $18\frac{2}{5} - 5\frac{5}{6} =$ *Ans.* $12\frac{7}{15}$.
20. $\frac{5}{21} - \frac{1}{15}$.
21. $28\frac{16}{63} - 3\frac{9}{14} =$ *Ans.* $24\frac{11}{18}$.
22. $91\frac{1}{3} - 2\frac{3}{4}$.
23. $165\frac{7}{8} - 77\frac{8}{9} =$ *Ans.* $87\frac{71}{72}$.
24. $147\frac{1}{9} - 131\frac{1}{12}$.
25. $3\frac{7}{15} - \frac{43}{125} =$ *Ans.* $3\frac{446}{375}$.
26. $17\frac{1}{7} - 8\frac{1}{4}$.
27. $7\frac{1}{4} - 2\frac{5}{8} =$ *Ans.* $4\frac{25}{38}$.
28. $47 - 1\frac{3}{20}$.
29. $9\frac{2}{7} - 3\frac{3}{5} =$ *Ans.* $5\frac{24}{35}$.
30. $101\frac{7}{8} - 93\frac{4}{5}$.
31. $634 - 342\frac{2}{9} =$ *Ans.* $291\frac{7}{9}$.
32. $25\frac{5}{7} - 13\frac{2}{3}$.
33. From $\frac{2}{3}$ of $\frac{5}{6}$ take $\frac{1}{5}$ of $\frac{7}{8}$. *Ans.* $\frac{137}{360}$.
34. From $\frac{1}{2}$ of $\frac{2}{3}$ take $\frac{1}{6}$ of $\frac{6}{7}$. *Ans.* $\frac{4}{21}$.
35. From $\frac{2}{3}$ of $\frac{7}{8}$ take $\frac{2}{5}$ of $\frac{5}{20}$. *Ans.* $\frac{29}{60}$.
36. From $\frac{2}{3}$ of $\frac{9}{11}$ take $\frac{1}{4}$ of $\frac{2}{3}$.
37. From $\frac{3}{5}$ of $\frac{5}{7}$ of $3\frac{1}{2}$ take $\frac{1}{2}$ of $1\frac{1}{2}$.
38. What is the value of $\frac{1}{4}$ of $3 - \frac{1}{3}$ of 2. *Ans.* $\frac{1}{12}$.
39. From 72 lbs. there were taken at one time $17\frac{5}{8}$ lbs., and at another, $28\frac{7}{12}$ lbs.; what quantity remains? *Ans.* $25\frac{19}{24}$ lbs.
40. From \$15, \$$3\frac{1}{8}$ were given to A, \$$4\frac{1}{4}$ to B, \$$2\frac{1}{2}$ to C, and the remainder to D; what did D receive?

MULTIPLICATION OF FRACTIONS.

159. CASE I.—*To multiply a fraction by an integer.*

Ex. Multiply $\frac{7}{9}$ by 3.

FIRST OPERATION.

$\frac{7}{9} \times 3 = \frac{21}{9} = 2\frac{3}{9}$

ANALYSIS.—In the first operation, we multiply the numerator of the fraction by the integer, 3, and obtain $2\frac{3}{9}$ for the answer. It is evident that

158. *What is the* rule *for subtracting fractions?*

SECOND OPERATION.

$\frac{7}{9} \times 3 = \frac{7}{3} = 2\frac{1}{3}$

THIRD OPERATION.

$$\frac{7}{\cancel{9}} \times \frac{\cancel{3}}{1} = \frac{7}{3} = 2\frac{1}{3}$$
$$\cancel{3}$$

the fraction $\frac{7}{9}$ is multiplied by multiplying its numerator by 3, since the parts taken, 21, are 3 times as many as before, while the parts into which the unit of the fraction is divided remain the same.

In the second operation, we divide the denominator of the fraction by the integer, 3, and obtain $2\frac{1}{3}$ for the answer, as before. It is evident, also, that the fraction $\frac{7}{9}$ is multiplied by 3 by dividing its denominator by 3, since the parts into which the unit of the fraction is divided are only $\frac{1}{3}$ as many, and consequently 3 times as large, as before, while the parts taken remain the same. Hence,

Multiplying the numerator or dividing the denominator of a fraction by any integer multiplies the fraction by that integer.

NOTE.—In the third operation, we express the multiplier in the form of a fraction, indicate the multiplication, and obtain the result by cancellation.

160. CASE II.—*To multiply an integer by a fraction, or to find a fractional part of an integer.*

Ex. Multiply 24 by $\frac{5}{6}$.

FIRST OPERATION.

$24 \times \frac{5}{6} = \frac{120}{6} = 20.$

SECOND OPERATION.

$24 \times \frac{5}{6} = 4 \times 5 = 20.$

THIRD OPERATION.

$$\frac{\cancel{24}}{1} \times \frac{5}{\cancel{6}} = 20.$$

ANALYSIS.—In the first operation, we first multiply the integer, 24, by the numerator of the fraction, then divide the product by the denominator, and obtain 20 for the answer.

In the second operation, we divide the integer, 24, by the denominator of the fraction, and obtain $\frac{1}{6}$ of 24 = 4, which multiplied by 5, the numerator of the fraction, gives $\frac{5}{6}$ of 24 = 20. Hence,

Multiplying by a fraction is taking the part of the multiplicand denoted by the multiplier.

NOTE.—In the third operation, we express the integer, 24, in the form of a fraction, indicate the multiplication, and obtain the result by cancellation.

161. CASE III.—*To multiply a fraction by a fraction.*

NOTE.—To multiply a fraction by a fraction is to find a fractional part of a fraction.

Ex. Multiply $\frac{5}{12}$ by $\frac{4}{5}$.

FIRST OPERATION.

$\frac{5}{12} \times \frac{4}{5} = \frac{20}{60} = \frac{1}{3}.$

SECOND OPERATION.

$$\frac{\cancel{5}}{\cancel{12}} \times \frac{\cancel{4}}{\cancel{5}} = \frac{1}{3}.$$
$$\cancel{3}$$

ANALYSIS.—To multiply $\frac{5}{12}$ by $\frac{4}{5}$ is to take $\frac{4}{5}$ of the multiplicand, $\frac{5}{12}$. Now, to obtain $\frac{4}{5}$ of $\frac{5}{12}$, we simply multiply the numerators together for a new numerator, and the denominators together for a new denominator (150). Therefore,

Multiplying one fraction by another is the same as reducing compound fractions to simple ones.

From the foregoing we deduce the following general

162. RULE.—I. *Reduce all integers and mixed numbers to improper fractions.*

II. *Multiply together the numerators for a new numerator, and the denominators for a new denominator.*

NOTES.—1. Cancel all factors common to numerators and denominators.

2. The word *of* between fractions is equivalent to the sign of multiplication.

EXAMPLES FOR PRACTICE.

1. $\frac{3}{4} \times 7 =$ *Ans.* $5\frac{1}{4}$.
2. $\frac{7}{8} \times 4.$
3. $\frac{5}{7} \times 8 =$ *Ans.* $5\frac{5}{7}$.
4. $\frac{8}{15} \times 5.$
5. $\frac{3}{11} \times 6 =$ *Ans.* $1\frac{7}{11}$.
6. $12 \times \frac{2}{3}.$
7. $13 \times \frac{4}{7} =$ *Ans.* $7\frac{3}{7}$.
8. $16 \times \frac{3}{19}.$
9. $19 \times \frac{13}{17} =$ *Ans.* $5\frac{12}{17}$.
10. $21 \times \frac{3}{5}.$
11. $\frac{17}{42} \times \frac{14}{17} =$ *Ans.* $\frac{1}{3}$.
12. $\frac{1}{9} \times \frac{8}{17}.$
13. $\frac{5}{6} \times \frac{3}{4} =$ *Ans.* $\frac{5}{12}$.
14. $\frac{6}{11} \times \frac{4}{5}.$
15. $2\frac{1}{2} \times 2\frac{1}{5} =$ *Ans.* $5\frac{1}{2}$.
16. $3\frac{3}{9} \times \frac{3}{5} =$ *Ans.* $2\frac{1}{3}$.
17. $\frac{2}{7\frac{5}{5}} \times 15.$
18. $1\frac{17}{18} \times 1\frac{2}{7} =$ *Ans.* $2\frac{1}{2}$.
19. $9 \times 8\frac{11}{12}.$
20. $7\frac{1}{8} \times 8\frac{3}{7} =$ *Ans.* $60\frac{3}{56}$.
21. $\frac{11}{16} \times \frac{7}{12}.$
22. $\frac{8}{9} \times 71\frac{4}{5} =$ *Ans.* $63\frac{37}{45}$.
23. $47\frac{7}{8} \times 9\frac{1}{4}.$
24. $12\frac{3}{4} \times 11\frac{5}{9} =$ *Ans.* $147\frac{1}{3}$.
25. $4\frac{1}{5} \times \frac{6}{7}.$
26. $\frac{2}{3} \times \frac{3}{4} \times \frac{3}{5} =$ *Ans.* $\frac{3}{10}$.
27. $\frac{9}{10} \times \frac{2}{7} \times \frac{5}{9} \times \frac{1}{4}.$
28. $\frac{7}{8} \times 1\frac{1}{7} \times \frac{2}{3} \times \frac{3}{5} =$ *Ans.* $\frac{2}{5}$.
29. $1\frac{5}{6} \times \frac{3}{5} \times 2 \times 5\frac{1}{3}.$
30. $\frac{7}{360} \times \frac{43}{120} \times \frac{5409}{53} =$
31. Find the value of $2\frac{1}{2}$ times $\frac{3}{4}$ of $\frac{4}{5}$ of $1\frac{1}{3}$. *Ans.* 2.
32. Find the value of $\frac{3}{7}$ of $1\frac{2}{5}$ of $\frac{1}{12}$ of $\frac{9}{12}$.
33. What is the product of $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{7}{8}$ by 11? *Ans.* $3\frac{5}{21}$.
34. What is the product of $12\frac{1}{2}$ by $5\frac{1}{2}$ times $6\frac{3}{4}$?

PRACTICAL PROBLEMS.

NOTE.—In business transactions it is customary to add 1 cent when the fraction is equal to or greater than a half of a cent, and to omit it when it is less than the half of a cent. The fraction is retained in the following answers.

Required the cost of

1. $6\frac{7}{8}$ lbs. of ham, at $12\frac{1}{2}$ cts. per lb. *Ans.* $\$0.85\frac{15}{16}$.
2. $7\frac{1}{2}$ yds. of tape, at $5\frac{3}{4}$ cts. per yard.
3. $9\frac{7}{8}$ quarts of plums, at $7\frac{2}{3}$ cts. per qt. *Ans.* $\$0.75\frac{17}{24}$.
4. 56 lbs. of chalk, at $\frac{3}{4}$ of a cent per lb.
5. $7\frac{2}{3}$ yards of muslin, at $9\frac{3}{4}$ cts. per yard. *Ans.* $\$0.74\frac{3}{4}$.
6. $7\frac{6}{11}$ lbs. of beef, at 5 cts. per lb.
7. $6\frac{1}{2}$ bush. of apples, at $74\frac{1}{2}$ cts. per bush. *Ans.* $\$4.84\frac{1}{4}$.
8. $12\frac{1}{2}$ bush. of oats, at $62\frac{1}{2}$ cts. per bush.

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162. *What is the* rule *for the multiplication of fractions?*

9. 79 bush. of salt, at $\frac{7}{8}$ of a dollar per bush. *Ans.* $69\frac{1}{8}$.
10. $5\frac{1}{2}$ quarts of nuts, at $9\frac{3}{4}$ cts. per quart.
11. $2\frac{3}{5}$ yards of cloth, at $\frac{7}{8}$ of a dollar per yd. *Ans.* $2.27\frac{1}{2}$.
12. 9 barrels of vinegar, at $6\frac{3}{8}$ per bbl.
13. 15 lbs. of almonds, at $9\frac{1}{2}$ cts. per lb. *Ans.* $1.42\frac{1}{2}$.
14. $8\frac{3}{8}$ yds. of cloth, at $5 per yard.
15. 15 yds. of ribbon, at $26\frac{3}{5}$ cts. per yd. *Ans.* $3.99.
16. $7\frac{3}{4}$ lbs. of coffee, at $\frac{3}{5}$ of a dollar per lb.
17. $8\frac{3}{5}$ cords of wood, at $2\frac{3}{5}$ per cord. *Ans.* $22\frac{21}{40}$.
18. 12 cords of wood, at 6.37\frac{1}{2}$ per cord.
19. 42 bush. of apples, at $63\frac{3}{7}$ cts. per bush. *Ans.* $26.58.
20. 11 cwt. of sugar, at $9\frac{3}{5}$ per cwt.
21. $7\frac{3}{4}$ doz. of eggs, at $12\frac{1}{2}$ cts. per doz. *Ans.* $0.96\frac{7}{8}$.
22. $11\frac{2}{7}$ bbls. of salmon, at $8\frac{4}{5}$ per bbl.
23. $12\frac{4}{5}$ bush. of potatoes, at $37\frac{1}{2}$ cts. per bush. *Ans.*
24. $22\frac{3}{5}$ yds. of selicia, at $87\frac{3}{4}$ cts. per yard.
25. $7\frac{3}{4}$ cords of maple, at $5\frac{3}{8}$ per cord. *Ans.* $41\frac{21}{32}$.
26. $4\frac{3}{8}$ bush. of rye, at $1.75 per bush.
27. $10\frac{3}{5}$ yds. of calico, at $15\frac{3}{5}$ cts. per yd. *Ans.* $1.74\frac{9}{10}$.
28. $35\frac{4}{5}$ lbs. of raisins, at $18\frac{3}{4}$ cts. per lb.
29. $7\frac{3}{8}$ yds. of cloth, at $3\frac{1}{2}$ per yd. *Ans.* $25\frac{13}{16}$.
30. $75\frac{3}{4}$ bush. of wheat, at $1\frac{3}{5}$ per bush.
31. 9 doz. of adzes, at $10\frac{3}{8}$ per doz. *Ans.* $95\frac{5}{8}$.
32. $6\frac{3}{4}$ bush. of turnips, at $37\frac{1}{2}$ cts. per bush.
33. $23\frac{4}{5}$ cords of wood, at $3\frac{3}{8}$ per cord.
34. $75\frac{1}{2}$ lbs. of sugar, at $7\frac{3}{4}$ cts. per lb. *Ans.* $5.85\frac{1}{8}$.
35. $212\frac{3}{4}$ lbs. of beef, at $7\frac{1}{2}$ cts. per lb.
36. $3\frac{4}{5}$ tons of hay, at $12\frac{3}{5}$ per ton.
37. $14\frac{4}{5}$ bbls of vinegar, at $10\frac{2}{5}$ per bbl.
38. $6\frac{3}{7}$ gal. of molasses, at $23\frac{3}{4}$ cts. per gal. *Ans.* $1.52\frac{19}{28}$.
39. 18 handkerchiefs, at $\frac{3}{4}$ of a dollar each.
40. $13\frac{1}{2}$ lbs. of fish, at $9\frac{3}{4}$ cts. per lb.

DIVISION OF FRACTIONS.

163. Case I.—*To divide a fraction by an integer.*

Ex. Divide $\frac{12}{13}$ by 6.

FIRST OPERATION.

$\frac{12}{13} \div 6 = \frac{2}{13}$.

SECOND OPERATION.

$\frac{12}{13} \div 6 = \frac{12}{78} = \frac{2}{13}$.

Analysis.—In the first operation, we divide the numerator of the fraction by 6, and write the quotient, 2, over the denominator.

In the second operation, we multiply the denominator of the fraction by the divisor, 6, and write the product under the numerator, 12. Hence,

Dividing the numerator or multiplying the denominator of a fraction by any number divides the fraction by that number (134).

164. CASE II.—*To divide an integer by a fraction.*

Ex. How many times will 24 contain $\frac{6}{7}$?

FIRST OPERATION.

$24 \div \frac{6}{7} = 168 \div 6 = 28.$

SECOND OPERATION.

$24 \div \frac{6}{7} = 4 \times 7 = 28.$

ANALYSIS.—The integer 24 will contain $\frac{1}{7}$ as many times as there are *sevenths* in 24, equal 168 sevenths. Now, if 24 contains 1 seventh 168 times, it will contain $\frac{6}{7}$ as many times as 168 will contain 6, or 28

In the second operation, we divide the integer by the numerator of the fraction, and multiply the quotient by the denominator, which produces the same result as in the first operation. Hence,

Dividing by a fraction consists in multiplying by the denominator, and dividing by the numerator of the divisor.

165. CASE III.—*To divide a fraction by a fraction.*

Ex. Divide $\frac{7}{8}$ by $\frac{2}{3}$.

OPERATION.

$\frac{7}{8} \div \frac{2}{3} = \frac{7}{8} \times \frac{3}{2} = \frac{21}{16}.$

ANALYSIS.—We invert the terms of the divisor, and then proceed as in multiplication of fractions (162). The reason of this process will be seen, if we consider that the divisor, $\frac{2}{3}$, is an expression denoting that 2 is to be divided by 3. Now, regarding 2 as an integer, we divide the fraction $\frac{7}{8}$ by it, by multiplying the denominator; thus, $\frac{7}{8 \times 2} = \frac{7}{16}$. But the divisor, 2, is 3 times as large as it ought to be, since it was to be divided by 3, as seen in the original fraction; therefore the quotient, $\frac{7}{16}$, is 3 as large as it should be, and must be multiplied by 3; thus, $\frac{7 \times 3}{16} = \frac{21}{16}$, the answer. By this operation we have multiplied the denominator of the dividend by the numerator of the divisor, and the numerator of the dividend by the denominator of the divisor.

From the foregoing we derive the following general

166. RULE.—I. *Reduce integers and mixed numbers to improper fractions.*

II. *Invert the terms of the divisor, and proceed as in multiplication of fractions* (162).

NOTES.—1. The dividend and divisor may be reduced to a common denominator, and the numerator of the dividend be divided by the numerator of the divisor; this will give the same result as the rule.

2. Apply cancellation where practicable.

EXAMPLES FOR PRACTICE.

1. $\frac{6}{7} \div 3 =$ *Ans.* $\frac{2}{7}$.
2. $\frac{5}{6} \div 6.$
3. $4 \div \frac{2}{3} =$ *Ans.* 6.
4. $23 \div \frac{1}{4}.$
5. $\frac{5}{9} \div \frac{1}{6} =$ *Ans.* $3\frac{1}{3}$.
6. $\frac{7}{8} \div \frac{4}{7}.$

166. *What is the* general rule *for dividing fractions?*

7. $17\frac{3}{5} \div 7 =$ *Ans.* $2\frac{8}{35}$.
8. $\frac{5}{27} \div \frac{5}{3}$.
9. $2\frac{1}{2} \div \frac{5}{6} =$ *Ans.* 3.
10. $3\frac{1}{3} \div \frac{10}{21}$.
11. $75 \div 13\frac{4}{5} =$ *Ans.* $5\frac{10}{23}$.
12. $\frac{16}{5} \div \frac{2}{5}$.
13. $7\frac{7}{8} \div 3\frac{1}{3} =$ *Ans.* $2\frac{1}{5}$.
14. $\frac{5}{6} \div 167$.
15. $63 \div \frac{7}{13} =$ *Ans.* 117.
16. $3\frac{1}{2} \div 7\frac{1}{2}$.
17. $\frac{15}{21} \div 28 =$ *Ans.* $\frac{15}{616}$.
18. $\frac{16}{29} \div 49$.
19. $\frac{2}{3} \div \frac{3}{10} =$ *Ans.* $2\frac{2}{9}$.
20. $11\frac{1}{4} \div 5\frac{3}{7}$.
21. $\frac{1}{2} \div \frac{7}{13} =$ *Ans.* $\frac{13}{14}$.
22. $8\frac{1}{4} \div 6\frac{3}{5}$.
23. $\frac{31}{97} \div \frac{31}{3001} =$ *Ans.* $30\frac{1}{97}$.
24. $15 \div \frac{1}{15}$.
25. $19 \div \frac{14}{23} =$ *Ans.* $31\frac{3}{14}$.
26. $\frac{14}{23} \div 19$.
27. $92\frac{5}{6} \div 47 =$ *Ans.* $\frac{1}{2}$.
28. $4\frac{3}{5} \div 17\frac{1}{5}$.
29. $81\frac{1}{7} \div 91\frac{1}{5} =$ *Ans.* $81\frac{32}{161}$.
30. $\frac{9}{25} \div 7\frac{3}{4}$.

31. Divide $\frac{2}{3}$ of $\frac{7}{8}$ by $\frac{1}{7}$ of $\frac{2}{3}$.

$\frac{2}{3} \times \frac{7}{8} = \frac{7}{12}$
$\frac{1}{7} \times \frac{2}{3} = \frac{2}{63}$
$\frac{7}{12} \times \frac{63}{2} = \frac{441}{24} = 18\frac{3}{8}$, *Ans.*

Or,

$\frac{2}{3} \times \frac{7}{8} \times \frac{7}{1} \times \frac{9}{2} = 18\frac{3}{8}$.

ANALYSIS.—The dividend, reduced to a simple fraction, is $\frac{7}{12}$; the divisor, reduced in like manner, is $\frac{2}{63}$; and $\frac{7}{12}$ divided by $\frac{2}{63}$ is $18\frac{3}{8}$, the quotient required. Or, we may apply the general rule directly by inverting both factors of the divisor.

The second method of solution has the twofold advantages of giving the answer by a single operation, and of affording greater facility for cancellation.

32. Divide $\frac{7}{11}$ of $\frac{4}{5}$ by $\frac{8}{5}$ of $\frac{8}{11}$. *Ans.* $1\frac{1}{6}$.
33. Divide $\frac{4}{9}$ of $\frac{5}{11}$ by $\frac{8}{11}$ of $\frac{5}{18}$.
34. Divide $\frac{1}{6}$ of $\frac{5}{6}$ by $\frac{1}{7}$ of $\frac{7}{8}$.
35. Divide $\frac{5}{9}$ of $7\frac{3}{11}$ by $\frac{4}{11}$ of $17\frac{3}{7}$. *Ans.* $\frac{350}{594}$.
36. Divide $\frac{9}{10}$ of 4 by $\frac{5}{8}$ of $3\frac{1}{4}$.
37. Divide $\frac{2}{7}$ of $\frac{14}{16}$ of $\frac{8}{12}$ by $\frac{7}{8}$ of $\frac{16}{14}$ of $\frac{2}{8}$. *Ans.* $\frac{1}{4}$.
38. Divide $\frac{3}{8}$ of $5\frac{1}{2}$ of 7 by $\frac{5}{8}$ of $3\frac{3}{10}$. *Ans.* $11\frac{1}{5}$.
39. Divide $\frac{3}{4}$ of $\frac{5}{7}$ of $\frac{4}{5}$ by $\frac{3}{8}$ of $\frac{6}{7}$ of $\frac{2}{15}$.
40. Divide $\frac{1}{8}$ of $\frac{7}{9}$ of 36 by $1\frac{7}{8}$ of $\frac{3}{5}$.

41. What is the value of $\frac{6\frac{2}{3}}{8\frac{2}{3}}$?

OPERATION.

$$\frac{6\frac{2}{3}}{8\frac{2}{3}} = \frac{\frac{56}{3}}{\frac{26}{3}} = \frac{56}{9} \div \frac{26}{3} = \frac{\overset{28}{\cancel{56}}}{9} \times \frac{3}{\underset{13}{\cancel{26}}} = \frac{28}{39}, \text{ Ans.}$$

OBS.—This example is only another form for expressing division of fractions; it is called a *complex fraction*. We simply reduce the upper number or dividend to an improper fraction, and the lower number, or divisor, to an improper fraction, and then divide as before.

42. What is the value of $\frac{4\frac{3}{7}}{\frac{2}{3}}$? *Ans.* $6\frac{9}{14}$.
43. What is the value of $\frac{\frac{5}{11}}{4\frac{2}{5}}$? *Ans.* $\frac{25}{242}$.
44. What is the value of $\frac{7\frac{4}{11}}{8}$?

45. What is the value of $\frac{\frac{3}{5} \text{ of } \frac{3}{4}}{\frac{1}{2}}$? *Ans.* 1.

46. What is the value of $\frac{4\frac{1}{2}}{\frac{2}{7} \text{ of } 3\frac{1}{4}}$?

47. What is the value of $\frac{8}{\frac{7}{8} \text{ of } \frac{4}{7}}$?

48. What is the value of $\frac{\frac{2}{5} \times \frac{10}{12}}{4\frac{1}{2} \text{ of } \frac{2}{9}}$?

49. Reduce $\frac{\frac{5}{9} \text{ of } \frac{3}{7}}{6\frac{1}{5} - 5\frac{4}{15}}$ to its simplest form. *Ans.* $\frac{25}{56}$.

50. Reduce $\frac{\frac{3}{5} \times 5\frac{1}{2} \times 3\frac{1}{3}}{7\frac{1}{8} \times \frac{6}{11} \times \frac{5}{16}}$ to its simplest form.

PRACTICAL PROBLEMS.

1. If $\frac{3}{4}$ of an acre of land sell for \$63, what will an acre sell for at the same rate? *Ans.* \$147.

2. At \$$\frac{3}{4}$ per bushel, how many bushels of onions can be bought for \$12? *Ans.* 16.

3. How many times will $16\frac{3}{4}$ gallons of vinegar fill a vessel that holds 3 gallons? *Ans.* $5\frac{7}{12}$.

4. At $\frac{4}{5}$ of a cent each, how many apples can be bought for $8\frac{4}{5}$ cents? *Ans.* 11.

5. If 15 pounds of raisins can be obtained for \$$3\frac{2}{7}$, what will 1 pound cost? *Ans.* \$$0.21\frac{12}{21}$.

6. A butcher expended \$$56\frac{2}{5}$ for sheep, giving \$$1\frac{1}{5}$ per head; how many sheep did he buy? *Ans.* 47.

7. At \$5 per yard of broadcloth, what part of a yard can be bought for $\frac{6}{7}$ of a dollar? *Ans.* $\frac{6}{35}$.

8. If I pay $5\frac{2}{3}$ cents for riding 1 mile, how many miles can I ride for $113\frac{1}{3}$ cents? *Ans.* 20.

9. How many pounds of tea, at \$$1\frac{1}{5}$ per pound, can be obtained for \$$13\frac{1}{2}$? *Ans.* 12.

10. If 9 men consume $\frac{3}{4}$ of $9\frac{3}{5}$ pounds of meat in a day, how much does each man consume? *Ans.* $\frac{4}{5}$ of a lb.

11. A man bought $37\frac{2}{5}$ yards of calico for \$5.61, how much did it cost per yard? *Ans.* \$0.15.

12. How many tons of coal, at \$$5\frac{3}{4}$ per ton, can be bought for \$57?

13. A horse eats $\frac{3}{8}$ of a bushel of oats in a day, in how many days will he eat $15\frac{3}{4}$ bushels? *Ans.* 42.

14. A merchant bought 97 sheep for \$$100\frac{2}{25}$, how much did he give per head? *Ans.* \$1.04.

15. If a boy earn $\frac{3}{8}$ of a dollar a day, how many days will it take him to earn \$$9\frac{3}{4}$? *Ans.* 26.

16. Peter paid \$$548\frac{3}{4}$ for a farm, giving \$$21\frac{3}{4}$ per acre; of how many acres did the farm consist? *Ans.* 25.

17. If \$$2\frac{3}{5}$ is paid for $5\frac{1}{5}$ pounds of grapes, how much is that per pound? *Ans.* \$0.50.

18. How many tons of hay can be purchased for $119\frac{1}{16}$, at $9\frac{3}{8}$ per ton? *Ans.* $12\frac{7}{10}$.

19. At $1\frac{2}{5}$ per day, how many days must a man work for $37\frac{1}{10}$?

20. At $\frac{1}{4}$ of $\frac{3}{4}$ of a dollar per gallon, how much beer can be bought for $\frac{9}{10}$? *Ans.* $2\frac{2}{5}$ gal.

21. If $2\frac{1}{2}$ apples are worth $3\frac{1}{8}$ cents, what part of an apple can you get for 1 cent? *Ans.* $\frac{3}{4}$.

22. If 2 yards of merino cost $3\frac{1}{4}$, how much less than $17 will 9 yards cost? *Ans.* $2\frac{3}{8}$.

23. If 3 turkeys cost $4\frac{1}{8}$, how many can be bought for $38\frac{1}{2}$?

24. If 3 horses eat $3\frac{3}{5}$ bushels of oats in a day, how many horses would $8\frac{2}{3}$ bushels supply for the same time? *Ans.* 7.

25. A young man, having $10. gave $\frac{2}{3}$ of his money for paper at $3\frac{1}{3}$ per ream; how much did he buy? *Ans.* 2 reams.

26. How many feet of carpet $2\frac{2}{3}$ feet in width will be required to cover a floor $14\frac{1}{2}$ feet in length and $10\frac{2}{3}$ feet in width? *Ans.*

27. How many bottles will be required to hold $8\frac{4}{15}$ gal. of wine, if each bottle will hold $\frac{1}{5}$ of $\frac{2}{3}$ of $\frac{4}{5}$ of 1 gal.? *Ans.* 31.

28. If 5 barrels of flour cost $48\frac{3}{4}$, how many barrels can be purchased for $263\frac{1}{4}$? *Ans.* 27.

29. How much more than $8\frac{3}{4}$ yards of tape, at 4 cts. a yard, will $4\frac{1}{4}$ yards of calico cost at 11 cts. a yard? *Ans.* $0.11\frac{3}{4}$.

30. A farmer gave $46 for some calves, 5 of which he sold for $5\frac{3}{5}$ each, and traded the rest for $5\frac{1}{2}$ bbls. of flour, at $4 a bbl.; how much did he gain? *Ans.* $4.

GREATEST COMMON DIVISOR OF FRACTIONS.

167. The **Greatest Common Divisor** of two or more fractions is the greatest number which will exactly divide each of them, giving a whole number for a quotient.

168. *To find the greatest common divisor of two or more fractions.*

Ex. What is the greatest common divisor of $3\frac{1}{5}$, $1\frac{5}{7}$, and $\frac{24}{35}$?

OPERATION.

$$3\tfrac{1}{5},\ 1\tfrac{5}{7},\ \tfrac{24}{35} = \tfrac{16}{5},\ \tfrac{12}{7},\ \tfrac{24}{35} = \tfrac{112}{35},\ \tfrac{60}{35},\ \tfrac{24}{35}.$$

$$\left.\begin{array}{l}\text{Greatest common divisor of the numerators} = 4\\ \text{Least common denominator of the fractions} = 35\end{array}\right\}\ \text{Greatest common divisor required.}$$

ANALYSIS.—Having reduced the fractions to equivalent fractions having the least common denominator, we find the greatest common divisor of the numerators 112, 60, and 24 to be 4. Now, since the 112, 60, and 24 represent *thirty-fifths*, their greatest common divisor is not 4, a whole number, but 4 thirty-fifths; therefore we write the 4 over the least common denominator, 35, and have $\frac{4}{35}$ as the answer.

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167. *What is the* greatest common divisor *of fractions?*

169. RULE.—*Reduce the fractions, if necessary, to their least common denominator. The greatest common divisor of the numerators, written over the least common denominator, will give the greatest common divisor required.*

EXAMPLES FOR PRACTICE.

Required the greatest common divisor of

1. $\frac{2}{3}$, $\frac{12}{15}$, and $\frac{4}{9}$. *Ans.* $\frac{2}{45}$.
2. $\frac{3}{4}$, $\frac{1}{4}$, and $1\frac{1}{8}$.
3. $\frac{14}{27}$, $\frac{7}{9}$, and $\frac{28}{45}$. *Ans.* $\frac{7}{135}$.
4. $\frac{4}{7}$, $\frac{8}{9}$, $\frac{16}{21}$, and $\frac{4}{5}$.
5. $3\frac{4}{5}$, $5\frac{7}{10}$, and $2\frac{8}{15}$. *Ans.* $\frac{1}{30}$.
6. $2\frac{1}{4}$, 4, $\frac{15}{16}$, and $5\frac{1}{3}$.
7. $8\frac{1}{4}$, $12\frac{3}{8}$, and $9\frac{3}{4}$. *Ans.* $\frac{3}{8}$.
8. $2\frac{2}{3}$, 4, $\frac{4}{30}$, and $2\frac{2}{5}$.

LEAST COMMON MULTIPLE OF FRACTIONS.

170. The **Least Common Multiple** of two or more fractions is the least number which can be exactly divided by each of them, giving a whole number for a quotient.

171. *To find the least common multiple of two or more fractions.*

Ex. What is the least common multiple of $7\frac{7}{8}$, $5\frac{1}{4}$,, and $3\frac{15}{16}$?

OPERATION.

$$7\tfrac{7}{8},\ 5\tfrac{1}{4},\ 3\tfrac{15}{16} = \tfrac{63}{8},\ \tfrac{21}{4},\ \tfrac{63}{16}.$$

Least common mult. of the numer. $= 63$
Greatest com. div. of the denom. $= 4$
$\Big\}$ Least common multip. required. $= \frac{63}{4} = 15\frac{3}{4}$

ANALYSIS.—Having reduced the fractions to their simplest form, we find the least common multiple of the numerators, 63, 21, and 63, to be 63. Now, since the 63, 21, and 63 are, from the nature of a fraction, dividends, of which their respective denominators, 8, 4, and 16, are the divisors (118), the least common multiple of the fractions is not 63, a whole number, but so many fractional parts of the greatest common divisor of the denominators. This common divisor we find to be 4, which, written as the denominator of the 63, gives $\frac{63}{4} = 15\frac{3}{4}$ as the least number that can be exactly divided by the given fractions.

172. RULE.—*Reduce the fractions, if necessary, to their lowest terms. Then find the least common multiple of the numerators, which, written over the greatest common divisor of the denominators, will give the least common multiple required.* Or,

Reduce the fractions, if necessary, to their least common denominator. Then find the least common multiple of the numerators, and write it over the least common denominator.

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169. *What is the* rule *for finding the greatest common divisor of fractions?*— 170. *What is the* least common multiple *of fractions?*— 172. *What is the* rule *for finding the least common multiple of fractions?*

EXAMPLES FOR PRACTICE.

Required the least common multiple of

1. $\frac{6}{16}$, $\frac{3}{4}$, and $2\frac{1}{16}$. *Ans.* $8\frac{1}{4}$.
2. $\frac{6}{7}$, $\frac{15}{35}$, and $\frac{10}{28}$.
3. $\frac{8}{9}$, $\frac{6}{7}$, and $\frac{4}{5}$. *Ans.* 24.
4. $\frac{7}{10}$, $\frac{14}{15}$, $\frac{8}{25}$, and $\frac{2}{5}$.
5. $5\frac{1}{4}$, $\frac{2}{12}$, and $1\frac{1}{2}$. *Ans.* $10\frac{1}{2}$.
6. $1\frac{3}{7}\frac{7}{5}$, $\frac{63}{100}$, and $2\frac{2}{2}\frac{2}{5}$.
7. $\frac{15}{16}$, $\frac{3}{4}$, and $\frac{5}{12}$. *Ans.* $3\frac{3}{4}$.
8. $\frac{1}{11}$, 5, $6\frac{1}{3}$, $\frac{1}{15}$, and $2\frac{1}{2}$.

PRACTICE, OR ANALYSIS BY ALIQUOT PARTS.

173. An **Aliquot Part** of any number or quantity is such a part as will exactly divide that number or quantity; thus, 2, 3, 4, and 6 are aliquot parts of 12.

NOTE.—An *aliquot part* may be a whole or a mixed number, while a factor must be a whole number.

ALIQUOT PARTS OF ONE DOLLAR.

50 cents = $\frac{1}{2}$ of 1 dollar.
$33\frac{1}{3}$ cents = $\frac{1}{3}$ of 1 dollar.
25 cents = $\frac{1}{4}$ of 1 dollar.
20 cents = $\frac{1}{5}$ of 1 dollar.
$16\frac{2}{3}$ cents = $\frac{1}{6}$ of 1 dollar.
$12\frac{1}{2}$ cents = $\frac{1}{8}$ of 1 dollar.
10 cents = $\frac{1}{10}$ of 1 dollar.
$8\frac{1}{3}$ cents = $\frac{1}{12}$ of 1 dollar.
$6\frac{1}{4}$ cents = $\frac{1}{16}$ of 1 dollar.
5 cents = $\frac{1}{20}$ of 1 dollar.

174. *To find the cost of any number or quantity, when the price of a unit is an aliquot part of one dollar.*

Ex. At $12\frac{1}{2}$ cents a yard, what will 416 yards of muslin cost?

OPERATION.

8) 416
Ans. $52

ANALYSIS.—If the price were $1 a yard, the cost would be as many dollars as there are yards. But since the price is $\frac{1}{8}$ of a dollar a yard, the whole cost will be $\frac{1}{8}$ as many dollars as there are yards; or, $\frac{1}{8}$ of 416 = 416 ÷ 8 = $52. Hence, the

175. RULE.—*Take such a fractional part of the given number as the price is part of one dollar.*

EXAMPLES FOR PRACTICE.

1. What will be the cost of 724 pounds of coffee at $33\frac{1}{3}$ cts. a pound? *Ans.* $241.33\frac{1}{3}$.
2. What cost 376 yards of calico, at 25 cts. per yd.?
3. At $6\frac{1}{4}$ cts. a pound, what will 1056 lbs. of nails cost? *A.* $66.
4. At $8\frac{1}{3}$ cts. a dozen, what cost 387 doz. of eggs?
5. What cost 384 yards of cloth, at $4.33\frac{1}{3}$ per yd.? *Ans.* $1664.
6. At $3.16\frac{2}{3}$ each, what will 93 hats cost?

173. *What is an* aliquot part of a number?— 175. *What is the* rule *for finding the cost of any number or quantity, when the price of a unit is an aliquot part of a dollar?*

QUESTIONS

INVOLVING THE RELATION OF PRICE, COST, AND QUANTITY.

176. CASE I.—The price and the quantity being given, to find the cost.

ANALYSIS.—The cost of 5 units must be 5 times the price of 1 unit; of 6 units, 6 times the price of 1 unit; of $\frac{5}{6}$ of a unit, $\frac{5}{6}$ times the price of 1 unit, etc. Hence, the

177. RULE.—*Multiply the price of* ONE *by the quantity.*

178. CASE II.—The cost and the quantity being given, to find the price.

ANALYSIS.—By Case I, the cost is the product of the price multiplied by the quantity. Now, having the cost, which is a product, and the quantity, which is one of two factors, we have the product and one of two factors given, to find the other factor. Hence, the

179. RULE.—*Divide the cost by the quantity.*

180. CASE III.—The price and the cost being given, to find the quantity.

ANALYSIS.—Reasoning as in Case II, we find that the cost is the product of two factors, and the price is one of the factors. Hence, the

181. RULE.—*Divide the cost by the price.*

182. CASE IV.—The quantity, and the price of 100 or 1000, being given, to find the cost.

ANALYSIS.—If the price of 100 units be multiplied by the number of units in a given quantity, the product will be 100 times the required result, because the multiplier used is 100 times the true multiplier. For a similar reason, it will be the same if the given price be 1000 units. The true value will be obtained either by dividing the product by 100 or 1000, as the case may be, or, by reducing the given quantity to hundreds and decimals of a hundred, or to thousands and decimals of a thousand. Hence, the

183. RULE.—I. *Reduce the given quantity to hundreds and decimals of a hundred, or to thousands and decimals of a thousand.*

II. *Multiply the price by the quantity, and point off in the result as in multiplication of decimals.*

184. CASE V.—To find the cost of articles sold by the ton of 2000 pounds.

ANALYSIS.—If the price of 1 ton or 2000 pounds be divided by 2, the quotient will be the price of $\frac{1}{2}$ ton or 1000 pounds. We then have the quantity and the price of 1000 to find the cost. Hence, the

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177. *What is the* rule *for finding the cost of articles, the price and the quantity being given?*— 179. *For finding the price of articles, the cost and the quantity being given?*—181. *For finding the quantity, the price and the cost being given?*—183. *For finding the cost of articles, the quantity, or the price of* 100 *or* 1000, *being given?*

185. RULE.—*Divide the price of* 1 *ton by* 2, *and multiply the quotient by the number of pounds expressed as thousandths.*

EXAMPLES FOR PRACTICE IN THE PRECEDING CASES.

1. At $7.50 per barrel, how many barrels of flour can be obtained for $217.50? *Ans.* 29 barrels.

2. If 1 yard of calico cost 23 cents, what will 31½ yards cost?

3. What cost 15 tubs of butter, each containing 70½ lbs., at $\$\frac{3}{16}$ a pound?

4. What is the freight on 1244½ pounds from Montreal to Quebec, at $0.85 per 100 lbs.? *Ans.* $10.578+.

5. If board for a family be $342.18¾ for 1 year, how much is it per day? *Ans.* $0.93¾.

6. How many dozen of eggs can be bought for $9.24, at 10½ cts. a dozen? *Ans.* 88.

7. What will 3921 feet of pine boards cost, at $17.25 per 1000?

8. What is the value of 210 kegs of nails, each weighing 162½ lbs., at $\$17\frac{3}{4}$ a ton?

9. At $1½ a bushel, how many bushels of oats can be bought for $113.06¼? *Ans.* 75⅜ bushels.

10. At 5 cents a pound, how many barrels of codfish, each containing 90 lbs., can be purchased for $94.50? *Ans.* 21 bbls.

11. What will be the cost of 1620 apple trees at $16½ per hundred?

12. At 37½ cts. a bushel, what will ⅜ of 456 bushels of potatoes cost?

13. How much must be paid for 486 feet of boards, at $20.25 per 1000; 787½ feet of scantling, at $2.87½ per 100; and 4378 feet of lath, at $7.50 per 1000? *Ans.* $65.317½.

14. What will be the cost of 4344 pounds of Paris plaster, at $3.87½ per ton? *Ans.* $8.416½.

15. If 32½ barrels of Montreal apples cost $97.50, what is the price per barrel? *Ans.* $3.

16. How many acres of land can be bought for $2117.18¾, at $\$5\frac{5}{8}$ an acre? *Ans.* $376\frac{7}{16}$.

17. At 37½ cts. per bushel, how many barrels of potatoes, each containing 2½ bushels, can be purchased for $50.62½? *Ans.* 54.

18. If .625 of a barrel of eels be worth $6.42, what is a barrel worth?

19. What must be paid for 523 lbs. of meat, at $\$4\frac{5}{8}$ per hundred pounds? *Ans.* $24.18⅞.

20. What cost 1080 lbs. of hay, at $12.75 a ton, and 1368 lbs. of mill feed at $15.50 a ton? *Ans.* $17.487.

21. What will be the cost of 654 feet of boards, at $15½ per 1000; 1344 feet of siding, at $1.62½ per 100; and 2216 bricks, at $4¼ per 1000? *Ans.* $41.39½.

22. A grocer bought 108 gallons of oil for $145.80, and lost 12 gal. of it by leakage. He sold the remainder at $1.70 per gallon; how much did he gain?

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185. *What is the* rule *for finding the cost of articles by the ton of* 2000 *lbs.?*

23. A lumber dealer bought 106250 feet of lumber at \$14.375 per 1000, and retailed it out at \$1.75 per 100; how much was his whole gain? *Ans.* \$332.03+.

24. A load of plaster weighing 3360 pounds cost \$5.71$\frac{1}{5}$, how much will a ton cost?

25. If \$6.97$\frac{1}{2}$ be paid for 0.93 of a hundred pounds of beef, how much will one hundred pounds cost?

26. A farmer exchanged 42$\frac{3}{4}$ bushels of barley worth 37$\frac{1}{2}$ cts. per bushel, and 679$\frac{1}{2}$ lbs. of hay worth 75 cts. per hundred, for 18780 lbs. of plaster; how much was the plaster worth per ton?

27. If 42 yards of cassimere cost \$147, what will be the cost of 34$\frac{4}{5}$ yards? *Ans.* \$121.80.

28. What is the value of 12 pieces of black cloth, each piece containing 27$\frac{3}{8}$ yards, worth \$2$\frac{7}{8}$ a yard? *Ans.* \$954.50.

29. At \$$\frac{7}{8}$ per bushel, how many bushels of wheat may be bought for \$18.90? *Ans.* 21$\frac{3}{5}$.

30. A farmer sold to a merchant three loads of hay weighing respectively 2739, 2217, and 2881$\frac{1}{2}$ lbs., at \$8.80 per ton, and 421$\frac{1}{8}$ lbs. of pork, at \$5.25 per hundred. He received in exchange 46$\frac{1}{2}$ yards of muslin at \$0.09, 9$\frac{3}{4}$ yards of carpet at \$4.50, and the balance in money; how much money did he receive?

Let the pupils make out, in proper form, as the case may be, the following:

1. Sold by R. S. Graham, Montreal, to E. Dudley, as follows: 1870, Jan. 3, 109$\frac{1}{2}$ yds. calico, at 18$\frac{1}{4}$ cts.; Feb 11, 430 yds. muslin, at 15$\frac{1}{2}$ cts.; March 2, 37$\frac{1}{4}$ yds. sheeting, at 23$\frac{1}{4}$ cts.; May 16, 75$\frac{3}{5}$ yds. Irish linen, at 42 cts.; 43$\frac{1}{8}$ yds. lace, at 78$\frac{3}{4}$ cts.

Footing of the bill, \$161.007+.

2. T. E. Clark bought of F. Larose & Co., Quebec: 1870, June 10, 73$\frac{1}{2}$ gal. Irish whisky, at 86 cts.; 108$\frac{1}{4}$ gal. fine old rum, at \$2.12$\frac{1}{2}$; 67$\frac{5}{8}$ gal. Holland gin, at \$1.45; Aug. 14, 89$\frac{1}{2}$ gal. old cognac, at \$2.67$\frac{1}{2}$; 107 gal. brandy, at \$1.37$\frac{1}{2}$; Sept. 7, 201$\frac{1}{8}$ gal. Scotch gin, at \$1.20. T. E. Clark gave in part payment, Sept. 11, 4 chests green tea, each 67$\frac{1}{2}$ lbs., at 56 cts. per lb. What balance was due F. L. & Co., Sept. 12, when the bill was made out? *Ans.* \$867.71$\frac{1}{4}$.

3. J. N. Webster, butcher, Kingston, sold to A. O'Regan, Oct. 6, 1870: A fillet of veal, weight 16$\frac{3}{4}$ lbs., at 10$\frac{1}{2}$ cts.; a loin of lamb, weight 7$\frac{1}{4}$ lbs., at 17$\frac{3}{4}$ cts.; a leg of mutton, weight 13$\frac{3}{4}$ lbs., at 6$\frac{3}{4}$ cts.; a leg of pork, weight 16$\frac{1}{4}$ lbs., at 9$\frac{1}{2}$ cts.; a pig, weight 24$\frac{1}{2}$ lbs., at 12$\frac{1}{4}$ cts.; a buttock of beef, weight 37$\frac{1}{4}$ lbs., at 7$\frac{1}{2}$ cts.

Footing of the bill, \$11.31$\frac{1}{4}$.

4. E. Lemay & Co. bought of Messrs. A. Roche & Son, Toronto, Sept. 3, 1870: 123$\frac{3}{5}$ lbs. gum lac, at \$1.15; 65$\frac{5}{8}$ lbs. quinquina, at \$14.10; 107$\frac{1}{4}$ lbs. rhubarb, at \$2.40; 120$\frac{1}{8}$ lbs. sassafras, at 11$\frac{1}{4}$ cts.; 356$\frac{1}{2}$ lbs. mastic, at 21$\frac{3}{4}$ cts. Footing of the bill, \$1415.91$\frac{3}{8}$.

5. Sold by B. H. Porter, Ottawa, to Miss D. Valcour, Aug. 20, 1870: 27$\frac{5}{8}$ yds. Dresden lace, at \$3.09; 19$\frac{1}{4}$ yds. Flanders lace, at \$1.62$\frac{1}{2}$; 83$\frac{1}{2}$ yds. gauze, at 45$\frac{1}{4}$ cts.; 36$\frac{1}{4}$ yds. muslin, at 18$\frac{1}{2}$ cts.; 50 pair kid gloves, at 32 cts.; 25$\frac{5}{8}$ dozen napkins, at \$6.12$\frac{1}{2}$.

Footing of the bill, \$335.36$\frac{1}{8}$

6. Invoiced, per Canadian Express, by S. Blanchard & Co., Quebec, to J. Butler, Kingston, July 6, 1870: 25 sacks tares, No. 3, each $2\frac{1}{2}$ bush., at 54 cts. per bush.; 32 sacks pease, No. 4, each 3 bush., at $87\frac{1}{2}$ cts. per bush.; 20 sacks oats, No. 6, each $3\frac{1}{3}$ bush., at $56\frac{1}{2}$ cts. per bush.; 8 sacks malt, No. 5, each $2\frac{3}{4}$ bush., at $\$1.37\frac{1}{2}$ per bush.; 16 sacks beans, No. 7, each $2\frac{1}{2}$ bush., at 85 cts. per bush. Insurance and cartage, $3.40. Amount of Invoice, $221.56.

7. T. McCullen & Co., wholesale merchants, Halifax, sold to Lenoir & O'Neil, Montreal, as follows: May 19, 1870, 85 pieces Norwich crapes, at $8.32; 102 pieces Liverpool cottons, at $7.63; June 5, $175\frac{1}{3}$ yds. Antwerp sheeting, at $24\frac{3}{4}$ cts.; $698\frac{3}{8}$ yds. Amiens velvet, at $1.80; Aug. 8, $375\frac{1}{8}$ yds. Yorkshire drab, at 65 cts.; $872\frac{3}{4}$ yds. Abbeville merino, at $\$1.12\frac{1}{2}$. On this are the following credits: July 10, by 18 bbls. Canadian flour, at $7.50; Aug. 12, by draft, at 3 day's sight, for $500. What balance was due T. McC. & Co., Sept. 3, when the account was settled? *Ans.* $3377.01.

8. C. N. Stonehouse of Montreal, sold to Mrs. F. Stephens, April 6, 1870, and Ed. Noonan, his clerk, collected the amount of the bill: $39\frac{1}{4}$ yds. camblet, at $31\frac{1}{4}$ cts.; $47\frac{3}{8}$ yds. shalloon, at 32 cts.; $27\frac{3}{4}$ yds. drugget, at $46\frac{1}{2}$ cts.; $19\frac{1}{8}$ yds. calico, at $11\frac{1}{2}$ cts.; $41\frac{1}{2}$ yds. chintz, at $90\frac{1}{2}$ cts.; $34\frac{1}{4}$ yds. calimanco, at $37\frac{1}{2}$ cts. Amt. of the bill, $\$93.02\frac{13}{16}$.

9. L. Rogers & Son, Quebec, sold to Messrs. O. Cooper & Co., Sorel, as follows: 1870, April 5, $12\frac{1}{2}$ doz. palm sack, at $9.42; May 12, Port wine, red, $65\frac{3}{8}$ gal., at $1.68; $42\frac{1}{2}$ gal. Claret, at $\$2.17\frac{1}{4}$; June 10, Lisbon wine, white, $31\frac{1}{4}$ gal., at 45 cts.; $32\frac{3}{4}$ gal. Rhenish wine, at $56\frac{1}{2}$ cts.; July 8, $25\frac{1}{4}$ gal. Sherry wine, at $1.33. Received in part payment, July 9, 150 bush. oats, at $57\frac{1}{2}$ cts., and $60 in cash. What was the balance due to L. R. & Son, July 10? *Ans.* $240.63.

10. T. J. Rinfret, bought of Tessier & Gray, Montreal, as follows: 1870, June 18, $4\frac{1}{2}$ pieces muslin, each $37\frac{1}{4}$ yds., at $2.15; $7\frac{3}{4}$ pieces chintz, each $47\frac{1}{2}$ yds., at $92\frac{1}{2}$ cts.; July 12, $4\frac{1}{2}$ pieces Holland linen, each $25\frac{1}{2}$ yds., at $57\frac{1}{4}$ cts.; $10\frac{3}{8}$ pieces serge, each $19\frac{1}{3}$ yds., at 48 cts.; Aug. 2, $1749\frac{1}{8}$ yds., Rendal cottons, at 17 cts.; $947\frac{1}{4}$ yds. Lowell cotton, at $18\frac{1}{2}$ cts. What was the amount due, Aug. 3, to T. & G? *Ans.* $\$1335.47\frac{5}{8}$.

MISCELLANEOUS PROBLEMS.

1. What will be the cost of $15\frac{1}{2}$ pounds of honey, at $16\frac{2}{3}$ cts. per pound? *Ans.* $\$2.58\frac{1}{3}$.

2. At $\$4\frac{1}{2}$ per yard, how many yards may be bought for $\$11\frac{1}{4}$?

3. Reduce $\frac{179}{14}$ to a mixed number. *Ans.* $12\frac{11}{14}$.

4. Reduce $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{5}$, and $\frac{7}{8}$ to equivalent fractions having a common denominator. *Ans.* $\frac{60}{120}$, $\frac{80}{120}$, $\frac{72}{120}$, $\frac{105}{120}$.

5. The less of two numbers is $2378\frac{2}{9}$, and their difference, $64\frac{3}{5}$; what is the greater number? *Ans.* $2442\frac{3}{7}\frac{3}{2}$.

6. At $28\frac{4}{7}$ cts. per bushel, how many bushels of barley may be bought for $16\frac{2}{3}$ cts.? *Ans.* $\frac{7}{12}$ bush.

7. John has $6\frac{1}{3}$ times $\$9\frac{3}{4}$, James has $2\frac{1}{7}$ times $\$8\frac{5}{6}$; how much more has John than James? *Ans.* $\$44\frac{25}{56}$.

8. What will $15\frac{1}{2}$ cords of wood cost at $\frac{1}{8}$ of $9\frac{1}{2}$ per cord?

9. How many pounds in 4 bags, the first containing $366\frac{7}{8}$, the second $580\frac{4}{5}$, the third $296\frac{3}{4}$, and the fourth $375\frac{9}{10}$? *Ans.* $1614\frac{13}{40}$.

10. Andrew spent $\frac{2}{5}$, $\frac{1}{8}$, and $\frac{1}{8}$ of his money, and had $54.50 left; how much had he at first? *Ans.* 384.70\frac{1}{2}$.

11. A servant had $\frac{1}{5}$ of his savings in one bank, $\frac{1}{4}$ in another, and the remainder, which was $77, in a third bank; how much money had he? *Ans.* $140.

12. Leo had $\frac{4}{5}$ of $\frac{5}{6}$ of $7\frac{1}{2}$ times $7862, and paid $\frac{1}{2}$ of $\frac{1}{5}$ of it for a farm; how much had he remaining? *Ans.* $35379.

13. In 5 hogheads of sugar containing, respectively, $945\frac{1}{2}$ lbs., $1054\frac{9}{10}$ lbs., $963\frac{3}{4}$ lbs., $901\frac{3}{8}$, and $899\frac{5}{8}$, how many pounds?

14. Henry bought a bale of cloth for $96.37\frac{1}{2}$; he disposes of it for $\frac{5}{8}$ of the cost, and by so doing, loses $2 on a yard; required the number of yards in the bale. *Ans.* $18\frac{8}{12\frac{1}{2}}$.

15. What is the value of $376\frac{11}{18}$ acres of land, at 75\frac{3}{8}$ per acre?

16. If the transportation of $18\frac{3}{8}$ tons of iron costs 48.15\frac{3}{4}$, what is it per ton? *Ans.* 2.62\frac{4}{9}$.

17. A man purchased $\frac{7}{8}$ of a yard of velvet at the rate of 3.62\frac{1}{2}$ per yard; what did it cost him? *Ans.* 3.17\frac{3}{16}$.

18. Charles has 634 sheep, which is 94 more than $\frac{2}{3}$ of $3\frac{1}{3}$ times David's number; how many has David? *Ans.* 243.

19. A man travels 4 miles in $\frac{2}{3}$ of an hour, how far will he travel in $3\frac{1}{2}$ hours at the same rate? *Ans.* 10 miles.

20. A merchant owned $\frac{3}{4}$ of a ship, and sold $\frac{1}{2}$ of $\frac{3}{5}$ of his share for $2400. At that rate, what was the whole worth? *Ans.* $19200.

21. What will $\frac{1}{2}$ of $10\frac{7}{8}$ tons of coal cost, at $\frac{4}{5}$ of $42 per ton?

22. If $\frac{1}{5}$ of $\frac{3}{4}$ of $3\frac{1}{5}$ be multiplied by $\frac{1}{2}$ of itself, and the product divided by $\frac{1}{2}$, what will be the result? *Ans.* $\frac{225}{1024}$.

23. B and C own 3144 sheep; how many has each, if B has $1\frac{3}{5}$ times as many as C? *Ans.* B 1834, C 1310.

24. Edward has $\frac{3}{4}$ of a dollar; he gives Louis $\frac{1}{3}$ of this amount, and then divides the remainder equally among three boys; what part does each of the 3 boys receive? *Ans.* $\frac{1}{8}$.

25. James obtains from two fields 344 bushels of oats; if the first yielded $\frac{3}{5}$ as much as the second, required the yield of each field?

26. How long will it take a man to travel 553 miles, provided he travels $3\frac{1}{2}$ miles per hour, and $9\frac{7}{8}$ hours per day? *Ans.* 16 days.

27. I bought 15 loads of wood, each containing $11\frac{2}{5}$ feet, cord measure, and divided it equally among 9 persons: what did each receive?

28. A tree, whose length was 136 feet, was broken into two pieces by falling; $\frac{3}{5}$ of the length of the longer piece equaled $\frac{3}{4}$ of the length of the shorter. What was the length of each piece? *Ans.* 72 and 64 ft.

29. How many bushels of wheat worth 80 cts. a bushel, will pay for $\frac{5}{8}$ of a barrel of flour at $7\frac{1}{2}$ a barrel? *Ans.* $7\frac{1}{2}$ bush.

30. Bought $\frac{3}{8}$ of $\frac{5}{9}$ of $5\frac{1}{2}$ yards of blue cloth at the rate of $3.50 per yard; what is the cost? *Ans.* 8.02\frac{1}{2}$.

31. If $\frac{5}{8}$ of a barrel of eels costs $5, how much will 2 tubs of eels cost, one containing $\frac{2}{3}$ of a barrel, and the other $\frac{5}{6}$ of a barrel?

32. If $\frac{3}{5}$ of a gal. of porter is worth $\frac{3}{4}$ of a gal. of ale, and ale is worth $\frac{2}{3}$ per gal., how many gal. of porter will $20 buy? *Ans.* 24.

33. A certain quantity of apples is to be divided among 5 boys; William is to have $\frac{1}{4}$, John $\frac{1}{5}$, Peter $\frac{1}{10}$, Thomas $\frac{1}{20}$, and Paul the remainder, which is 24; what is the whole quantity to be divided?

34. What will be the cost of $7\frac{1}{2}$ yds. of calico, at $12\frac{1}{2}$ cts. per yd., and $12\frac{1}{2}$ yds. of muslin, at $18\frac{3}{4}$ cts. per yard? *Ans.* 3.28\frac{1}{8}$.

35. Philip owns $\frac{3}{17}$ of a ship's cargo, valued at $493000; Daniel owns $\frac{15}{28}$ of the remainder; Joseph owns $\frac{3}{10}$ as much as Philip and Daniel; and Henry owns the remainder. How much does each own? *Ans.* P, owns $87000; D, $210000; J, $89100; and H, $106900.

36. I own $\frac{7}{8}$ of a steamboat, and sell $\frac{3}{4}$ of my share to Owen for $45000. What part of the steamboat have I left, and what is it worth at that rate? *Ans.* $\frac{5}{32}$ left, worth $15000.

37. If $4\frac{3}{5}$ pounds of maple sugar cost $34\frac{1}{2}$ cts., how much must be paid for $80\frac{1}{4}$ pounds?

38. A grocer bought $9\frac{1}{4}$ tons of coal at $5\frac{1}{4}$ per ton, and paid for it in coffee at $\frac{3}{8}$ of a dollar a pound; how many pounds were required to pay for the coal? *Ans.* 133 lbs.

39. I have $800 and wish to lay out $346\frac{2}{3}$ of it in sugar at $8\frac{1}{3}$ cts. a pound, and the remainder in tea at $52\frac{3}{4}$ cts. a pound; how many pounds of tea do I buy? *Ans.* $859\frac{115}{1899}$ lbs.

40. A merchant expended $840 for dry goods, and then had remaining only $\frac{37}{40}$ as much money as he had at first; how much money had he at first?

41. A farmer has three fields: the first contains $73\frac{7}{11}$ acres, the second $88\frac{4}{11}$ acres, the third $139\frac{10}{11}$ acres. What is the largest-sized house-lots of the same extent into which the three fields can be divided, and also the number of lots? *Ans.* Size of each lot, $7\frac{4}{11}$ a.; 41 lots.

42. A man owning $135\frac{3}{4}$ acres of land, sold $\frac{1}{3}$ of it, and gave $\frac{3}{8}$ of it to his son; what was the value of the remainder, at $57.80 per acre? *Ans.* 2288.51\frac{7}{8}$.

43. A merchant owns $\frac{7}{8}$ of a factory worth $48000. He sells $\frac{2}{5}$ of his share to A, and $\frac{1}{2}$ the remainder to B. How much does he receive from A and B respectively, and what part has he remaining?
Ans. From A, $25200; From B, $8400; has left, $\frac{7}{40}$.

44. A drover bought 267 sheep, at $2.25 per head; he afterward bought 348 at 1.87\frac{1}{2}$ per head; then sold $\frac{2}{3}$ of the whole number at $1.75 per head, and the remainder at 2.12\frac{1}{2}$; did he gain or lose, and how much? *Ans.* Lost $35. 87$\frac{1}{2}$.

45. A mother divided a basket of oranges among her three daughters; to the first she gave 12 oranges, to the second $\frac{3}{8}$ of the whole, and to the third as much as to the other two; how many oranges did the third have? *Ans.* 48 oranges.

46. What is the smallest sum of money with which a farmer could purchase a number of sheep at $2\frac{1}{4}$ each, a number of calves at $4\frac{1}{2}$ each, and a number of yearlings at $9\frac{3}{8}$ each? and how many of each could he buy with this money?
Ans. $112.50, 50 sheep, 25 calves, 12 yearlings.

47. In selling $46\frac{1}{2}$ yards of merino for $50\frac{3}{4}$ I lost $\frac{1}{8}$ of the buying price. What was the cost of one yard? *Ans.* $1.318+.

48. Bought $\frac{5}{8}$ of a yard of cotton for $\frac{3}{5}$ of 20 cents, and gave in payment $\frac{1}{15}$ of a yard of cloth worth $3 a yd. Did I gain or lose by the bargain?

49. The $\frac{5}{11}$ of a farm are sown with corn; the $\frac{3}{14}$ with barley; and the remainder, containing $10\frac{1}{4}$ acres, planted with potatoes; how many acres does the farm contain? *Ans.* $30\frac{97}{102}$ acres.

50. How many bushels of oats at $62\frac{1}{2}$ cents per bushel are required to pay for 31 yards of cotton at $8\frac{1}{4}$ cents a yd., and $7\frac{1}{2}$ yards cloth, at $2.75 per yard? *Ans.* $37\frac{23}{250}$ bush.

51. If it required $3\frac{4}{7}$ days for a mason and his son to make $2\frac{2}{3}$ cubic yds. of masonry, how long will it take them to make a cubic yard?

52. If the $\frac{3}{5}$ of a hundred bottles of Rhenish wine cost $9.36; how much will 3482 bottles come to? *Ans.* $543.192.

53. What will be the price of $97\frac{5}{9}$ bushels of rye, if $17\frac{3}{11}$ bushels of the same quality cost 5\frac{7}{8}$? *Ans.* $30.66 + .

54. A piece of silk velvet would bring $210 were it $\frac{1}{8}$ longer; knowing the price of a yard to be $7.50, required the length of the whole piece? *Ans.* 24 yds.

55. A market woman sold the $\frac{3}{5}$ of a basket of eggs, in adding 28 eggs to the remainder, the number she had at first would be augmented $\frac{1}{5}$: how many had she? *Ans.* 35 eggs.

56. A man has an income such, that if it were augmented by the price he paid for a mahogany writing desk, that is $54, he could spend 2.02\frac{1}{2}$ per day. What is his income? *Ans.* 685.12\frac{1}{2}$.

57. A weaver can weave a yard of linen in $1\frac{7}{8}$ hours; how long will it take him to weave: 1st. 15 yds.; 2nd. $2\frac{6}{7}$ yds.; 3rd. $4\frac{5}{8}$ yds.; 4th. $\frac{9}{11}$ of a yd.; 5th. $\frac{13}{16}$ of a yd.? *Ans.* 1° $28\frac{1}{8}$ h.; 2° $5\frac{3}{14}$ h., etc.

58. What is the price of a lb. of sponge, if the difference between the $\frac{1}{4}$ and the $\frac{2}{3}$ of the sum paid for $9\frac{3}{5}$ lbs. be 60 cts.? *Ans.* $2.25.

59. In mixing 10 lbs. of bismuth with 6 lbs. of pewter and 4 lbs. of lead, we obtain an alloy which melts at the temperature of boiling water; required 1st. what quantity of each metal enters into the mixture of 2 lbs.; 2nd. $1\frac{1}{2}$ lbs.; 3rd. $3\frac{5}{6}$ lbs.; 4th. $1\frac{3}{5}$ lbs.; 5th. $27\frac{1}{11}$ lbs.; 6th. 1 lb.; 7th. $1\frac{7}{9}$ lbs.; 8th. $43\frac{1}{3}$ lbs.; 9th. $144\frac{6}{7}$ lbs.; 10th. $97\frac{3}{25}$ lbs.? *Ans.* 1° 1 lb. of bismuth, $\frac{3}{5}$ lb. of pewter, and $\frac{2}{5}$ lb. of lead;

2° $\frac{3}{4}$ lb. of bismuth, $\frac{9}{20}$ lb. of pewter, and $\frac{3}{10}$ lb. of lead, etc.

60. A weaving machine makes $13\frac{7}{8}$ yards of cloth per day; how many yards will it make, 1st. in 3 days; 2nd. in $\frac{7}{16}$ of a day; 3rd. in $4\frac{3}{5}$ days; 4th. in $1\frac{7}{8}$ days; 5th. in $32\frac{1}{12}$ days; 6th. in $47\frac{1}{2}\frac{2}{5}$ days; and 7th. in $274\frac{19}{24}$ days? *Ans.* 1° $41\frac{5}{8}$ yds.; 2° $6\frac{9}{125}$ yds., etc.

61. It would require 1800 yards of cloth $\frac{6}{4}$ yds. wide to make clothes for a regiment; but, on delivery, the cloth is found to be too narrow and the purveyor is obliged to buy 2000 yards: what is the width of the cloth? *Ans.* $1\frac{7}{20}$ yds.

62. Paid $2235.45 for 8 pieces of broadcloth of equal length and a remnant of $15\frac{3}{10}$ yards: required the length of a piece knowing that one yard costs $10.50? *Ans.* 24.7 yds.

63. The breadth of a painting is but the $\frac{7}{11}$ of its height. If the breadth equal the $\frac{4}{7}$ of $2\frac{7}{10}\frac{3}{0}$ yards, what is the height? *A.* $2\frac{7}{11}\frac{9}{5}$ yds.

64. A teacher of a select school has 60 pupils; 24 of them pay $1.25 a month each, the $\frac{5}{9}$ of the remainder, $1.75, and the rest $2.50. How much does he receive from his pupils in 8 months? *Ans.* $840.

65. The difference of time between two watches is $\frac{3}{4}$ of an hour; one of them gains $4\frac{3}{5}$ minutes per day, while the other loses $5\frac{7}{8}$ in the same time: in how many days will they again mark the same time?

66. How many herrings were there in a barrel of which 243 were sold at one time, then the $\frac{3}{8}$, and if there still remain $\frac{2}{5}$. Required also the value of the whole barrel if the herrings were sold on an average of 80 cents per hundred? *Ans.* 1080 herrings; $8.64.

67. A dealer in porcelain bought a certain quantity of plates; he sells $\frac{1}{3}$ of them at 36 cents a doz., $\frac{1}{8}$ at 38 cents a doz., and the remainder at 41 cents. How many dozen of plates did he buy, knowing that he paid 31 cents per doz n and gained $1.05 by his bargain?

68. A man having bought 84 bushels potatoes, forgets how much he paid per bushel; but remembers that there was a difference of $4 between the $\frac{4}{7}$ and the $\frac{4}{5}$ of the sum laid out. How much did he pay per bushel? *Ans.* 0.37\frac{1}{2}$.

69. A dealer in furs sold a certain number of astrakhan skins at the rate of $1.70 a piece. Now, in adding to the proceeds of his sales the $\frac{5}{13}$ of the same proceeds less $9.60, he could buy 25 fox skins at $19.20. How many astrakhan skins did he sell?

70. A farmer sold 4 sheep and expended the $\frac{5}{9}$ of the sum in purchasing 5 lambs; the remainder of his money is equal to $\frac{1}{2}$ of the sum itself less $2.00. Required the price of a sheep and of a lamb?

Ans. $9, the price of a sheep; $4, the price of a lamb.

DENOMINATE NUMBERS.

186. A **Simple Number** is either an abstract or a denominate number of but one denomination; as 18, $12, 40 rods, 15 oranges (9).

187. A **Compound Number** is a collection of concrete units of different denominations (10); as, 3 feet 4 inches, 5 pounds 6 ounces, 2 days 8 hours 24 minutes.

NOTE.—In simple numbers and decimals the scale is uniform, and the law of increase and decrease is by 10. In compound numbers, the scale of increase and decrease is varying.

188. A **Denominate Number** is any concrete number which expresses some particular kind or quantity; as 3 yards, 7 dollars.

189. A **Denominate Fraction** is a concrete fraction whose integral unit is *one* of a denomination of some compound number. Thus, $\frac{3}{8}$ of a bushel is a denominate fraction, the integral unit being one bushel; so are $\frac{3}{7}$ of a day, $\frac{3}{4}$ of a yard, etc., denominate fractions.

190. Denominate Numbers express **Currencies, Weights, and Measures.**

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186. *What is a* simple number?— 187. *A* compound number?— 188. *A* denominate number?— 189. *A* denominate fraction?— 190. *What do* denominate numbers *express?*

CURRENCIES.

I. Dominion of Canada Money (77).

II. Old Canadian Money, or Halifax Currency.

TABLE.

4 farthings	make	1 penny,	*d.*
12 pence	"	1 shilling,	*s.*
5 shillings	"	1 dollar,	$.
4 dollars	"	1 pound,	£.

		s.	*d.*	*qr.*
			1 =	4.
	$	1 =	12 =	48.
£	1 =	5 =	60 =	240.
1 =	4 =	20 =	240 =	960.

NOTE:—Every 3*d.* of the old coinage is equal to 5 cents of the new.

III. English Money.

TABLE.

4 farthings (*far.* or *qr.*)	make	1 penny	*d.*
12 pence	"	1 shilling	*s.*
20 shillings	"	1 pound or sovereign	£ or *sov.*

	s.	*d.*	*far.*
		1 =	4.
£	1 =	12 =	48.
1 =	20 =	240 =	960.

NOTES.—1. Farthings are generally expressed as fractions of a penny; thus, 1 far., sometimes called one quarter, (*qr.*) = ¼*d.*; 3 far. = ¾*d.*

2. The old *f*, the original abbreviation for shillings, was formerly written between shillings and pence, and *d*, the abbreviation for pence, was omitted. Thus 3*s.* 6*d.* was written 3/6. A straight line is now used in place of the *f*, and shillings are written on the left of it, and pence on the right. Thus, 3/6, 7/3, etc.

3. The present value of the sterling pound in the Dominion of Canada is $4.8666, and hence the value of an English shilling is 24⅓ cents.

4. The coins of England in general circulation are: the sovereign (= £1), and the half-sovereign (= 10*s.*), made of *gold;* the crown (= 5*s.*), the half-crown (= 2*s.* 6*d*), the florin (= 2*s.*), the shilling, the six-pence, the four-pence, and the three-pence, made of *silver;* the penny, the half-penny, and the farthing, made of *copper.*

5. The standard gold coin of England is 11 parts *pure gold* and 1 part *alloy.* The standard silver coin is 37 parts ($\frac{37}{40}$ = .925) *pure silver* and 3 parts ($\frac{3}{40}$ = .075) *copper.* 24 pence, in copper coin, weigh a pound avoirdupois.

IV. United States Money (78).

V. French Money.

191. French Currency is decimal. The *Franc* is the unit of the currency, and is equal in value to $0.186 Dominion of Canada money.

TABLE.

10 millimes make 1 centime.
10 centimes " 1 decime.
10 decimes " 1 franc.

Coins.— Gold pieces of 100, 50, 20, 10, and 5 francs.
Silver pieces of 5, 2, and 1 francs; 50 and 20 centimes.
Copper or bronze pieces of 10, 5, 2, and 1 centimes.

DOMINION OF CANADA, ENGLISH, AND FRENCH MONEYS COMPARED.

English.		D. C.	French.		D. C.
1*d.*	=	$0.020275.	1 millime	=	$0.000186.
1*s.*	=	$0.2433.	1 centime	=	$0.00186.
£1	=	$4.866.	1 franc	=	$0.186.

WEIGHTS.

192. Weight is the measure of the quantity of matter a body contains, determined according to some fixed standard. Three scales of weight are used in the Dominion of Canada, Great Britain, and the United States, viz.: Troy, Apothecaries', and Avoirdupois.

I. Troy Weight.

193. Troy Weight is used in weighing gold, silver, and jewels; in philosophical experiments, &c.

TABLE.

24 grains (*gr.*) make 1 pennyweight, *pwt.* or *dwt.*
20 pennyweights " 1 ounce, *oz.*
12 ounces " 1 pound, *lb.*

	pwt.	*gr.*
oz.	1 =	24.
lb.	1 = 20 =	480.
1 = 12 =	240 =	5760.

Notes.—1. Diamonds, etc., are weighed by *carats*, and *fractions* of a carat. A carat weighs 4 *grains* Troy weight.

2. In speaking of the purity of gold, a *carat* means $\frac{1}{24}$ part; as, 18 carats fine, meaning $\frac{18}{24}$ pure gold and $\frac{6}{24}$ alloy.

3. A Troy pound is equal to 372.965 French grammes.

II. Apothecaries' Weight.

194. Apothecaries' Weight is used by apothecaries and physicians in mixing medicines; but medicines, in the quantity, are bought and sold by Avoirdupois weight.

TABLE.

20 grains (*gr.*)	make	1 scruple,	*sc.* or ℈.
3 scruples	"	1 dram,	*dr.* or ʒ.
8 drams	"	1 ounce,	*oz.* or ℥.
12 ounces	"	1 pound,	*lb.* or ℔.

						sc.		*gr.*
				dr.		1	=	20
		oz.		1	=	3	=	60
lb.		1	=	8	=	24	=	480
1	=	12	=	96	=	288	=	5760

III. Avoirdupois Weight.

195. Avoirdupois Weight is used for all the ordinary purposes of weighing.

TABLE.

16 drams (*dr.*)	make	1 ounce,	*oz.*
16 ounces	"	1 pound,	*lb.*
25 pounds	"	1 quarter,	*qr.*
4 quarters	"	1 hundred weight,	*cwt.*
20 cwt., or 2000 lbs.,	"	1 ton,	*T.*

							oz.		*dr.*
					lb.		1	=	16.
			qr.		1	=	16	=	256.
		cwt.	1	=	25	=	400	=	6400.
	T.	1	=	4	=	100	=	1600	=	25600.
1	=	20	=	80	=	2000	=	32000	=	512000.

NOTE.—The *long* or *gross ton*, hundred weight, and quarter, were formerly in common use; but they have now fallen into disuse among merchants in Canada. The Custom-Houses continue to use it. Farmers and others weigh still some few articles by the *long ton*.

LONG TON TABLE.

28 lbs.	make	1 quarter,	marked *qr.*
4 qr. = 112 lbs.	"	1 hundred weight,	" *cwt.*
20 cwt. = 2240 lbs.	"	1 ton,	" *T.*

COMPARATIVE TABLE OF WEIGHTS.

	Troy.	Apothecaries'.	Avoirdupois.
1 pound =	5760 grains,	= 5760 grains,	= 7000 grains.
1 ounce =	480 "	= 480 "	= 437.5 "
	175 pounds,	= 175 pounds,	= 144 pounds.

NOTES.—1. 7560 grains Troy make one pound marc weight, or old French weight. This pound contains 16 ounces; the ounce, 8 drams; and the dram, 72 grains of the marc weight. It is also divided into two marcs of 8 ounces each. 100 lbs. marc weight make 108 lbs. Avoirdupois weight, or 131¼ lbs. Troy; and, 16 lbs. marc weight make 21 lbs. Troy. It is proper to remark that the old French weights and measures are yet in general use and legally recognised in the Province of Quebec.

2. To reduce or change the English pound into French, *multiply by* 100 and *divide by* 108, and *vice versa*.

3. To change a quantity from one weight to its equivalent in another weight, *reduce the given quantity to Troy grains, and then find their value in denominations of the weight required.*

MEASURES.

196. Measure is that by which extent, dimension, capacity or amount is ascertained, determined according to some fixed standard. It may be properly divided into two classes.—Measures of Extension, and Measures of Capacity.

MEASURES OF EXTENSION.

197. Extension has three dimensions— length, breadth and thickness.

A **Line** has only one dimension— length.

A **Surface** or **Area** has two dimensions— length and breadth.

A **Solid** or **Body** has three dimensions— length, breadth, and thickness.

I. LINEAR OR LONG MEASURE.

198. Linear or **Long Measure**, is used in measuring lines or distances.

TABLE.

1	inch (*in.*) =	0.3363 French inch.		
12	inches	make 1 foot,		*ft.*
3	feet	" 1 yard,		*yd.*
5½	yd., or 16½ ft.	" 1 rod,		*rd.*
40	rods	" 1 furlong,		*fur.*
8	furlongs, or 320 rods	" 1 mile,		*mi.*
3	miles	" 1 league,		*lea.*
69½	miles (nearly)	" 1 degree on the equator,		*deg.* or °
360	degrees	" 1 great circle of the earth.		

										ft.		*in.*	
						yd.			1	=	12		
				rd.		1	=		3	=	36		
		fur.		1	=	5½	=		16½	=	198		
mi.		1	=	40	=	220	=		660	=	7920		
1	=	8	=	320	=	1760	=		5280	=	63360		

NOTES.—1. For the purpose of measuring cloth and other goods sold by the yard, the yard is divided into halves, fourths, eighths, and sixteenths. The old table of cloth measure is practically obsolete.

2. In Mariners' Measure, 12 lines make 1 inch; 4 inches, 1 hand; 6 feet, 1 fathom; 120 fathoms, 1 cable-length; 7⅓ cable-lengths, 1 mile; $\frac{1}{60}$ of a degree of the circumference of the earth, 1 knot, or geographical mile, equal to $1\frac{11}{72}$ statute miles.

3. The length of a degree of latitude varies, being 68.72 miles at the equator, 68.9 to 69.05 miles in middle latitudes, and 66.30 to 69.34 miles in the polar regions. The mean or average length is as stated in the table. A degree of longitude is greatest at the equator, where it is 69.16 miles, and it gradually decreases toward the poles, where it is 0.

Table of the Old French Linear Measures.

1 line	=	0.089	Engl. inch.	
12 lines (*l.*)	make	1	inch,	*in.*
12 inches	"	1	foot,	*ft.*
6 feet	"	1	toise,	*to.*
3 toises	"	1	perch,	*per.*
10 perches	"	1	arpent,	*arp.*
84 arpents	"	1	league,	*lea.*
1000 French feet	"	1068	Engl. feet.	

NOTES.—1. The French linear measures are in frequent use in the Province of Quebec.

2. The Engl. league = 15840 Engl. feet, and the French league of Canada = 15120 French ft., or 16148.16 Engl. ft.; the difference between the two = 308.16 Engl. ft., or $288\frac{48}{49}$ French ft.

Surveyors' Linear or Long Measure.

199. A **Gunter's Chain**, used by land surveyors, is 4 rods or 66 feet long, and consists of 100 links.

TABLE.

7.92	inches (*in.*)	make	1 link,	*l.*
25	links	"	1 rod,	*rd.*
4	rods, or 66 feet,	"	1 chain,	*ch.*
10	chains	"	1 furlong,	*fur.*
8	furlongs	"	1 mile,	*mi.*

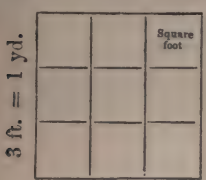
mi.		*fur.*		*ch.*		*rd.*		*l.*		*in.*
								1	=	7.92
						1	=	25	=	198.
				1	=	4	=	100	=	792.
		1	=	10	=	40	=	1000	=	7920.
1	=	8	=	80	=	320	=	8000	=	63360.

II. Square Measure.

200. A **Square** is a figure bounded by four equal lines, perpendicular to each other. It is the *Unit of Measure* for com-

puting areas or surfaces; as of land, boards, painting, plastering, paving, etc.

201. An **Area** or **Surface** is that which has length and breadth, without thickness.



The square in the margin is called *three feet square*, as it is three feet on each side. Each of the small squares, within the large square, represents 1 *square foot*, or 1 *foot square*. Since there are 3 square feet in each row, and 3 rows in the square, there are 3 times 3 square feet, equal to 9 square feet in 3 feet square. Hence,

The area of a square or rectangle is found by multiplying its length by its width.

NOTE.—From the above it will be observed that the difference between 3 *feet square* and 3 *square feet* is 6 square feet.

TABLE.

1	square inch (*sq. in.*)	= 0.8767 French inch.		
144	square inches	make 1 square foot,	*sq. ft.*	
9	square feet	" 1 square yard,	*sq. yd.*	
$30\frac{1}{4}$	square yards	" 1 square rod,	*sq. rd.*	
40	square rods	" 1 rood,	*R.*	
4	roods	" 1 acre,	*A.*	
640	acres	" 1 square mile,	*sq. mi.*	

						sq. ft.		*sq. in.*
				sq. yd.		1	=	144
		sq. rd.		1	=	9	=	1296
	R.	1	=	$30\frac{1}{4}$	=	$272\frac{1}{4}$	=	39204
A.	1	=	40	=	1210	=	10890	=	1568160
sq. mi. 1	=	4	=	160	=	4840	=	43560	=	6272640
1	= 640	=	2560	=	102400	=	3097600	=	27878400	=	4014489600

TABLE OF THE OLD FRENCH SQUARE MEASURES.

1	square inch (*sq. in.*)	= 0.007921 Engl. foot.		
144	square inches	make 1 square foot,	*sq. ft.*	
36	feet	" 1 square toise,	*sq. to.*	
9	toises	" 1 square perch,	*sq. per.*	
100	perches	" 1 square arpent,	*sq. arp.*	
7056	arpents	" 1 square league,	*sq. L.*	

NOTES.—1. Artificers estimate their work as follows, viz.: glazing and stone-cutting, by the square yard; painting, plastering, paving, ceiling, and paper-hanging, by the square yards; flooring, partitioning, roofing, slating, tiling, by the square of 100 square feet; brick-laying is estimated by the thousand bricks, by the square yard, and by the square of 100 square feet.

2. In estimating the painting of mouldings, cornices, etc., the mensuring-line is carried into all the mouldings and cornices.

3. In estimating brick-laying by either the square yard or the square of 100 feet, the work is understood to be 12 inches or $1\frac{1}{2}$ brick thick.

4. A thousand shingles are estimated to cover 1 square, being laid 5 inches to the weather.

SURVEYORS' SQUARE MEASURE.

202. This measure is used by surveyors in computing the area or contents of land.

TABLE.

625 square links (*sq. l.*)	make	1 pole,	*P.*
16 poles	"	1 square chain,	*sq. ch.*
10 square chains	"	1 acre,	*A.*
640 acres	"	1 square mile,	*sq. mi.*
36 square miles (6 miles square)	"	1 township,	*Tp.*

NOTES.—1. Canal and railroad engineers commonly use an engineer's chain, which consists of 100 links, each 1 foot long.

2. The contents of land are commonly estimated in square miles, acres, and hundredths; the denomination, *rood*, is rapidly going into disuse. A square mile of land is also called a *section*.

III. CUBIC OR SOLID MEASURE.

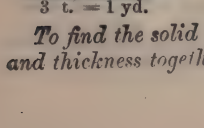
203. A **Cube** is a solid, or body, bounded by six equal square sides or faces. The sides of the squares are called its *edges*.

204. Cubic Measure is used in estimating the contents of solids, or bodies; as timber, wood, stone, etc.

205. The **Contents**, or **Solidity**, of a volume, is the number of times it contains a given unit of measure.

The measurements for computing solidity are always taken in the denominations of linear measure.

If each of the sides of a cube is 1 foot, it is called a *cubic foot*. If each of the sides of a cube is 3 feet = 1 yard, it is called a *cubic yard*.



The annexed cube represents a cubic yard. Since each of the edges of a cubic yard is 3 feet, each of its faces will contain 3 times 3 equal to 9 square feet. If, from one face of this cube, we cut off a piece 1 foot in thickness, we evidently have 9 *solid feet*; and as the whole block is 3 feet thick, it must contain 3 times 9 = 27 solid feet. Hence,

To find the solid contents of a cube, multiply its length, breadth, and thickness together.

TABLE.

1728 cubic inches (*cu. in.*)	make	1 cubic foot,	*cu. ft.*
27 cubic feet	"	1 cubic yard,	*cu. yd.*
40 cubic feet of round timber, or 50 " " " hewn "	"	1 ton or load,	*T.*
16 cubic feet	"	1 cord foot,	*cd. ft.*
8 cord feet, or 128 cubic feet	"	1 cord of wood,	*Cd.*
24¾ cubic feet	"	1 perch of stone or masonry.	*Pch.*

TABLE OF FRENCH MEASURES.

1728 cubic inches	make	1 cubic foot,	*cu. ft.*
216 cubic feet	"	1 " toise,	*cu. to.*
1000 French cubic feet	"	1218. 186432 Engl. cub. feet.	
1000 cubic toises	"	9745. 491456 cub. yd.	

NOTES.—1. Railroad and transportation companies estimate light freight by the space it occupies in cubic feet; and heavy freight, by weight.

2. A pile of wood 8 feet long, 4 feet wide, and 4 feet high, contains one cord; and a cord foot is one foot in length of such a pile.

3. A perch of stone or of masonry is 16½ feet long, 1½ feet wide, and 1 foot high.

4. Joiners, bricklayers, and masons, make an allowance for windows, doors, etc., of one half the openings or vacant spaces. Bricklayers and masons, in estimating their work by cubic measure, make no allowance for the corners of the walls of houses, cellars, etc., but estimate their work by the *girt*, that is, the entire length of the wall on the *outside*.

5. Engineers, in making estimates for excavations and embankments, take the dimensions with a line or measure divided into feet and decimals of a foot. The computations are made in feet and decimals, and the results are reduced to cubic yards. In civil engineering, the cubic yard is the unit to which estimates for excavations and embankments are finally reduced.

6. In scaling or measuring timber for shipping or freighting, $\frac{1}{5}$ of the solid contents of round timber is deducted for waste in hewing or sawing. Thus, a log that will make 36 feet of hewn or sawed timber, actually contains 45 cubic feet by measurement: but its market value is only equal to 36 cubic feet of hewn or sawed timber. Hence, the cubic contents of 36 feet of round and 45 feet of hewn timber, as estimated for market, are identical.

7. Sawed timber, joists, planks, and scantlings are now generally bought and sold by what is called *board measure*.

8. A cubic foot of distilled water at the maximum density, at the level of the sea, and the barometer at 30 inches, is equal in weight to 62½ lbs. or 1000 oz. Avoirdupois.

MEASURES OF CAPACITY.

206. Measures of Capacity are all cubic measures, solidity and capacity being referred to different units. *Capacity* signifies extent of space.

207. Measures of capacity may be properly subdivided into two classes, *Measures of Liquids* and *Measures of Dry Substances*.

I. Liquid Measure.

208. Liquid Measure, also called **Wine Measure,** is now used for measuring all kinds of liquids.

TABLE.

4	gills (*gi.*)	make 1 pint,	*pt.*
2	pints	" 1 quart,	*qt.*
4	quarts	" 1 gallon,	*gal.*
31½	gallons	" 1 barrel,	*bbl.*
2	barrels	" 1 hogshead,	*hhd.*
2	hogsheads	" 1 pipe,	*pi.*
2 pipes, or 4 hogsheads		" 1 tun,	*tun.*

									pt.		*gi.*
						qt.		1	=	4.
				gal.		1	=	2	=	8.
			bbl.	1	=	4	=	8	=	32.
		hhd.	1	= 31½	=	126	=	252	=	1008.
	pi.	1	= 2	= 63	=	252	=	504	=	2016.
tun.	1	= 2	= 4	= 126	=	504	=	1008	=	4032.
1	= 2	= 4	= 8	= 252	=	1008	=	2016	=	8064.

Notes.—1. The English Imperial gallon contains 277.274 cubic inches or 10 lbs. Avoirdupois of pure distilled water, weighed at a temperature of 62° Fahrenheit, and under a barometer pressure of 30 inches.

2. In the United States the wine gallon contains 231 cubic inches, and the beer gallon 282 cubic inches. The gallon of England is therefore about equal to 1.2 gallons United States Wine Measure.

3. By an Act of the Imperial Parliament, 1826, the Imperial gallon of 277.274 cubic inches, was adopted as the only gallon, and is therefore the standard for both liquid and dry measures.

4. Beer is usually sold by the gallon; sometimes, however, in casks of 5, 10, 20 gals. etc. The beer barrel contains 36 gallons, and the hogshead, 54 gallons.

II. Dry Measure.

209. Dry Measure is used in measuring articles not liquid, as grain, salt, fruit, roots, &c.

TABLE.

2	pints (*pt.*)	make 1 quart,	*qt.*
4	quarts	" 1 gallon,	*gal.*
2	gallons	" 1 peck,	*pk.*
4	pecks	" 1 bushel,	*bush.*
36	bushels	" 1 chaldron,	*ch.*

									qt.		*pt.*
						gal.		1	=	2.
				pk.		1	=	4	=	8.
		bush.		1	=	2	=	8	=	16.
ch.		1	=	4	=	8	=	32	=	64.
1	= 36	=	144	=	288	=	1152	=	2304.		

NOTES.—1. The English or Winchester bushel is an upright cylinder whose internal diameter is 18½ inches, and depth 8 inches. It contains 2150.4 cubic inches, or 77.627 lbs. Avoirdupois of pure distilled water, at 62° Fahr. and 30 in. barometer. The bushel of Canada is 18½ inches in diameter, and 8.701 inches deep, and must contain 2338.917 Engl. cubic inches, or 1920 French cubic inches. The standard unit of Dry Measure in the United States is the Winchester bushel. The standard unit of Dry Measure in Great Britain is the Imperial bushel, which is 18.789 inches in diameter, and 8 inches deep. It contains 2218.192 cubic inches, or 80 lbs. Avoirdupois of pure distilled water at 62° Fahr. and 30 in. barometer.

2. Grain is frequently bought and sold by weight. The standard per bushel is, viz.: of wheat, 60 lbs.; of rye, 56 lbs.; of Indian corn, 56 lbs.; of barley, 48 lbs.; of oats, 34 lbs.; of peas, 60 lbs.; of beans, 50 lbs.; of buckwheat, 40 lbs.; of flax seed, 56 lbs.; of Timothy-seed or red clover-seed, 60 lbs.

3. The old French Weights and Measures are legal in the Province of Quebec.

MEASURE OF TIME.

210. Time is the measure of duration. The unit is the day, and the table is made up of its divisors and multiples.

TABLE.

60 seconds (*sec.*)	make	1 minute,	*min.*
60 minutes	"	1 hour,	*h.*
24 hours	"	1 day,	*da.*
7 days	"	1 week,	*wk.*
4 weeks	"	1 lunar month,	*mo.*
365 days	"	1 common year,	*yr.*
366 days	"	1 leap year,	*yr.*
12 calendar months	"	1 year,	*yr.*
100 years	"	1 century,	*C.*

The calendar year is divided as follows:—

No. of months.	Seasons.	Names of months.	Abbreviations.	No. of days.
1	Winter,	January,	Jan.	31.
2		February,	Feb.	28 or 29.
3		March,	Mar.	31.
4	Spring,	April,	Apr.	30.
5		May,	May.	31.
6		June,	Jun.	30.
7	Summer,	July,	July.	31.
8		August,	Aug.	31.
9		September,	Sept.	30.
10	Autumn,	October,	Oct.	31.
11		November,	Nov.	30.
12	Winter,	December,	Dec.	31.

NOTES.—1. The true *Solar* or *Tropical Year* is the time measured from the sun's leaving either equinox or solstice to its return to the same again, and is 365 d. 5h. 48 min. 49 $\frac{7}{10}$ sec.

2. The *Julian Year*, so called from the calendar instituted by Julius Cæsar, contains 365¼ days, as a medium; three years in succession containing 365 days, and the fourth year 366 days; which, as compared with the true solar year, produces a yearly error of 11m. $10\frac{3}{10}$ sec., or of 1 whole day in about 120 years.

3. The *Gregorian Year*, or that instituted by Pope Gregory XIII, in the year 1582, and which is now the *Civil* or *Legal Year* in use among the different nations of the earth, contains 365 days for three years in succession, and 366 days for the fourth, *excepting centennial* years whose number cannot be exactly divided by 400. The Gregorian year gives an error of only 1 day in 3866 days.

4. The *civil day* begins and ends at 12 o'clock, midnight. The *astronomical day*, used by astronomers in dating events, begins and ends at 12 o'clock, noon.

6. In most business transactions 30 days are called 1 month.

TABLE

SHOWING THE NUMBER OF DAYS FROM ANY DAY OF ONE MONTH TO THE SAME DAY OF ANY OTHER MONTH IN THE SAME YEAR.

FROM ANY DAY OF	TO THE SAME DAY OF											
	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
January	365	31	59	90	120	151	181	212	243	273	304	334
February	334	365	28	59	89	120	150	181	212	242	273	303
March	306	337	365	31	61	92	122	153	184	214	245	275
April	275	306	334	365	30	61	91	122	153	183	214	244
May	245	276	304	335	365	31	61	92	123	153	184	214
June	214	245	273	304	334	365	30	61	92	122	153	183
July	184	215	243	274	304	335	365	31	62	92	123	153
August	153	184	212	243	273	304	334	365	31	61	92	122
September	122	153	181	212	242	273	303	334	365	30	61	91
October	92	123	151	182	212	243	273	304	335	365	31	61
November	61	92	120	151	181	212	242	273	304	334	365	30
December	31	62	90	121	151	182	212	243	274	304	335	365

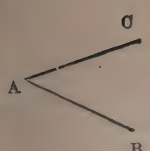
For example, to find the number of days from April 4th to November 4th, we look for April in the left vertical column, and November at the top, and, where the lines intersect, is 214, the number sought. Again, to find the number of days from June 10th to September 16th, we find the difference between June 10th and September 10th to be 92 days, and add 6 days for the excess of the 16th over the 10th of September, so we have 98 days as the exact difference.

If the end of February be included between the points of a time, a day must be added in leap year.

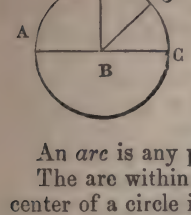
When the time exceeds one year, there must be added 365 days for each year.

CIRCULAR MEASURE.

211. Circular Measure, called also *Angular Measure*, is used principally in surveying, navigation, astronomy, and geography; for reckoning latitude and longitude, determining locations of places and vessels, and computing difference of time.



212. An **Angle** *is the difference of direction of two lines which meet at a point;* thus, A, B, C, is an angle. The lines are called the *sides* of the angle, and the point where they meet is called the *vertex*.



213. A **Circle** is a plane figure bounded by a curved line, all the parts of which are equally distant from a point within called the *center*.

A *circumference* is the curve line which bounds a circle, and always contains 360 degrees.

An *arc* is any part of the circumference, as C D, D E.

The arc within the sides of an angle whose vertex is on the center of a circle is the measure of the angle; thus, the arc C E is one fourth of the circumference, and measures the angle E B C, which contains 90 degrees.

TABLE.

60 seconds (″)						make 1 minute,					′.
60 minutes						“ 1 degree,					°.
30 degrees						“ 1 sign,					S.
12 signs, or 360°						“ 1 circle,					C.
				°		′					″
						1	=				60.
		s.		1	=	60	=				3600.
c.		1	=	30	=	1800	=				108000.
1	=	12	=	360	=	21600	=				1296000.

NOTE.—A *quadrant*, or right angle, is one-fourth of a circumference, or an arc of 90°; as A B. 60° is called a sextant, or $\frac{1}{6}$ of a circle.

MISCELLANEOUS TABLES.

COUNTING.

12 units	make	1 dozen.		12 gross	make	1 great gross.
12 dozen	“	1 gross.		20 units	“	1 score.

PAPER.

24 sheets	make	1 quire.		2 reams	make	1 bundle.
20 quires	“	1 ream.		5 bundles	“	1 bale.

BOOKS.

A sheet folded in

2	leaves is called a	folio.		16	leaves is called a	16mo.
4	“ “	a quarto, or 4to.		18	“ “	an 18mo.
8	“ “	an octavo, or 8vo.		24	“ “	a 24mo.
12	“ “	a 12mo.		32	“ “	a 32mo.

THE METRIC SYSTEM OF WEIGHTS AND MEASURES.

The metric system of weights and measures—so called, because the metre is the unit from which the other units of the system, whether of length, area, solidity, capacity, or weight, are derived—originated in France in 1790. It was determined and established as follows: a very accurate survey of that portion of the terrestrial meridian, or north and south circle, between Dunkirk in the north of France, and Barcelona in Spain, was made under the direction of Government, and from this measurement the exact length of a quadrant of the entire meridian, or the distance from the equator to the north pole, was computed. The ten millionth part of this arc was denominated a metre, and from this all the standard units of measure and weight are derived and determined.

The metric system was finally made the only legal system throughout the whole of France in 1841. Since that time, it has been adopted by Spain, Belgium, and Portugal, to the exclusion of other weights and measures. In Holland, other weights are used only in compounding medicines. In 1864, the system was legalized in Great Britain; and its use, either as a whole or in some of its parts, has been authorized in Greece, Italy, Norway, Sweden, Mexico, Guatemala, Venezuala, Ecuador, United States of Columbia, Brazil, Chili, San Salvador, and Argentine Republic. In 1866, the use of the metric system of weights and measures, was authorized by Congress for the whole of the United States.

TABLES AUTHORIZED BY CONGRESS OF THE UNITED STATES.

MEASURES OF LENGTHS.

Metric Denominations and Values.		Equivalents in Denominations in use.
Myriametre,...	10,000 metres,..	6.2137 miles.
Kilometre,.....	1,000 metres,.....	0.62137 miles, or 3280 feet, 10 inches.
Hectometre,...	100 metres,.....	328 feet and 1 inch.
Decametre,.. ...	10 metres,..	393.7 inches.
Metre,.....	1 metre,.....	39.37 inches.
Decimetre,.....	$\frac{1}{10}$ of a metre,...	3.937 inches.
Centimetre,....	$\frac{1}{100}$ of a metre,...	0.3937 inch.
Millimetre,.....	$\frac{1}{1000}$ of a metre,...	0.0394 inch.

MEASURES OF SURFACES.

Metric Denominations and Values.		Equivalents in Denominations in use.
Hectare,.....	10,000 square metres,	2.471 acres.
Are,.....	100 square metres,	119.6 square yards.
Centiare,.....	1 square metre,	1550 square inches.

MEASURES OF SOLIDS.

Metric Denominations and Values.		Equivalents in Denominations in use.
Decastere,.....	10 cubic metres,.....	13.079 cubic yards.
Stere,.....	1 cubic metre,.....	0.2759 of a cord of wood.
Decistere,.....	100 cubic decimetres,.	3.53144 cubic feet.

MEASURES OF CAPACITY.

Metric Denominations and Values.			Equivalents in Denominations in use.	
Names.	No. of litres	Cubic Measure.	Dry Measure.	Liquid or wine measure.
Kilolitre, or stere,	1000	1 cubic metre,.....	1.308 cubic yd.	264.17 gallons..
Hectolitre,.....	100	$\frac{1}{10}$ of a cubic metre,....	2 bu. 3.35 pk...	26.417 gallons..
Decalitre,.....	10	10 cubic decimetres,....	9.08 quarts,.....	2.6417 gallons..
Litre, ...	1	1 cubic decimetre,.....	0.908 quart,....	1.0567 quarts..
Decilitre,.....	$\frac{1}{10}$	$\frac{1}{10}$ of a cubic decimetre,	6.1022 cubic in.	0.845 gill.....
Centilitre, ...	$\frac{1}{100}$	10 cubic centimetres,...	0.6102 cubic in.	0.338 fluid oz...
Millilitre,.....	$\frac{1}{1000}$	1 cubic centimetre,.....	0.061 cubic in..	0.27 fluid dr....

WEIGHTS.

Metric Denominations and Values.			Equivalents in Denominations in use.
Names.	Number of grammes.	Weight of what quantity of water at maximum density	Avoirdupois weight.
Millier, or tonneau,.	1,000,000	1 cubic metre,.....	2204.6 pounds.
Quintal,.....	100,000	1 hectolitre,.....	220.46 pounds.
Myriagramme,.....	10,000	10 litres,.....	22.046 pounds.
Kilogramme, or kilo,	1,000	1 litre,.....	2.2046 pounds.
Hectogramme,.....	100	1 decilitre,.....	3.5274 ounces.
Decagramme, ...	10	10 cubic centimetres,.....	0.3527 ounce.
Gramme, ...	1	1 cubic centimetre, ...	15.432 gr. Tr. W.
Decigramme.	$\frac{1}{10}$	$\frac{1}{10}$ of a cubic centimetre,...	1.5432 grains.
Centigramme,.....	$\frac{1}{100}$	10 cubic millimetres,.....	0.1543 of a grain.
Milligramme,.....	$\frac{1}{1000}$	1 cubic millimetre,.....	0.0154 of a grain.

MEASURES OF ANGLES.

Metric Denominations and Values.		Equivalents in Denominations in use.
Circle,.....	400 grades, ...	1 circle or 360°.
Quadrant,..	100 grades,.....	1 quadrant or 90°.
Grade, ...	1 grade,.....	54 minutes.
Minute,.....	$\frac{1}{100}$ of a g.,.....	32.4 seconds.
Second,.....	$\frac{1}{10000}$ of a g.,.....	0.324 of a second.

NOMENCLATURE AND TABLES.

There are eight kinds of quantities for which tables are usually constructed; viz., Lengths, Surfaces, Volumes or Solids, Capacities, Weights, Values, Times, and Angles or Arcs. The table for Times is the same in the metric as in the ordinary system. The table for Angles is constructed upon a centesimal scale. The tables for the other six kinds of quantities are constructed upon a decimal scale. In each of the tables for Lengths, Surfaces, Volumes, Capacities, and Weights, there are eight denominations of units,—one principal and seven derivative. The principal units are the *metre*, which is the base of the system, and those derived directly from it. The two following tabular views present the facts regarding the principal and derivative units, which should be fixed in the memory.

PRINCIPAL UNITS.

I. METRE, ...
1. Principal unit of lengths.
2. The base of the metric system, and nearly one ten-millionth part of a quadrant of the earth's meridian.
3. Equivalent, 39.3708 inches.

II. ARE, ...
1. Principal unit of surfaces.
2. A square whose side is ten metres.
3. Equivalent, 119.6 square yards.

III. STERE, ...
1. Principal unit of volumes or solids.
2. A cube whose edge is one metre.
3. Equivalent, 1.308 cubic yards.

IV. LITRE,.....
1. Principal unit of capacities.
2. A vessel whose volume is equal to a cube whose edge is one-tenth of a metre.
3. Equivalent, .908 quart dry measure, or 1.0567 quarts wine measure.

V. GRAMME, ...
1. Principal unit of weights.
2. The weight of a cube of pure water whose edge is .01 of a metre.
3. The water must be weighed in a vacuum 4° C., or 39.2° F.
4. Equivalent, 15.432 grains.

DERIVATIVE UNITS.

I. How Derived.

1. Three orders of small units, or submultiples of each kind, are formed by dividing each of the principal units into tenths, hundredths, and thousandths.

2. Four orders of larger units, or multiples of each kind, are formed by considering as a unit ten times, one hundred times, one thousand times, and ten thousand times, each of the principal units.

II. Names how Formed.

1. General Principle. The names of derivative units are formed by attaching a prefix to the name of the principal unit from which they are derived, which indicates their relation to the principal unit.

2. For Submultiples, Latin Ordinals are used as Prefixes.

1. Millesimus, one thousandth, contracted Milli. *Example*, Millilitre = $\frac{1}{1000}$ of a litre; 8 millilitres = $\frac{8}{1000}$ of a litre.

2. Centesimus, one hundredth, contracted centi. *Ex.*, Centiare = $\frac{1}{100}$ of an are; 4 centiares = $\frac{4}{100}$ of an are.

3. Decimus, tenth, contracted deci. *Ex.*, Decimetre = $\frac{1}{10}$ metre; 3 decimetres = $\frac{3}{10}$ metre.

3. For multiples, Greek Cardinals are used as Prefixes.

1. Deca, ten. *Example*, Decametre, = 10 metres; 5 decametres = 50 metres.

2. Hecaton, one hundred, contracted hecto. *Ex.*, Hectolitre = 100 litres; 7 hectolitres = 700 litres.

3. Kilioi, one thousand, contracted kilo. *Ex.* Kilogramme = 1000 grammes.

4. Myria, ten thousand. *Ex.*, Myriastere = 10,000 steres; 3 myriasteres = 30,000 steres.

5. The *a* in deca and myria, and the *o* in hecto and kilo, are dropped when prefixed to *are*.

III. Order of Progression in Tables.

The tables being constructed upon a decimal scale, ten units of a lower order make one of the next higher, thus: 10 millimetres = 1 centimetre; 10 centimetres = 1 decimetre; 10 decimetres = 1 metre; 10 metres = 1 decametre, &c.

The facts in the preceding views being mastered, the tables can be constructed by the pupil at sight. For example: The names of the derivative units are formed by attaching the seven prefixes,

in their order, to the principal units of the tables. The order of pragression being ten, the table of capacities will be written thus:—

10 Millilitres = 1 Centilitre. 10 Litres = 1 Decalitre.
10 Centilitres = 1 Decilitre. 10 Decalitres = 1 Hectolitre.
10 Decilitres = 1 Litre. 10 Hectolitres = 1 Kilolitre.
10 Kilolitres = 1 Myrialitre.

All the tables peculiar to the Metric System are presented together in a convenient form in the two following tables:—

TABLE OF SUBMULTIPLES AND PRINCIPAL UNITS.

Names of Units.		Pronunciation.	Symbols.
prefix.	base.		
10 Milli- Equal 1 Centi-	Metre	Mill'-e-mee'-ter	$_3$M
	Are	Mill'-e-âre	$_3$A
	Stere	Mill'-e-stêr	$_3$S
	Litre	Mill'-e-li'-ter	$_3$L
	Gramme	Mill'-e-gram	$_3$G
10 Centi- Equal 1 Deci-	Metre	Sent'-e-mee'-ter	$_2$M
	Are	Sent'-e-âre	$_2$A
	Stere	Sent'-e-stêr	$_2$S
	Litre	Sent'-e-li'-ter	$_2$L
	Gramme	Sent'-e-gram	$_2$G
10 Deci- Equal 1 Principal Unit.	Metre	Des'-e-mee'-ter	$_1$M
	Are	Des'-e-âre	$_1$A
	Stere	Des'-e-stêr	$_1$S
	Litre	Des'-e-li'-ter	$_1$L
	Gramme	Des'-e-gram	$_1$G
10 Principal Units Equal 1 Deca-	Metre	Mee'ter	M
	Are	Are	A
	Stere	Stêr	S
	Litre	Li'-ter	L
	Gramme	Gram	G

TABLE OF MULTIPLES.

Names of Units.		Pronunciation.	
PREFIX.	BASE.		
	Metre	Dek'-a-mee-ter	^{1}M
	Are	Dek'-âre	^{1}A
10 Deca- Equal 1 Hecto-	Stere	Dek'-a-stêr	^{1}S
	Litre	Dek'-a-li'-ter	^{1}L
	Gramme	Dek'-a-gram	^{1}G
	Metre	Hec'-to-mee-ter	^{2}M
	Are	Hec'-târe	^{2}A
10 Hecto- Equal 1 Kilo-	Stere	Hec'-to-stêr	^{2}S
	Litre	Hec'-to-li'-ter	^{2}L
	Gramme	Hec'-to-gram	^{2}G
	Metre	Kill'-o-mee-ter	^{3}M
	Are	Kill'-âre	^{3}A
10 Kilo- Equal 1 Myria-	Stere	Kill'-o-stêr	^{3}S
	Litre	Kill'-o-li'-ter	^{3}L
	Gramme	Kill'-o-gram	^{3}G
	Metre	Mir'-e-a-mee-ter	^{4}M
	Are	Mir'-e-âre	^{4}A
Myria-	Stere	Mir'-e-a-stêr	^{4}S
	Litre	Mir'-e-a-li'-ter	^{4}L
	Gramme	Mir'-e-a-gram	^{4}G

ABBREVIATED NOMENCLATURE.

To secure the fullest advantage to business men by the universal adoption of the new system of weights and measures, it is necessary that the names used should be short and easy to write and pronounce, that they should express clearly the relation of the different denominations of the same table to each other, and that they should be identical in all languages.

The last two of these requirements would be secured by the universal use of the nomenclature adopted by the French. It is

cosmopolitan in its character: it belongs to their language no more than to any other. The former, however, is not secured. It is evident to all, that for business purposes, the long names of the metric system are inconvenient, and that to shorten them would prove a great advantage. Efforts have been made to introduce short names; but these efforts have invariably sacrificed their universal and expressive character, which is of more importance to the business world than their shortness.

The only true course which seems to be open, is to abbreviate the names already introduced, in such a way as to retain their peculiar characteristics.

To secure this, the following plan of abbreviation is suggested:—

First. Let the prefixes be abbreviated thus: Myr, kil, hect, dec, des, cent, mil.

Second. Let the initial letter of the names of the five principal units be used, instead of the names themselves, thus: For metre, use a capital M; for are, use a capital A; for stere, a capital S; for litre, a capital L; and, for gramme, a capital G.

Third. For the names of multiples and sub-multiples, attach to these initial capital letters the abbreviated prefixes, thus: Kil M, pronounced kill-em'; Kil S, pronounced kill-ess', &c.

By this method of abbreviation, the elements of the original terms are retained in such a form that each part is clearly indicated. The capital letter used after the prefix will always point to the base-word of which it is the initial, although the pronunciation is changed.

TABLES WITH ABBREVIATED NOMENCLATURE.

MEASURES OF LENGTHS.

Written.	Pronounced.		
10 Mil M,	Mill-em',	make	1 Cent M.
10 Cent M,	Cent-em',	"	1 Des M.
10 Des M,	Des-em',	"	1 M.
10 M,	Em,	"	1 Dec M.
10 Dec M,	Dek-em',	"	1 Hect M.
10 Hect M,	Hect-em',	"	1 Kil M.
10 Kil M,	Kill-em',	"	1 Myr M.
Myr M,	Mir-em'.		

MEASURES OF SURFACES.

Written.	Pronounced.		
10 Mil A,	Mill-a',	make	1 Cent A.
10 Cent A,	Cent-a',	"	1 Des A.
10 Des A,	Des-a',	"	1 A.
10 A,	A,	"	1 Dec A.
10 Dec A,	Dek-a',	"	1 Hect A.
10 Hect A,	Hect-a',	"	1 Kil A.
10 Kil A,	Kill-a',	"	1 Myr A.
Myr A,	Mir-a'.		

MEASURES OF VOLUMES, OR SOLIDS.

Written.	Pronounced.		
10 Mil S,	Mill-ess',	make	1 Cent S.
10 Cent S,	Cent-ess',	"	1 Des S.
10 Des S,	Des-ess',	"	1 S.
10 S,	Ess,	"	1 Dec S.
10 Dec S,	Dek-ess',	"	1 Hect S.
10 Hect S,	Hect-ess',	"	1 Kil S.
10 Kil S,	Kill-ess',	"	1 Myr S.
Myr S,	Mir-ess'.		

MEASURES OF CAPACITY.

Written.	Pronounced.		
10 Mil L,	Mill-ell',	make	1 Cent L.
10 Cent L,	Cent-ell',	"	1 Des L.
10 Des L,	Dess-ell',	"	1 L.
10 L,	Ell,	"	1 Dec L.
10 Dec L,	Dek-ell',	"	1 Hect L.
10 Hect L,	Hect-ell',	"	1 Kil L.
10 Kil L,	Kill-ell',	"	1 Myr L.
Myr L,	Mir-ell'.		

MEASURES OF WEIGHTS.

Written.	Pronounced.		
10 Mill G,	Mill-gee,	make	1 Cent G.
10 Cent G,	Cent-gee',	"	1 Des G.
10 Des G,	Des-gee',	"	1 G.
10 G,	Gee,	"	1 Dec G.
10 Dec G,	Dek-gee',	"	1 Hect G.
10 Hect G,	Hect-gee',	"	1 Kil G.
10 Kil G,	Kill-gee',	"	1 Myr G.
Myr G,	Mir-gee'.		

REDUCTION OF COMPOUND DENOMINATE NUMBERS.

214. Reduction is the process of changing numbers from one denomination to another, without altering their value.

Reduction is of two kinds, *Descending* and *Ascending*.

215. Reduction Descending is changing numbers to lower denominations without altering their value; as pounds to shillings, yards to feet, etc. It is performed by *Multiplication*.

216. Reduction Ascending is changing numbers to higher denominations without altering their value; as farthings to pence, inches to feet, etc. It is performed by *Division*.

REDUCTION DESCENDING.

217. CASE I.—*To reduce a compound number to lower denominations.*

Ex. Reduce £45 7*s.* 8*d.* to pence.

OPERATION.

£45 7*s.* 8*d.*
20
907*s.*
12
10892*d.*

ANALYSIS.—There are 20*s.* in £1; therefore, 20 times the number of £ = the number of shillings. 20 times 45 = 900*s.*, to which we add 7*s.*, and obtain 907*s.* There are 12*d.* in 1*s.*; therefore, 12 times the number of shillings equal the number of pence. 12 times 907 = 10884*d.*, to which we add 8*d.*, and obtain 10892*d.* Hence the following

218. RULE.—I. *Multiply the highest denomination of the given number by that number of the scale which will reduce it to the next lower denomination, and add to the product the given number, if any, of that lower denomination.*

II. *Proceed in like manner with the results obtained in each lower denomination, until the reduction is brought to the denomination required.*

EXAMPLES FOR PRACTICE.

1. In £35 6*s.* 8*d.*, how many pence? *Ans.* 8480.
2. In £28 12*s.* 8¾*d.*, how many farthings?
3. In 14*lb.* 10*oz.* 18*pwt.* 22*gr*, how many grains? *Ans.* 85894.
4. In 165*T.* 13*cwt.* 3*qr.* 19*lb.* 14*oz.*, how many ounces?
5. In 23℔ 9℥ 0℥ 2℈ 13 gr., how many grains?
6. In 12*rd.* 8*yd.* 2*ft.*, how many feet? *Ans.* 224.
7. How many inches in 2*mi.* 4*fur.* 32*rd.* 1*yd.*?
8. In 60*arp.* 7*per.* 1*to.* 5*ft.*, how many feet?

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214. *What is* reduction?— *How many kinds of reduction?*— 215. *What is* reduction descending?— 216. Reduction ascending?— 218. *What is the* rule *for reduction descending?*

9. How many links in 7*mi.* 5*fur.* 6*ch.* 30*l.*? *Ans.* 61630.
10. In 4*mi.* 49*ch.* 72*l.*, how many links?
11. Reduce 12*A.* 3*R.* 24*sq. rd.* 144*sq. ft.* 72*sq. in.*, to square inches? *Ans.* 80937864 square inches.
12. In 10*A.* 1*R.* 25*sq. rd.* 16 *sq. yd.* 4*sq. ft.* 136*sq. in.*, how many square inches? *Ans.* 65296108 square inches.
13. How many square links in 75*A.* 4*sq. ch.* 8*P.* 118*sq. l.*?
14. How many poles in 3 townships of land?
15. In 7*sq. arp.* 30*sq. per.* 4*sq. to.* 9*sq. ft.* 40*sq. in.*, how many square inches? *Ans.* 34080952.
16. How many cubic feet in 67 cords and 74 cubic feet of wood?
17. In 30 cords of wood, how many cubic inches?
18. In $4\frac{3}{4}$*gal.* 4.25*qt.* 4.75*gi.*, how many gills? *Ans.* $190\frac{3}{4}$.
19. In 57*tuns* 3*hhd.* 50*gal.* 3*qt.*, how many pints?
20. How many pints in 10*bu.* 3.5*pk.* $7\frac{2}{3}$*qt.* 1*pt.*?
21. How many quarts in 676 chaldrons, of 36 bushels each?
22. In 4*da.* 4*h.* 45*mi.*, how many seconds? *Ans.* 362700.
23. In 3*wk.* 2*da.* 1*h.* 1*min.*, how many minutes?
24. How many days from March 17th., 1870, to May 16th. 1871?
25. In 44S. 18° 57' 23", how many seconds? *Ans.* 4820243".
26. How many minutes in $1\frac{1}{2}$C. 1S. 1° 1'?
27. Reduce 38*lb*, 6ʒ 3ʒ 1ȝ, to grains.
28. How many days from August 30th 1771, to June 1st. 1872?
29. Louis has a lump of pure silver weighing 13*lb.* 9*oz.* What is its value at $\$1.385\frac{7}{10}$ per ounce? *Ans.* $\$228.640\frac{1}{2}$.
30. Change 13*lb.* 6*oz.* Avoirdupois weight to Troy weight.
31. Purchased 3*A.* 1*R.* 30*rd.* of land, at $1.25 per square foot; what did I pay for the land? *Ans.* $\$187171.87\frac{1}{2}$.
32. Bought 2 hogsheads of sirup at 40 cts. per gal., and sold it at 12 cts. per quart; what did I gain by the bargain? *Ans.* $10.08.

219. CASE II.—*To reduce a denominate fraction to one of a lower denomination.*

Ex. Reduce $\frac{5}{896}$ of a gallon to the fraction of a gill.

OPERATION.

gal. gi.
$\frac{5}{896} \times \frac{4}{1} \times \frac{2}{1} \times \frac{4}{1} = \frac{5}{28}$.
224
112
28

ANALYSIS.—To reduce gallons to gills, we multiply successively by 4, 2, and 4, the numbers in the scale. And, since the given number is a fraction, we indicate the process, as in multiplication of fractions; and, after cancelling, obtain $\frac{5}{28}$, the answer. Hence, the

220. RULE.—*Multiply the fraction of the higher denomination by the numbers in the descending scale successsively, between the given and the required denominations.*

EXAMPLES FOR PRACTICE.

1. What part of a farthing is $\frac{1}{12880}$ of a £? *Ans.* $\frac{1}{4}$*far.*
2. Reduce $\frac{1}{14400}$ of a week to the fraction of a minute.

3. What part of a square foot is $\frac{1}{54450}$ of an acre? *Ans.* $\frac{4}{5}$*sq. ft.*
4. Reduce $\frac{1}{9600}$ of a lb. Troy to the fraction of a grain.
5. Reduce $\frac{1}{6480}$ of a £ to a fraction of a penny. *Ans.* $\frac{3}{8}$*d.*
6. Reduce $\frac{3}{88000}$ of a cwt. to the fraction of an ounce.
7. What part of a pound is $\frac{3}{14000}$ of a ton?
8. What part of a link is $\frac{1}{45}$ of a rod? *Ans.* $\frac{5}{9}$*l.*
9. Reduce $\frac{1}{19800}$ of a furlong to a fraction of a foot.
10. What part of a pint is $\frac{5}{8888}$ of a bushel? *Ans.* $\frac{5}{14}$*pt.*
11. Reduce $\frac{2}{3}$ of $\frac{1}{8}$ of 2*lb.* to the fraction of an ounce Troy.
12. What part of a square rod is $\frac{3}{13860}$ of $4\frac{1}{4}$ times $\frac{2}{19}$ of an acre?
13. What fraction of a yard is $\frac{6}{7}$ of $\frac{4}{11}$ of a rod?
14. What part of a dram is $\frac{1}{45960}$ of a hundred weight? *A.* $\frac{640}{11149}$ *dr.*
15. Reduce 0.03125 of a mile to feet.

221. CASE III.—*To reduce a denominate fraction to integers of lower denominations.*

Ex. What is the value of $\frac{3}{7}$ of a £?

OPERATION.

£	s.	d.	far.
7) 3	0	0	0
0	8	6	$3\frac{3}{7}$, *Ans.*

ANALYSIS.—$\frac{3}{7}$ of £1 is the same as $\frac{1}{7}$ of £3 = 8*s.* 6*d.* $3\frac{3}{7}$*far.* Hence, the

222. RULE.—*Consider the numerator of the fraction as so many units of the given denomination, and divide them by the denominator.*

EXAMPLES FOR PRACTICE.

What is the value of

1. $\frac{3}{11}$ of a £? *Ans.* 5*s.* 5*d.* $1\frac{9}{11}$*far.*
2. $\frac{2}{5}$ of a bushel? *Ans.* 1*pk.* 4*qt.* $1\frac{3}{5}$*pt.*
3. $\frac{3}{5}$ of a shilling?
4. $\frac{5}{9}$ of a cwt.? *Ans.* 3*qr.* 2*lb.* 12*oz.* $7\frac{1}{9}$*dr.*
5. $\frac{5}{7}$ of a yard?
6. $\frac{4}{9}$ of a lb. Avoirdupois? *Ans.* 7*oz.* $1\frac{7}{9}$*dr.*
7. $\frac{9}{75}$ of a day?
8. $\frac{6}{7}$ of 15 cwt.? *Ans.* 12*cwt.* 85*lb.* 11*oz.* $6\frac{6}{7}$*dr.*
9. $\frac{3}{8}$ of $2\frac{1}{2}$ pounds Apothecaries' weight?
10. $\frac{7}{13}$ of an acre? *Ans.* 2*R.* 6*rd.* 4*sq. yd.* 5*sq. ft.* $127\frac{5}{13}$*sq. in.*
11. $\frac{1}{7}$ of $5\frac{1}{2}$ tons?
12. $\frac{2}{19}$ of a hhd. of wine? *Ans.* 6*gal.* 2*qt.* 1*pt.* $0\frac{4}{19}$*gi.*
13. $\frac{4}{5}$ of $3\frac{3}{8}$ cords of wood?
14. $\frac{3}{5}$ of a sign? *Ans.* 12° 51' 25''$\frac{5}{7}$.
15. From a piece of velvet containing 8*yd.* 3*qr.* I cut 2*yd.* 2*qr.*; what part of the whole piece did I take?

223. CASE IV.—*To reduce a denominate decimal to integers of lower denominations.*

Ex. Reduce 0.628125 of a £ to shillings and pence.

OPERATION.

£0.628125
20
12.562500*s.*
12
6.750000*d.*
4
3.000000*far.*
£0 12*s.* 6¾*d. Ans.*

ANALYSIS.—We first multiply the given decimal, 0.628125 of a £, by 20 to reduce it to shillings, and the result is 12*s.* and the decimal .5625 of a shilling. We then multiply this decimal by 12 to reduce it to pence, and obtain 6*d.* and .75 of a *d.* This last decimal we multiply by 4, to reduce it to *far.* or *qr.*, and the result is 3 *far.* or ¾ of a *d.* Hence, the answer is £0 12*s.* 6¾d.

224. RULE.—I. *Multiply the given decimal by that number in the scale which will reduce it to the next lower denomination, and point off as in multiplication of decimals.*

II. *Proceed with the decimal part of the product in the same manner until reduced to the required denominations. The integers at the left will be the answer required.*

EXAMPLES FOR PRACTICE.

What is the value of

1. 0.45¾ of a £? *Ans.* 9*s.* 1*d.* 2⅖*far.*
2. 0.748 of a bushel? *Ans.* 2*pk.* 7*qt.* 1*pt.* 3.488*gi.*
3. 0.765 of a pound Troy?
4. 0.7525 of a mile? *Ans.* 6*fur.* 0*rd.* 4*yd.* 1*ft.* 2⅖*in.*
5. 0.659 of a week?
6. 0.217°? *Ans.* 13' 1.2".
7. 0.875 of a hhd.?
8. 0.865 of an acre? *Ans.* 3*R.* 18⅖*sq. rd.*
9. 7.88125 acres?
10. 0.625 of a fathom? *Ans.* 3¾*ft.*
11. 0.78875 of a long ton? *Ans.* 15*cwt.* 3*qr.* 2*lb.* 12.8*oz.*
12. 0.8469 of a degree?

REDUCTION ASCENDING.

225. CASE I.—*To reduce a denominate number to a compound number of higher denominations.*

Ex. In 78692*gr.*, how many pounds Troy weight?

OPERATION.

24) 78692*gr.*
20) 3278*pwt.* 20*gr.*
12) 163*oz.* 18*pwt.*
13*lb.* 7*oz.*

13*lb.* 7*oz.* 18*pwt.* 20*gr.*, *Ans.*

ANALYSIS.—24*gr.* = 1*pwt.*; therefore, $\frac{1}{24}$ of the number of grains = the number of pennyweights. $\frac{1}{24}$ of 78692 = 3278*pwt.*, and 20*gr.* remaining. 20*pwt.* = 1*oz.*; therefore, $\frac{1}{20}$ of the number of pennyweights = the number of ounces. $\frac{1}{20}$ of 3278 = 163*oz.*, and 18*pwt.* remaining. 12*oz.* = 1*lb.*; therefore, $\frac{1}{12}$ of the number

of ounces = the number of pounds. $\frac{1}{12}$ of 163 = 13*lb.*, and 7*oz.* remaining; therefore, 78692*gr.* = 13*lb.* 7*oz.* 18*pwt.* 20*gr.* Hence, the

226. RULE.—I. *Divide the given number by that number of the ascending scale which will reduce it to the next higher denomination.*

II. *Divide in like manner the quotient thus obtained, and so proceed until it is brought to the denomination required. The last quotient, with the several remainders annexed in a reversed order, will be the answer.*

EXAMPLES FOR PRACTICE.

1. In 16452*far.*, how many £? *Ans.* £17 2*s.* 9*d.*
2. In 90720 pence, how many £?
3. How many pounds in 4253? *Ans.* 4℔ 5℥ 1ʒ.
4. In 78692*gr.*, how many pounds Troy weight?
5. A physician who averages daily 5 prescriptions of 20 grains each, how many pounds of medicine will he use in one year, or 365 days? *Ans.* 6℔ 4℥ 1ɔ.
6. How many pounds of standard silver can be purchased for $1099.88, at the rate of $0.062 per *pwt.*?
7. In 87320*lb.*, how many tons? *Ans.* 43*T.* 13*cwt.* 20*lb.*
8. How much will 230*lb.* of hay cost, at $10 per ton?
9. In 1265 pints, how many bushels? *Ans.* 19*bu.* 3*pk.* 1*pt.*
10. At 6 cts. a *pt.*, how much sirup can be bought for $3.84?
11. How many francs in $176.70? *Ans.* 950.
12. In 2468 pence, how many half crowns?
13. In 90060 seconds, how many days? *Ans.* 1*d.* 1*h.* 1*mi.*
14. What would be the cost of plastering a room 18*f*. long, 16½*ft.* wide, and 9*ft.* high, at 22 cts. a *sq. yd.*? *Ans.* $22.44.
15. In a pond measuring 28*ft.* 6*in.*, how many fathoms deep is there?
16. How many bushels of oats in 27072*qt.*? *Ans.* 846*bush.*
17. How many days in 98950 seconds?
18. The extent of a certain farm is found, by survey, to be 1377*sq. ch.* How many acres does it contain? *Ans.* 137*A.* 2*R.* 32*per.*
19. A load of wood is 12 feet long and 3 feet wide, how high must it be to make a cord? *Ans.* 3⅝*ft.* high.
20. How many tons of round timber in 622080 *cu. in.*?
21. A cellar wall, 32*ft.* by 24*ft.* is 6*ft.* high and 1½*ft.* thick. How much did it cost at $1.25 a perch? *Ans.* $50.909+.
22. Reduce 16936 links to miles. *Ans.* 2*mi.* 9*ch.* 36*l.*
23. In 161384 inches, how many miles?
24. How many beer gallons are there in 1*bbl.* 1*gal.* 2*qt.*, wine measure? *Ans.* 27$\frac{3}{31}$.
25. In 5832000 square inches, how many roods?
26. Reduce 20937 minutes to signs. *Ans.* 11S. 18° 57'.
27. Change 16*lb.* 3*oz.* 1*pwt.* 1*gr.* Troy weight, to Avoirdupois weight. *Ans.* 13*lb.* 6*oz.*

28. A ship, during 3 days' storm at sea, changed her latitude 412 geographical miles; how many degrees and minutes did she change?

29. How many acres of land can be purchased in the city of Montreal for $147500, at 65 cts. a square foot? *Ans.* 5*A.* 33*per.* 15*sq. yd.* 3*sq. ft.* $119\frac{242}{1573}$*sq. in.*

30. In 13360128 drams, how many tons?

227. CASE II.—*To reduce a denominate fraction from a lower to a higher denomination.*

Ex. Reduce $\frac{4}{9}$ of a farthing to the fraction of a £.

OPERATION.

$$\frac{4}{9}\text{far.} \times \frac{1}{4} \times \frac{1}{12} \times \frac{1}{20} = \frac{1}{2160}\,£ \quad \textit{Ans.}$$

ANALYSIS.—There are 4*far.* in 1*d.*, therefore $\frac{1}{4}$ of the number of farthings equals the number of pence. There are 12*d.* in 1*s.*, therefore $\frac{1}{12}$ of the number of pence equals the number of shillings. There are 20*s.* in £1, therefore $\frac{1}{20}$ of the number of shillings equals the number of £. Hence $\frac{4}{9}$*far.* $= \frac{4}{9} \times \frac{1}{4} \times \frac{1}{12} \times \frac{1}{20} = \frac{1}{2160}$ of a £.

228. RULE.—*Divide the fraction by the numbers in the scale, successively, between the given and the required denomination.*

EXAMPLES FOR PRACTICE.

What part of

1. a pound Troy is $\frac{3}{5}$ of a grain? *Ans.* $\frac{1}{9600}$*lb.*
2. a pound is $\frac{5}{9}$ of a scruple?
3. a rod is $\frac{1}{8}$ of a foot? *Ans.* $\frac{1}{132}$*rd.*
4. a mile is $\frac{8}{9}$ of a rod?
5. a hundred-weight is $\frac{3}{8}$ of an ounce? *Ans.* $\frac{3}{3200}$*cwt.*
6. an hour is $\frac{4}{9}$ of 20 seconds?
7. an acre is $\frac{4}{5}$ of a square foot? *Ans.* $\frac{1}{54450}$*A.*
8. 3 hhd. is $\frac{4}{5}$ of a quart?
9. 4 days is $\frac{3}{4}$ of a minute? *Ans.* $\frac{1}{7680}$.
10. a cord of wood is a pile $7\frac{1}{2}$*ft.* long, 2*ft.* high, and $3\frac{1}{4}$*ft.* wide?
11. a rod is $2\frac{3}{4}$ of $\frac{7}{12}$ of an inch? *Ans.* $\frac{7}{864}$.
12. an acre is $\frac{3}{19}$ of $\frac{4}{17}$ of $9\frac{1}{2}$ square rods?
13. Reduce 9.312*far.* to the decimal of a £. *Ans.* £0.0097.
14. Reduce 517.44*ft.* to the decimal of a mile.

229. CASE III.—*To reduce a compound number to a fraction of a higher denomination.*

Ex. Reduce 8*s.* 6*d.* 2*far.* to the fraction of a £.

OPERATION.

8*s.* 6*d.* 2*far.* $= 410$*far.* $= \frac{41}{96}$£.
1£ $= 960$*far.*

ANALYSIS.—By reduction of denominate numbers (217), we find 8*s.* 6*d.* 2*far.* $= 410$*far.*, and that £1 $= 960$ *far.* One farthing is $\frac{1}{960}$ of a £, and 410*far.* $=$ 410 times $\frac{1}{960} = \frac{410}{960} = \frac{41}{96}$ of a £.

230. RULE.—*Reduce the given number to its lowest denomination for the numerator, and a unit of the required denomination to the same denomination for the denominator of the required fraction.*

EXAMPLES FOR PRACTICE.

What part of

1. a £ is 10*s.* 10*d.*? *Ans.* $\frac{13}{24}$.
2. a ton is 4*cwt.* 3*qr.* 12*lb.*? *Ans.* $\frac{487}{2000}$.
3. an acre is 2*R.* 20*per.*?
4. a mile is 1*fur.* 12*rd.* 4*yd.* 2*ft.*? *Ans.* $\frac{103}{660}$.
5. a hogshead of wine is 18*gal.* 2*qt.*?
6. a square rod is 144*ft.* $19\frac{1}{7}$*in.*? *Ans.* $\frac{9}{17}$.
7. 2*cwt.* 3*qr.* is 1*cwt.* 2*qr.* 20*lb.*?
8. 30 days is 8*da.* 17*h.* 20*min.*? *Ans.* $\frac{157}{640}$.
9. a bushel is $1\frac{3}{8}$ pecks?
10. a pound Troy is 10*oz.* 13*pwt.* 8*gr.*?

231. CASE IV.—*To reduce a compound number to a decimal of a higher denomination.*

Ex. Reduce 12*s.* 9*d.* 3*far.* to the decimal of a pound.

OPERATION.

4	3.00*far.*
12	9.7500*d.*
20	12.81250*s.*
	0.640625£. *Ans.*

Or, 12*s.* 9*d.* 3*far.* = 615*far.*
£1 = 960*far.*
$\frac{615}{960}$ = £0.640625, *Ans.*

ANALYSIS.—Since there are 4 farthings in 1*d.*, $\frac{1}{4}$ of the number of farthings equals the number of pence. $\frac{1}{4}$ of 3 = 0.75*d.* which added to 9*d.* = 9.75*d.* There are 12*d.* in 1*s.*, therefore, $\frac{1}{12}$ of the number of pence equals the number of shillings. $\frac{1}{12}$ of 9.75*d.*=0.8125*s.* which added to 12*s.*= 12.8125*s.* There are 20*s.* in £1, therefore, $\frac{1}{20}$ of the number of shillings equals the number of pounds, $\frac{1}{20}$ of 12.8125 *s.*= £0.640625. Hence, the

232. RULE.—*Divide the lowest denomination given by that number in the scale which will reduce it to the next higher denomination, and annex the quotient as a decimal to that higher. Proceed in the same manner until the whole is reduced to the denomination required.* Or,

Reduce the given number to a fraction of the required denomination, and reduce this fraction to a decimal.

EXAMPLES FOR PRACTICE.

What decimal part of

1. a gallon is 3*qt.* 1*pt.* 2*gi.*? *Ans.* 0.9375*gal.*
2. a week is 5*da.* 9*h.* 46*min.* 48*sec.*?
3. a mile is 5*fur.* 35*rd.* 2*yd.* 2*ft.* 9*in.*? *Ans.* 0.73603219 + *mi.*
4. a bushel is 3*pk.* 6*qt.* 1*pt.*?

5. a pound Troy is 10*oz.* 12*pwt.* 18*gr.*? *Ans.* 0.886458$\frac{1}{3}$*lb.*

6. a fathom is 3$\frac{3}{4}$*ft.*?

7. a ton is 16*cwt.* 3*qr.* 16.45*lb.*? *Ans.* 0.885725*T.*

8. 1$\frac{1}{4}$ bushels is 0.45 of a peck?

9. Reduce 12*T.* 3*cwt.* 2*qr.* 20*lb.* to hundred-weights and the decimal of a hundred-weight. *Ans.* 243.7.

10. Reduce to the decimal of a pound, 19*s.* 11$\frac{3}{4}$*d.*, 16*s.* 9$\frac{1}{4}$*d.*, and 17*s.* 5$\frac{1}{2}$*d.*, and find their sum. *Ans.* £2.710416+.

REDUCTION OF THE OLD CANADIAN CURRENCY TO THE NEW OR DECIMAL CURRENCY.

Ex. Reduce £72 13 9$\frac{3}{4}$ to cents.

OPERATION.

£72 × 400 = 28800 cents.
13*s.* × 20 = 260 "
9$\frac{3}{4}$ = 39*far.* × 5 ÷ 12 = 16$\frac{1}{4}$ "
£72 13 9$\frac{3}{4}$ = 29076$\frac{1}{4}$ "
or 290.76\frac{1}{4}$, *Ans.*

ANALYSIS.—We multiply £72 by 400, because each pound is equal to 4 dollars or 400 cents; next we multiply 13, the number of shillings, by 20, because each shilling is equal to 20 cents; lastly, we multiply the number of farthings in the pence and farthings by 5, and divide the remainder by 12, because each farthing is equal to $\frac{5}{12}$ of a cent.

That each farthing is equal to $\frac{5}{12}$ of a cent, is evident from the fact that 48 farthings (or one shilling) are equal to 20 cents; or 12 farthings equal 5 cents, and one farthing equals $\frac{5}{12}$ of a cent. Hence, the following

233. RULE.—I. *Multiply the pounds by* 400, *the shillings by* 20, *and take five-twelfths of the number expressing how many farthings there are in the given pence and farthings.*

II. *Add the three results together, and their sum will be the number of cents required.*

III. *Consider the last two figures as cents, and the result will be dollars and cents.*

EXAMPLES FOR PRACTICE.

How many dollars and cents in

1.	£ 4	3	1$\frac{1}{4}$?	*Ans.* 16.62\frac{1}{12}$.	10.	£16	6	2 ?	*Ans.* 65.23\frac{1}{3}$.
2.	27	16	3$\frac{1}{2}$?		11.	97	3	11$\frac{1}{3}$?	
3.	27	16	11$\frac{3}{4}$?	*Ans.* 111.38\frac{3}{4}$.	12.	46	17	7$\frac{1}{2}$?	*Ans.* 187.52\frac{1}{2}$.
4.	69	15	6 ?		13.	121	0	7 ?	
5.	0	14	8$\frac{1}{2}$?	*Ans.* 2.94\frac{1}{6}$.	14.	12	9	11 ?	*Ans.* 49.98\frac{1}{3}$.
6.	77	19	4$\frac{1}{2}$?		15.	1	12	9$\frac{1}{2}$?	
7.	17	16	5$\frac{3}{4}$?	*Ans.* 71.29\frac{7}{12}$.	16.	173	13	4 ?	*Ans.* 694.66\frac{3}{4}$.
8.	18	18	10$\frac{1}{4}$?		17.	91	8	8 ?	
9.	9	3	5$\frac{1}{4}$?	*Ans.* 36.69\frac{1}{6}$.	18.	19	11	4$\frac{3}{4}$?	*Ans.* 78.27\frac{11}{12}$.

REDUCTION OF THE DECIMAL CURRENCY TO THE OLD CANADIAN CURRENCY.

Ex. Reduce $246.88 to the old Canadian currency.

OPERATION.

4) 246.88

£61.72

20

14.40*s.*

12

4.80*d.*

4

3.20*far.*

Ans. £61 14 $4\frac{3}{4} + \frac{2}{10} = \frac{1}{5}$*far.*

ANALYSIS.—We divide 246.88 by 4, the number of dollars in a pound, and the result is £61 and 72 hundredths of a pound. We multiply 72 by 20 (224), the number of shillings in a pound, and the result is 14*s.* and 40 hundredths of a shilling. Again, we multiply 40 by 12, the number of pence in a shilling, and the result is 4*d.* and 80 hundredths of a penny. Lastly, we multiply 80 by 4, the number of farthings in a penny, and the result is 3*far.* and 20 hundredths or $\frac{1}{5}$ of a farthing. Hence, the

234. RULE.—*Divide the given number by* 4, *and the quotient will be pounds and decimals of a pound. Then proceed as in* No. 224.

EXAMPLES FOR PRACTICE.

Reduce to the old Canadian currency:—

1.	$162.30 =	*Ans.*	£40	11	6		10.	319.13\frac{1}{2}$ =	*Ans.*	£79	15	$8\frac{1}{10}$
2.	716.12						11.	933.04$\frac{1}{4}$				
3.	391.37 =	*Ans.*	97	16	$10\frac{1}{5}$		12.	601.53 =	*Ans*	150	7	$7\frac{4}{5}$
4.	537.37$\frac{1}{2}$						13.	293.17				
5.	82.19 =	*Ans.*	20	10	$11\frac{2}{5}$		14.	39.06$\frac{1}{2}$ =	*Ans.*	9	15	$3\frac{9}{10}$
6.	207.16						15.	436.99				
7.	569.09$\frac{1}{4}$ =	*Ans.*	142	5	$5\frac{11}{20}$		16.	152.18$\frac{1}{2}$ =	*Ans.*	38	0	$11\frac{1}{10}$
8.	17.35$\frac{1}{3}$						17.	846.07$\frac{3}{4}$				
9.	924.08 =	*Ans.*	231	0	$4\frac{4}{5}$		18.	719.11 =	*Ans.*	179	15	$6\frac{3}{5}$

ADDITION OF COMPOUND NUMBERS.

235. Addition, Subtraction, Multiplication, and Division of Denominate Numbers are performed by the same general methods, as are employed for like operations in Abstract Numbers. The only difference arises from *varying*, instead of *uniform scales*.

Ex. 1. What is the sum of £5 10*s.* 4*d.*, £6 16*s.* 10*d.*, £8 15*s.* 6*d.*, and £4 13*s.* 9*d.*?

OPERATION.

£	*s.*	*d.*
5	10	4
6	16	10
8	15	6
4	13	9
25	16	5

ANALYSIS.—Having written units of the same denomination in the same column, we find the sum of pence in the right-hand column to be 29 pence = 2*s.* 5*d.* We write the 5*d.* under the column of pence, and carry the 2*s.* to the column of shillings; the sum of which is 56*s.* = £2 16*s.* Having written the 16*s.* under the column of shillings, we carry the £2 to the column of pounds, and find the entire amount sought to be £25 16*s.* 5*d.*

Ex. 2. Add $\frac{7}{15}$ of a £ to $\frac{5}{7}$ of a shilling.

OPERATION.

$\frac{7}{15}$ of a £ = 9*s.* 4*d.*
$\frac{5}{7}$ of a *s.* = 0*s.* 8*d.* $2\frac{2}{7}$*far.*
Ans. 10*s.* 0*d.* $2\frac{2}{7}$*far.*

Or,

$\frac{5}{7} \times \frac{1}{20} = 1\frac{5}{140}$£.
$\frac{7}{15} + \frac{5}{140}$£ $= \frac{211}{420}$£;
$\frac{211}{420}$£ = 10*s.* 0*d.* $2\frac{2}{7}$*far.*

ANALYSIS.—We first find the value of each fraction in integers of less denominations (221), and then add the resulting or equivalent compound numbers.

Or, we may reduce the given fractions to fractions of the same denomination (219), then add them, and find the value of their sum in lower denominations. Hence, the following

236. RULE.—I. *If any of the numbers are denominate fractions, or if any of the denominations are mixed numbers, reduce the fractions to integers of lower denominations.*

II. *Write the numbers so that units of the same denominations will stand in the same column.*

III. *Beginning with the lowest denomination, add as in simple numbers, carrying to each succeeding denomination one for as many units as it takes of the denomination added, to make one of the next higher denomination.*

EXAMPLES FOR PRACTICE.

(1.)

T.	*cwt.*	*qr.*	*lb.*	*oz.*	*dr.*
71	19	3	27	14	13
14	13	2	15	15	15
14	13	1	11	13	12
11	17	3	16	15	11
13	18	2	13	11	13
127	3	2	11	8	0

(2.)

yr.	*da.*	*h.*	*min.*	*sec.*
12	10	13	42	27
16	102	18	24	36
19	8	21	54	57
23	13	19	49	48
29	18	23	58	56

(3.)

deg.	*mi.*	*fur.*	*rd.*	*ft.*	*in.*
18	19	7	15	11	1
61	47	6	39	10	11
78	32	5	14	9	9
17	59	7	36	16	10
28	56	1	30	16	1
205	$8\frac{1}{2}$	5	17	$14\frac{1}{2}$	8
	$\frac{1}{2}=4$			$\frac{1}{2}=6$	
205	9	1	17	15	2

(4.)

A.	*R.*	*per. sq. yd.*	*sq. ft.*
140	3	17	27	6
320	1	30	14	2
111		7	3	
214	2	15	22	7
100	3		6	1
25	1	36		8
104	2	9	1	4

5. What is the sum of 20*lb.* 9*oz.* 19*pwt.* 23*gr.*, 10*lb.* 7*oz.* 15*pwt.* 13*gr.*, 11*oz.* 8*gr.*, and 1*lb.* 8*oz.* 17*pwt.* 21*gr.*? *Ans.* 34*lb.* 1*oz.* 13*pwt.*

6. Find the sum of 81℔ 11ʒ 6ʒ 1ɔ 19*gr.*, 75℔ 10ʒ 7ʒ 2ɔ 13*gr.*, 14℔ 9ʒ 7ʒ 1ɔ 12*gr.*, 37℔ 8ʒ 1ʒ 1ɔ 11*gr.*, 61℔ 11ʒ 3ʒ 2ɔ 3*gr.*

Ans. 272℔ 4ʒ 3ʒ 18*gr.*

7. Add 197*sq. yd.* 4*sq. ft.* 104$\frac{1}{2}$*sq. in.*, 122*sq. yd.* 2*sq. ft.* 27$\frac{3}{4}$*sq. in.*, 5*sq. yd.* 8*sq. ft.* 2$\frac{3}{8}$*sq. in.*, and 237*sq. yd.* 7*sq. ft.* 128$\frac{1}{5}$*sq. in.*

Ans. 563*sq. yd.* 4*sq. ft.* 118.825*sq. in.*

8. What is the sum of 17*mi.* 5*fur.* 8*ch.* 3*rd.* 24*l.*, 16*mi.* 3*fur.* 7*ch.* 1*rd.* 21*l.*, 47*mi.* 7*fur.* 9*ch.* 3*rd.* 19*l.*, 19*mi.* 6*fur.* 6*ch.* 1*rd.* 16*l.*, 31*mi.* 7*fur.* 1*ch.* 20*l.*? *Ans.* 133*mi.* 7*fur.* 4*ch.*

9. Add 3S. 22° 50', 24° 36' 25.7", 17' 18.2", 1S. 3° 12' 15.5", 12° 36' 17.8", and 57.3". *Ans.* 6S. 3° 33' 14.5".

10. Find the sum of $\frac{4}{11}$ of a mile, $\frac{4}{9}$ of a mile, $\frac{3}{11}$ of a furlong, and $\frac{7}{11}$ of a yard. *Ans.* 6*fur.* 29*rd.* 3*yd.* 1*ft.* 0$\frac{10}{11}$*in.*

11. Add $\frac{4}{7}$ of a ton to $\frac{9}{17}$ of a cwt.

12. Add $\frac{5}{16}$ of a week to $\frac{1}{5}$ of a day. *Ans.* 2*da.* 9*h.* 18*min.*

13. What is the sum of $\frac{2}{5}$ of an acre and $\frac{8}{9}$ of a rood?

Ans. 3*R.* 10*sq. rd.* 8*sq. yd.* 5*sq. ft.* 113$\frac{1}{7}$*sq. in.*

14. Find the sum of $\frac{4}{5}$ of a cwt., 8$\frac{5}{8}$lb., and 3$\frac{9}{10}$oz. by long ton table.

15. A farmer received 60cts. a bushel for 4 loads of corn; the first contained 42.4bu.; the second, 2866lb.; the third, 36$\frac{3}{4}$bu.; and the fourth, 39bu. 29lb. How much did he receive for the whole? *Ans.* $101.90+.

16. Add $\frac{2}{3}$ of a yard, $\frac{5}{9}$ of a yard, and $\frac{4}{11}$ of a quarter.

SUBTRACTION OF COMPOUND NUMBERS.

Ex. 1. From £35 6*s.* 10*d.* 1*far.* take £14 15*s.* 8*d.* 3*far.*

OPERATION.

	£	*s.*	*d.*	*far.*
From	35	6	10	1
Take	14	15	8	3
Rem.	20	11	1	2

ANALYSIS.—Writing the subtrahend under the minuend, placing units of the same denomination under each other, we begin at the right-hand; since we cannot take 3*far.* from 1*far.*, we add 1*d.* or 4*far.* to 1*far.*, making 5*far.*; and taking 3*far.* from 1*far.*, we write the remainder, 2*far.*, underneath the column of farthings. Having added 1*d.* or 4*far.* to the minuend, we now add 1*d.* to the 8 in the subtrahend, making 9*d.*; and 9*d.* from 10*d.* leaves 1*d.*, which we write in the remainder. Next, as we cannot take 15*s.* from 6*s.*, we add £1 or 20*s.* to 6*s.*, making 26*s.*, and taking 15*s.* from 26*s.*, we write the remainder, 11*s.*, under the denomination of shillings. Adding £1 to £14, we subtract £15 from £35, as in simple numbers, and write the remainder, £20, under the column of £.

Ex. 2. From $\frac{5}{9}$ of a mile subtract $\frac{4}{7}$ of a furlong.

OPERATION.

$\frac{5}{9}$*mi.* =	4*fur.*	17*rd.*	4*yd.*	0*ft.*	10*in.*
$\frac{4}{7}$*fur.* =		22	4	2	1$\frac{5}{7}$
Ans.	3	34	4$\frac{1}{2}$	1	8$\frac{2}{7}$

ANALYSIS.—We perform the same reduction as in addition of denominate fractions, (234), and then subtract the less value from the greater.

Or. $\frac{5}{9}$*mi.* × 8 = $\frac{40}{9}$*fur.*

$\frac{40}{9} - \frac{4}{7} = \frac{244}{63}$ =

3*fur.* 34*rd.* 4$\frac{1}{2}$*yd.* 1*ft.* 8$\frac{2}{7}$*in.*

237. RULE.—I. *Write the subtrahend under the minuend, so that units of the same denomination shall stand under each other.*

II. *Beginning at the right-hand, subtract each denomination separately, as in simple numbers.*

III. *If any term of the minuend is less than the corresponding term of the subtrahend, add to that term as many units as are required of that denomination to make one of the next higher, and from the sum take the term of the subtrahend, and add* 1 *to the next term of the subtrahend before subtracting.*

IV. *Proceed in like manner with each denomination.*

EXAMPLES FOR PRACTICE.

(1.)

T.	*cwt.*	*qr.*	*lb.*	*oz.*	*dr.*
71	18	1	13	1	13
19	19	2	16	8	5
51	18	2	21	9	8

(2.)

lb	ʒ	ʒ	ȝ	*gr.*
15	7	3	1	14
11	9	7	2	19
3	9	3	1	15

(3.)

deg.	*mi.*	*fur.*	*rd.*	*yd.*	*ft.*	*in.*
95	3	7	31	1	1	3
18	17	1	39	1	2	7
76	55$\frac{1}{6}$	5	31	4$\frac{1}{2}$	1	8
	$\frac{1}{6}$=1	1	3	1	2	6
76	55	7	5	1	1	2

(4.)

A.	*R.*	*p.*	*ft.*	*in.*
96	1	13	100	113
89	3	17	200	117
6	1	35	171$\frac{1}{4}$	140
			$\frac{1}{4}$=	36
6	1	35	172	32

5. From £23 18*s.* 3$\frac{1}{2}$*d.* take £13 14*s.* 10$\frac{1}{4}$*d.* *Ans.* £10 3*s.* 5$\frac{1}{4}$*d.*

6. From 71*lb.* 3*oz.* 12*pwt.* 15*gr.* take 16*lb.* 10*oz.* 17*pwt.* 20*gr.*

7. Subtract 3lb 8ʒ 2ʒ 2ȝ 18*gr.* from 10lb 7ʒ 4ʒ 1ȝ 15*gr.*

8. From 171 *T.* 3*hhd.* 8*gal.* 1*qt.* 1*pt.* 1*gi.* take 99 *T.* 1*hhd.* 19*gal.* 3*qt.* 1*pt.* 3*gi.* *Ans.* 72 *T.* 1*hhd.* 51*gal.* 1*qt.* 1*pt.* 2*gi.*

9. From 56*A.* 1*R.* 19*p.* 119*ft.* 110*in.* take 17*A.* 3*R.* 13*p.* 127*ft.* 113*in.* *Ans.* 38*A.* 2*R.* 5*p.* 264*ft.* 33*in.*

10. From 16*mi.* 7*fur.* 18*rd.* 3*ft.* 1*in.* take 9*mi.* 7*fur.* 19*rd.* 16*ft.* 8*in.* *Ans.* 6*mi.* 7*fur.* 38*rd.* 2*ft.* 11*in.*

11. From $\frac{5}{8}$ of a bushel take $\frac{15}{16}$ of a peck. *Ans.* 1*pk.* 4*qt.* 1*pt.*

12. From $\frac{5}{7}$ of a week take $\frac{7}{8}$ of a day. *Ans.* 4*da.* 3*h.*

13. Subtract $\frac{2}{7}$ of 9*cwt.* from $\frac{1}{16}$ of 5 tons.

14. From 5$\frac{1}{2}$*bbl.* take $\frac{4}{5}$ of a hogshead. *Ans.* 4*bbl.* 11*gal.* 1*qt.*

15. Subtract 0.659 week from 2 weeks 3$\frac{5}{6}$ days.

16. From a hogshead of sirup containing 100 gallons, $\frac{3}{11}$ of it leaked out, and $\frac{3}{5}$ of the remainder was sold; what quantity still remained for sale? *Ans.* 24*gal.* 0*qt.* 1$\frac{8}{11}$*pt.*

PRACTICAL PROBLEMS IN COMPOUND ADDITION AND SUBTRACTION.

1. I had 10*A.* 3*R.* 10*per.* of land; and I have sold two house-lots, one containing 1*A.* 1*R.* 13*per.*, the other 2*A.* 2*R.* 5*per.*; how much have I remaining?

2. An excavation 58*ft.* long 37*ft.* wide, and 6*ft.* deep is to be made for a cellar; after 471*cu. yd.* 16*cu. ft.* 972*cu. in.* of earth have been removed, how much still remains to be taken out?

3. Bought a hogshead of sugar weighing 9*cwt.* 3*qr.* 21*lb.*; sold John 1*cwt.* 2*qr.* 15*lb.*; to Bernard 2*cwt.* 3*qr.* 24*lb.*; and to Thomas 3*cwt.* 1*qr.* 15*lb.*; how much remains unsold? *Ans.* 1*cwt.* 3*qr.* 17*lb.*

4. Joseph and Henry start from two places 120 miles apart, and travel toward each other; after Joseph travels $\frac{2}{3}$, and Henry $\frac{3}{7}$, of the distance, how far are they apart? *Ans.* 41*mi.* 7*fur.* 9*rd.* 8*ft.* $7\frac{5}{7}$*in.*

5. A man agrees to build 136 *rd.* and 15*ft.* of stone fence; at one time, he builds 36*rd.* 2*ft.*; at another time, 56*rd.* 3*ft.*; and at another time, 10*rd.* 1*ft.* How much still remains to be built?

6. A merchant sold goods to the amount of £397 18*s.* $6\frac{1}{2}$*d.*; and received in payment £199 19*s.* $10\frac{3}{4}$*d.*; how much remains due?

7. A hogshead of wine, lost by leakage, on an average, for 5 years, including two leap years, one gill of wine a day; how much remained? *Ans.* 5*gal.* 3*qt.* 1*pt.* 1*gi.*

8. Suppose a person was born February 29, 1792: how many anniversaries of his birthday will he have had on Feb. 29, 1844?

9. How long has a note to run, dated April 23, 1870, and made payable Dec. 9, 1874? *Ans.* 4*yr.* 7*mo.* 16*da.*

10. From a mass of silver weighing 106lb., a goldsmith made 36 spoons, weighing 5*lb.* 11*oz.* 12*pwt.* 15*gr.*; a tankard, 3*lb.* 0*oz.* 13*pwt.* 14*gr.*; a vase, 7*lb.* 11*oz.* 14*pwt.* 23*gr.*; how much unwrought silver remains? *Ans.* 88*lb.* 11*oz.* 18*pwt.* 20*gr.*

11. From a pile of wood containing 423 cords, I sold at one time, 56C. 112*cu. ft.*; at another time, 97C. 113*cu. ft.*; at another time, 126C. 97*cu. ft.* How many cords remain unsold?

12. Suppose a note given Sept. 10, 1856, to be paid March 5, 1868. How long was the note on interest, if we count 30 days to the month? How long, if the time is computed by days?

Ans. 1st. 11*yr.* 5*mo.* 25*da.*; 2nd. 4135 *days.*

MULTIPLICATION OF COMPOUND NUMBERS.

Ex. 1. Multiply £8 9*s.* 5*d.* by 6.

OPERATION.

£	*s.*	*d.*
8	9	5
		6
£50	16*s.*	6*d.*

ANALYSIS.—6 times 5*d.* are 30*d.* = 2*s.* 6*d.* We write the 6*d.* under the pence, and add the 2*s.* with the product of shillings. 6 times 9*s.* are 54*s.* and 2*s.*, are 56*s.* = £2 16*s.* We write the 16*s.* under the shillings and add the £2 with the product of pounds. 6 times £8 are £48, and £2 = £50 which we write under pounds. Therefore 6 times £8 9*s.* 5*d.* = £50 16*s.* 6*d.*

238. RULE.—I. *Write the multiplier under the lowest denomination of the multiplicand.*

II. *Multiply as in simple numbers, and carry as in addition of compound numbers.*

NOTES.—1. When the multiplier is large, and is a *composite* number, it is advisable to multiply by the component factors.

2. When the multiplier is large, and is not a *composite* number, it may be resolved into any convenient parts, and multiplication made by these several parts.

Ex. 2. What will 45 yards of cloth cost, at £2 3*s.* 6*d.* per yard?

OPERATION.

£	*s.*	*d.*	
2	3	6	= price of 1 yard.
		5	
10	17	6	= price of 5 yards.
		9	
£97	17*s.*	6*d.*	= price of 45 yds.

ANALYSIS.—We find the number 45 equal to the product of 5 and 9; we therefore multiply the price of 1 yard by 5, and then that product by 9; and the last product is the answer. Hence the

239. RULE.—When the multiplier is a composite number, *multiply by its factors in succession.*

Ex. 3. What cost 643 barrels of flour, at £2 5*s.* 7*d.* per bbl.?

OPERATION.

	£	*s.*	*d.*			£	*s.*	*d.*	
1 bbl. =	2	5	7,	× 3 =		6	16	9	= value of 3 bbl.
			10						
10 bbl. =	22	15	10,	× 4 =		91	3	4	= value of 40 bbl.
			10						
100 bbl. =	227	18	4,	× 6 =		1367	10	0	= value of 600 bbl.
					Ans.	1465	10	1	= value of 643 bbl.

ANALYSIS.—Since 643 is not a composite number, we cannot resolve it into factors; but we may separate it into parts, and find the value of each part separately: thus, 643 = 600 + 40 + 3. In the operation, we first multiply by 10, and obtain the value of 10 barrels, and this product we multiply by 10, and obtain the value of 100 barrels. Then, to find the value of 600 barrels, we multiply the last product by 6; and to find the value of 40 barrels, we multiply the value of 10 barrels by 4; and to find the value of 3 barrels, we multiply the value of 1 barrel by 3. Adding the several products, we obtain £1465 10*s.* 1*d.* for the answer. Hence the

240. RULE.—When the multiplier is not a composite number, *resolve it into any convenient parts, as of units, tens, etc., multiply by these several parts, and add together the products thus obtained for the required result.*

EXAMPLES FOR PRACTICE.

(1.)

cwt.	*qr.*	*lb.*	*oz.*
18	3	17	10
			6
113	2	5	12

(2.)

lb.	*oz.*	*pwt.*	*gr.*
32	8	17	12
			8
261	11	0	0

(3.)

℔	ʒ	ʒ	℈	*gr.*
38	10	5	2	14
				11
427	10	0	2	14

(4.)

mi.	*fur.*	*rd.*	*ft.*
14	6	36	14
			9

(5.)

A.	*R.*	*p.*	*sq. yd.*	*sq. ft.*
7	1	33	21	7
				6

(6.)

deg.	*mi.*	*fur.*	*rd.*
18	12	6	18
			8

7. How much cloth will it take for 8 suits of clothes, if each suit require 8*yd.* 1*qr.* 3*na.*? *Ans.* 67*yd.* 2*qr.*

8. A man gives each of his 9 sons 23*A.* 3*R.* 19⅓*p.*, what do they all receive? *Ans.* 214*A.* 3*R.* 12*p.*

9. How long will it take a man to saw eleven cords of wood, if it take him 8*h.* 45*min.* 50*sec.*, to saw 1 cord?

10. If 1 share in a certain stock be valued at £13 8*s.* 9½*d.*, what is the value of 96 shares? *Ans.* £1290 4*s.* 0*d.*

11. If a family consume 12*gal.* 3*qt.* 1*pt.* of molasses in one week, what quantity will they consume in 1 year?

12. If a man be 2*da.* 5*h.* 17*min.* 19*sec.* in walking 1 degree, how long would it take him to walk round the earth, allowing 365¼ days to a year? *Ans.* 2*y.* 68*da.* 19*h.* 54*min.*

13. What will be the value of 1 dozen gold cups, each cup weighing 9*oz.* 13*pwt.* 8*gr.*, at $212.38 a pound?

14. If a ship sails 3° 24′ 10″ per day, how far will she sail in 60 days? *Ans.* 204° 10′.

15. One ton of copper ore will buy 17*T.* 14*cwt.* 3*qr.* 18*lb.* 14*oz.* of iron ore; how much will 451 tons buy?

16. If $80 will buy 4*A.* 3*R.* 26*per.* 20*sq. yd.* 3*sq. ft.* of land, how much will $4800 buy? *Ans.* 295*A.* 10*sq. yd.*

17. If 1 cask of oil contains 86*gal.* 2*qt.* 1*pt.*, how much will 100 casks of the same size contain?

18. What is the cost of a board 18*ft.* 9*in.* long, and 2*ft.* 3½*in.* wide, at $0.053 per foot? *Ans.* $2.27 7½.

19. Bought 17 bags of hops, each weighing 4*cwt.* 3*qr.* 7*lb.*, at $5.87½ per cwt.; what was the cost?

20. What cost 27*T.* 15*cwt.* 1*qr.* 3½*lb.* of hemp, at $183.62 per ton? *Ans.* $5098.07+.

21. At $125.75 per acre, what cost 37*A.* 3*R.* 35*rd.*?

22. What cost the construction of 17*mi.* 6*fur.* 36*rd.* of railroad, at $3765.60 per mile? *Ans.* $67263.03+.

23. Bought a farm containing 144*A.* 3*R.* 30*per.*, at $97.62½ per acre; what was the cost of the farm? *Ans.* $14149.52+.

24. At $9.25 per cwt., what cost 19*cwt.* 3*qr.* 14*lb.* of iron?

MULTIPLICATION OF COMPOUND NUMBERS

SOLVED BY ALIQUOT PARTS.

TABLE OF ALIQUOT PARTS (173).

Parts of £1.	
10s.	$= \frac{1}{2}$
6s. 8d.	$= \frac{1}{3}$
5s.	$= \frac{1}{4}$
4s.	$= \frac{1}{5}$
3s. 4d.	$= \frac{1}{6}$
2s. 6d.	$= \frac{1}{8}$
2s.	$= \frac{1}{10}$
1s. 8d.	$= \frac{1}{12}$
1s. 4d.	$= \frac{1}{15}$
1s. 3d.	$= \frac{1}{16}$
1s.	$= \frac{1}{20}$

Parts of 1s.	
6 d.	$= \frac{1}{2}$
4 d.	$= \frac{1}{3}$
3 d.	$= \frac{1}{4}$
2 d.	$= \frac{1}{6}$
1½d.	$= \frac{1}{8}$
1 d.	$= \frac{1}{12}$

Parts of a cwt. (1) of 112lb.	
56 lb.	$= \frac{1}{2}$
28 lb.	$= \frac{1}{4}$
16 lb.	$= \frac{1}{7}$
14 lb.	$= \frac{1}{8}$
8 lb.	$= \frac{1}{14}$
7 lb.	$= \frac{1}{16}$
4 lb.	$= \frac{1}{28}$
3½lb.	$= \frac{1}{32}$
2 lb.	$= \frac{1}{56}$

Parts of a quarter of 28lb.	
14 lb.	$= \frac{1}{2}$
7 lb.	$= \frac{1}{4}$
4 lb.	$= \frac{1}{7}$
3½lb.	$= \frac{1}{8}$
1¾lb.	$= \frac{1}{16}$

Parts of 1lb. Avoirdupois.	
8oz.	$= \frac{1}{2}$
4oz.	$= \frac{1}{4}$
2oz.	$= \frac{1}{8}$
1oz.	$= \frac{1}{16}$

Parts of 1lb. Troy.	
6oz.	$= \frac{1}{2}$
4oz.	$= \frac{1}{3}$
3oz.	$= \frac{1}{4}$
2oz.	$= \frac{1}{6}$
1oz. 10pwt.	$= \frac{1}{8}$
1oz.	$= \frac{1}{12}$

Parts of 1oz. Troy.	
10pwt. 0gr.	$= \frac{1}{3}$
6 " 16 "	$= \frac{1}{3}$

Parts of 1oz. Troy.	
5pwt. 0gr.	$= \frac{1}{4}$
4 " 0 "	$= \frac{1}{5}$
3 " 8 "	$= \frac{1}{6}$
2 " 12 "	$= \frac{1}{8}$
2 " 0 "	$= \frac{1}{10}$
1 " 16 "	$= \frac{1}{12}$

Parts of 1 acre.	
2R.	$= \frac{1}{2}$
1R.	$= \frac{1}{4}$
20per.	$= \frac{1}{8}$
16per.	$= \frac{1}{10}$

Parts of 1 rood.	
10per.	$= \frac{1}{4}$
8per.	$= \frac{1}{5}$

Parts of a year.	
6 months	$= \frac{1}{2}$
4 "	$= \frac{1}{3}$
3 "	$= \frac{1}{4}$
2 "	$= \frac{1}{6}$
1½ "	$= \frac{1}{8}$
1 "	$= \frac{1}{12}$

Parts of a month.	
15 days	$= \frac{1}{2}$
10 "	$= \frac{1}{3}$
7½ "	$= \frac{1}{4}$
6 "	$= \frac{1}{5}$
5 "	$= \frac{1}{6}$
3 "	$= \frac{1}{10}$
2 "	$= \frac{1}{15}$
1 "	$= \frac{1}{30}$

(1) The aliquot parts of the short ton or new cwt. of 100lb. are the same as the aliquot parts of $1 (p. 105).

241. CASE I.—When the given price is: 1° *farthings*; 2° *pence*, or *pence* and *farthings*; 3° *shillings*, *shillings* and *pence*, or *shillings*, *pence* and *farthings*; 4° *pounds*, *shillings*, *pence* and *farthings*.

Ex. Find the price of 944 pens, at $\frac{3}{4}d.$ per pen.

OPERATION.

944 pens at $1d. = 944d. =$ £3 18 8

$\frac{1}{2}d. = \frac{1}{2}$ of $1d.$; $\frac{1}{2}$ of £3 18 8 = £1 19 4 = price of 944 pens at $\frac{1}{2}d.$

$\frac{1}{4}d. = \frac{1}{2}$ of $\frac{1}{2}d.$; $\frac{1}{2}$ of £1 19 4 = £0 19 8 = " " " " " $\frac{1}{4}d.$

Ans. £2 19 0 = " " " " " $\frac{3}{4}d.$

ANALYSIS.—In this example, the price being *farthings*, we multiply the given number by a penny; but, as $\frac{3}{4}d.$ is not an even part of a penny, we decompose it into $\frac{1}{2}d.$ and $\frac{1}{4}d.$; $\frac{1}{2}d.$ is the half of a penny, and $\frac{1}{4}d.$, the fourth of a penny, or the half of $\frac{1}{2}d.$ We then take the $\frac{1}{2}$ of £3 18 8 for $\frac{1}{2}d.$, giving for result £1 19 4; then $\frac{1}{4}d.$, or $\frac{1}{2}$ of $\frac{1}{2}d.$, that is, one half of £1 19 4 = 19*s.* 8*d.*, which we add to £1 19 4; the sum then gives £2 19 0, for the answer.

Ex. 2. What cost 1638lb. of sugar, at $8\frac{1}{2}$d. per lb.?

OPERATION.

1638lb. at 1*s.* = 1638*s.* = £81 18 0

6 *d.* = $\frac{1}{2}$ of 1*s.*;	$\frac{1}{2}$ of £81	18	0 =	£40	19	0 =	price of 1638lb. at 6 *d.*				
2 *d.* = $\frac{1}{3}$ of 6*d.*;	$\frac{1}{3}$ of £40	19	0 =	£13	13	0 =	"	"	"	"	2 *d.*
$0\frac{1}{2}d.$ = $\frac{1}{4}$ of 2*d.*;	$\frac{1}{4}$ of £13	13	0 =	£ 3	8	3 =	"	"	"	"	$\frac{1}{2}d.$
	Ans.			£58	0	3 =	"	"	"	"	$8\frac{1}{2}d.$

ANALYSIS.—The price being *pence* and *farthings*, we multiply the given number by a shilling. Now, as $8\frac{1}{2}d.$ is not an aliquot part of a shilling, we decompose it into 6*d.*, 2*d.*, and $\frac{1}{2}d.$, and then proceed as in the foregoing example.

Ex. 3. Find the price of 252 yards of merino, at 3*s.* $9\frac{1}{2}d.$ per yd.

OPERATION.

252 yards at £1 = £252

3*s.* 4 *d.* = $\frac{1}{6}$ of £1	£42	0	0 =	price at 3*s.* 4 *d.* per yd.							
0*s.* 5 *d.* = $\frac{1}{8}$ of 3*s.* 4*d.*	5	5	0 =	"	"	0*s.* 5 *d.*	"	"			
0*s.* $0\frac{1}{2}d.$ = $\frac{1}{10}$ of 5*d.*	0	10	6 =	"	"	0*s.* $0\frac{1}{2}d.$	"	"			
Ans.	£47	15	6 =	"	"	3*s.* $9\frac{1}{2}d.$	"	"			

ANALYSIS.—Here, the price being *shillings*, etc., we multiply the given number by a pound; then, we decompose 3*s.* $9\frac{1}{2}d.$ into 3*s.* 4*d.*, 5*d.*, and $0\frac{1}{2}d.$, and proceed as in the preceding examples.

Ex. 4. What cost 694 cwt. of butter, at £5 11 $6\frac{1}{2}$ per cwt.?

OPERATION.

£694..... = price of 694cwt. at £1 a cwt.

694cwt. × £5 =	£3470	0	0 =	"	"	694	"	"	£5	"	"
10*s.* 0 *d.* = $\frac{1}{2}$ of £1	347	0	0 =	"	"	"	"	"	0 10 0	"	"
1*s.* 3 *d.* = $\frac{1}{8}$ of 10*s.*	43	7	6 =	"	"	"	"	"	0 1 3	"	"
0*s.* 3 *d.* = $\frac{1}{5}$ of 1*s.* 3*d.*	8	13	6 =	"	"	"	"	"	0 0 3	"	"
0*s.* $0\frac{1}{2}d.$ = $\frac{1}{6}$ of 0*s.* 3*d.*	1	8	11 =	"	"	"	"	"	0 0 $0\frac{1}{2}$	"	"
Ans.	£3870	9	11 =	"	"	"	"	"	£5 11 $6\frac{1}{2}$	"	"

EXAMPLES FOR PRACTICE.

		Answers.					*Answers.*		
	s. *d.*	£	*s.*	*d.*		*s.* *d.*	£	*s.*	*d.*
1.	664 × 0 $0\frac{1}{4}$ =	0	13	10	5.	1078 × 0 $0\frac{1}{4}$ =	1	2	$5\frac{1}{2}$
2.	1732 × 0 $0\frac{1}{2}$ =	3	12	2	6.	1683 × 0 $2\frac{1}{2}$			
3.	1984 × 0 $0\frac{3}{4}$				7.	2142 × 0 $5\frac{3}{4}$ =	51	6	$4\frac{1}{2}$
4.	1896 × 0 $0\frac{1}{3}$ =	2	12	8	8.	1053 × 0 $5\frac{1}{4}$ =	23	0	$8\frac{1}{4}$

		Answers.					Answers.		
		s.	*d.*	£	*s.*	*d.*	£ *s.* *d.*	£	*s.*	*d.*
9.	5728 × 0	7¼ =		173	0	8	33. 1893 × 0 4 10½			
10.	5430 × 0	3 =		67	17	6	34. 604 × 0 8 2¾ =	248	10	5
11.	2436 × 0	6½					35. 2916 × 0 5 11½ =	868	14	6
12.	2147 × 0	3½ =		31	6	2½	36. 5348 × 0 7 8¼			
13.	7028 × 0	8¼ =		241	11	9	37. 3720 × 0 10 6½ =	1960	15	0
14.	2708 × 0	6¾					38. 1509 × 0 14 6 =	1094	0	6
15.	5491 × 0	7½ =		171	11	10½	39. 878 × 0 11 4½			
16.	4936 × 0	8¾ =		179	19	2	40. 4571 × 0 13 7½ =	3113	19	10½
17.	4967 × 0	10½					41. 54 × 1 2 9 =	61	8	6
18.	2522 × 0	11 =		115	11	10	42. 62 × 1 7 4½			
19.	2897 × 0	10¾ =		129	15	2¾	43. 17 × 4 3 11 =	71	6	7
20.	7509 × 0	11½					44. 24 × 3 13 5¼ =	88	2	6
21.	1870 × 0	9¾ =		75	19	4½	45. 472 × 5 10 3½			
22.	2244 × 0	11¼ =		105	3	9	46. 1958 × 1 18 8 =	3785	9	4
23.	392 × 1	8					47. 2471 × 5 14 9¼ =	14179	18	8¾
24.	576 × 1	9¼ =		51	0	0	48. 972 × 3 15 10			
25.	465 × 3	7½ =		84	5	7½	49. 1077 × 7 12 3 =	8198	13	3
26.	425 × 4	11¾					50. 3714 × 2 13 11¾ =	10023	18	7½
27.	1349 × 5	8 =		382	4	4	51. 1415 × 4 11 10½			
28.	7045 × 0	7					52. 2150 × 9 16 1½ =	21083	8	9
29.	2426 × 7	4¼ =		892	1	2½	53. 2175 × 5 17 10½ =	12818	18	1½
30.	1454 × 6	5½					54. 7251 × 8 7 7¾			
31.	3632 × 9	7 =		1740	6	8	55. 6494 × 6 19 5¾ =	45288	17	8½
32.	6741 × 2	6¾ =		863	13	9¾	56. 7122 × 9 13 4½ =	68860	16	9

242. Case II.—When there is a *fraction* in the given quantity.

Ex. Required the price of 158¾ yards of cloth, at £1 2 11 per yd.

OPERATION.

158¾ yards, at £1 2 11

2*s.* 6*d.* = £⅛	19	15	0	= price of 158 yd. at 2*s.* 6*d.*
5*d.* = ⅙ of 2*s.* 6*d.*	3	5	10	= " " " " " 0*s.* 5*d.*
½ of £1 2 11	0	11	5½	= " " ½ "
½ of 11*s.* 5½*d.*	0	5	8¾	= " " ¼ "
Ans....	£181	18	0¼	

Analysis.—In this process, the price of 158 yards is first found (or rather the parts composing it are found) according to the method of Case I.; and then, for ¼ yard, the half of £1 2 11 is taken, and for ¼ yard, the half of that is found: the sum of all which parts, is £181 18 0¼, the result required.

ANOTHER METHOD.

158¾ yards, at £1 2 11.

	£158	15	0	= price at £1.....per yard.
2*s.* 6*d.* = £⅛	19	16	10½	= " " 0 2*s.* 6*d.* " "
0*s.* 5*d.* = ⅙ of 2*s.* 6*d.*	3	6	1¾	= " " 0 0*s.* 5*d.* " "
Ans....	£181	18	0¼	= " " £1 2 11 " "

ANALYSIS.—In this method, we first find the price of 158¾ yards, at £1 per yard. This is £158 15 0; for the price of 158 yards is £158, and, the price of a quarter of a yard being evidently 5*s.* 0*d.*, that of ¾ of a yard is 15*s.* Then, the price at £1 per yard being £158 15, the price at 2*s.* 6*d.* will be one eighth of this, or £19 16 10½; and the price at 5*d.*, one sixth of the price at 2*s.* 6*d.*, or £3 6 1¾. The sum of these is £181 18 0¼, the whole price, as before.

EXAMPLES FOR PRACTICE.

		£ *s.* *d.*		£ *s.* *d.*
1. 187 ½	×	1 17 8	=	*Ans.* 353 2 6
2. 328 ¾	×	0 6 6	=	*Ans.* 106 16 10½
3. 208 ⅞	×	0 13 10		
4. 971 ¼	×	3 15 2	=	*Ans.* 3650 5 7½
5. 675 ½	×	1 7 2	=	*Ans.* 917 11 1
6. 371 ⅕	×	3 14 7½		
7. 538 ¾	×	0 4 8	=	*Ans.* 125 14 2
8. 495 ⅚	×	3 5 9¾	=	*Ans.* 1630 13 9
9. 917 ⅛	×	4 18 10½		
10. 515 ⅝	×	2 9 4	=	*Ans.* 1271 14 0⅜.
11. 63 ⅖	×	3 18 9	=	*Ans.* 249 12 9
12. 85 ⅓	×	2 7 6		
13. 172 7/10	×	3 15 10	=	*Ans.* 654 16 5
14. 176 ¾	×	0 11 8	=	*Ans.* 103 2 1
15. 785 6/7	×	7 6 3¾		
16. 239 ¾	×	1 10 10	=	*Ans.* 369 12 3½
17. 375 ⅝	×	4 19 11¾	=	*Ans.* 1877 7 8⅞
18. 759 1/11	×	2 15 9¼		
19. 774 ⅗	×	9 11 6	=	*Ans.* 7416 15 10⅘
20. 749 1/12	×	8 19 10¼	=	*Ans.* 6736 5 9 5/12

243. CASE III.—When the given quantity and price are both of several *denominations*.

Ex. What is the cost of 94*cwt.* 2*qr.* 15*lb.* of tobacco, at £5 12 6 per cwt.?

OPERATION.

£94 = cost of 94 cwt. at £1.

94*cwt.* × £5 =	£470	0	0	=	cost of 94*cwt.*	at	£5					per	cwt.	
10*s.* 0*d.* = £½.	47	0	0	=	"	"	"	"	10*s.*			"	"	
2*s.* 6*d.* = ¼ of 10*s.*	11	15	0	=	"	"	"	"	2*s.* 6*d.*			"	"	
2*qr.* = ½ of 1*cwt.*	2	16	3	=	"	"	2*qr.*	at	£5	12	6	"	"	
10*lb.* = ⅕ of 2*qr.*	0	11	3	=	"	"	10*lb.*	"	"	"	"	"	"	
5*lb.* = ½ of 10*lb.*	0	5	7½	=	"	"	5*lb.*	"	"	"	"	"	"	
Ans....	£532	8	1½	=	cost required									

ANOTHER METHOD.

£5 12 6 = cost of 1*cwt.*

£5 12 6 × 94 =	£528	15	0	=	cost of 94*cwt.*			at	£5	12	6	per	cwt.	
2*qr.* = ½*cwt.*	2	16	3	=	"	"	2*qr.*	"	"	"	"	"	"	
10*lb.* = ⅕ of 2*qr.*	0	11	3	=	"	"	10*lb.*	"	"	"	"	"	"	
5*lb.* = ½ of 10*lb.*	0	5	7½	=	"	"	5*lb.*	"	"	"	"	"	"	
Ans....	£532	8	1½	=	"	"	94 2 15	at	£5	12	6	per	cwt.	

EXAMPLES FOR PRACTICE.

1. 85*cwt.* 2*qr.* 7*lb.* at £0 17 4 per cwt. *Ans.* £ 74 3 $2\frac{14}{25}$.
2. 78*cwt.* 2*qr.* 21*lb.* at £4 14 8 " " *Ans.* £ 372 11 $2\frac{1}{2}$+
3. 19*cwt.* 3*qr.* 14*lb.* at £4 11 6 " "
4. 129*cwt.* 1*qr.* 16*lb.* at £2 12 6 " " *Ans.* £ 339 14 $0\frac{3}{10}$.
5. 144*cwt.* 3*qr.* 8*lb.* at £4 5 8 " " *Ans.* £ 620 7 $1\frac{6}{23}$.
6. 168*cwt.* 1*qr.* 17*lb.* at £2 15 9 " "
7. 285*cwt.* 3*qr.* 7*lb.* at £1 18 10 " " *Ans.* £ 554 19 4+.
8. 346*cwt.* 1*qr.* 4*lb.* at £1 12 7 " " *Ans.* £ 564 3 $1\frac{3}{4}$.
9. 181*cwt.* 3*qr.* 53*lb.* at £2 13 4 " "
10. 175*tons* 18*cwt.* 1*qr.* at £38 13 0 per ton. *Ans.* £6799 0 4+.
11. 93*oz.* 7*pwt.* 15*gr.* at £0 10 4 per oz. *Ans.* £ 48 4 $11\frac{1}{4}$.
12. 58*yd* 3*qr.* 1*na.* at £0 12 8 per yd.
13. 8*A.* 3*R.* 19*per.* at £0 18 10 per acre. *Ans.* £ 8 7 $0\frac{3}{8}$+
14. 58*arp.* 8*per.* 4*ft.* at £2 10 6 per arp. *Ans.* £ 148 10 $6\frac{4}{15}$.
15. 11*A.* 1*R.* 23*per.* at £1 3 7½ per acre.

DIVISION OF COMPOUND NUMBERS.

Ex. 1. If 5 barrels of sugar weigh 9*cwt.* 1*qr.* 10*lb.*, how much will 1 barrel weigh?

OPERATION.

	cwt.	qr.	lb.
5)	9	1	10
	1	3	12

ANALYSIS.—One fifth of 9*cwt.* is 1*cwt.* and 4*cwt.* = 16*qr.* remaining, to which we add the 1*qr.*, and have 17*qr.* 1 fifth of 17*qr.* is 3*qr.* and 2*qr.* = 50*lb.* remaining, to which we add the 10*lb.* and have 60*lb.* 1 fifth of 60*lb.* is 12*lb.* Therefore, 1 fifth of 9*cwt.* 1*qr.* 10*lb.* = 1*cwt.* 3*qr.* 12*lb.*

244. RULE.—I. *Divide the highest denomination as in simple numbers, and each succeeding denomination in the same manner, if there be no remainder.*

II. *If there be a remainder after dividing any denomination, reduce it to the next lower denomination, adding in the given number of that denomination, if any, and divide as before.*

III. *Proceed in like manner with all the denominations. The several partial quotients will be the quotient required.*

NOTES.—1. When the divisor is large, and is a *composite* number, we may shorten the work by dividing by the component factors.

2. When the divisor and dividend are both compound numbers, they must be reduced to the same denominations, and the division then is the same as in simple numbers.

Ex. 2. When 24 yards of silk velvet are sold for £57 10 0, what is the price of 1 yard?

OPERATION.

	£	s.	d.	
6)	57	10	0	= price of 24 yards.
4)	9	11	9	= price of 4 yards.
	2	7	11	= price of 1 yard.

ANALYSIS.—24 is equal to 6 × 4. We therefore divide the price by one of these factors, and the quotient arising by the other. Hence, the

245. RULE.—*Divide by the factors of the composite number in succession.*

Ex. 3. Divide £360 8 4 by 173.

OPERATION.

	£	s.	d.	
173)	360	8	4	(£2
	14			
	20			
173)	288			(1*s.*
	173			
	115			
	12			
173)	1384			(8*d.*
	1384			

ANALYSIS.—We divide the pounds by 173, and obtain £2 for the quotient, and £14 remaining, which we reduce to shillings, and add the 8*s.*; and again, divide by 173, and obtain 1*s.* for the quotient. The remainder, 115*s.*, we reduce to pence, and add the 4*d.*, and again divide by 173, and obtain 8*d.* for the quotient. Thus, the method is the same as by general rule (244). By uniting the several quotients, we obtain £2 1 8, for the answer.

Ex. 4. Divide £24 3 8 by £3 0 5½.

OPERATION.

$$\frac{£24\ 3s.\ 8d.\ 0far.}{£\ 3\ 0s.\ 5d.\ 2far.} = \frac{23216\,far.}{2902\,far.} = 8.$$

ANALYSIS.—Reducing both dividend and divisor to the lowest denomination mentioned in either, and then dividing as in simple numbers, we have 8 for the quotient.

EXAMPLES FOR PRACTICE.

(1.)

	T.	*cwt.*	*lb.*
7)	45	15	25
	6	10	75

(2.)

	lb.	*oz.*	*dr.*
9)	143	5	5
	15	14	13

(3.)

	hhd.	*gal.*	*qt.*	*pt.*
12)	9	28	2	
	49	2	1	

4. A man in 1 month travels 746*mi.* 5*fur.*, how far does he go in 1 day? *Ans.* 24*mi.* 7*fur.* 4*rd.*

5. If 21 yards of cloth cost £10 8 3, what is the price of 1 yard?

6. If 35 loads of coal weigh 72*T.* 14*cwt.* 2*qr.* 10*lb.*, what will 1 load weigh? *Ans.* 2*T.* 1*cwt.* 2*qr.* 6*lb.*

7. Divide 28° 51' 27.756" by 2.754. *Ans.* 10° 28' 42½".

8. Divide 1275*A.* 2*R.* 16*per.* 22*yd.* 8*ft.* 32*in.* equally among 32 persons. *Ans.* 39*A.* 3*R.* 18*per.* 0*yd.* 6*ft.* 64*in.*

9. When 96 shares of a certain stock are valued at £1290 4*s.* 0*d.*, what would be the cost of 1 share?

10. If a town 4 miles square be divided equally into 124 farms, how how much will each farm contain? *Ans.* 82*A.* 2*R.*. 12$\frac{28}{31}$*per.*

11. Divide 57*T.* 19*cwt.* 42*lb.* 14*oz.* by 123.

12. If a man walk round the earth in 2*yr.* 68*da.* 19*h.* 54*min.*, how long would it take him to walk 1 degree, allowing 365¼ days to a year? *Ans.* 2*da.* 5*h.* 17*min.* 19*sec.*

13. Divide 916*mi.* 3*fur.* 30*rd.* 10*ft.* 6*in.* by 47.

14. How many times are £5 10 10 contained in £537 10 10?

15. Divide 336*bu.* 3*pk.* 4*qt.* by 4*bu.* 3*pk.* 2*qt.* *Ans.* 70.

16. Divide 121℔ 3℥ 2℈ 1℘ 4*gr.* by $13\frac{3}{4}$.

17. A merchant sold to each of a certain number of farmers 6*bu.* 1*pk.* 7*qt.* of grass seed, and to them all he sold 71*bu.* 5*qt.* How many farmers were there? *Ans.* 11.

18. Divide 3794*cu. yd.* 20*cu. ft.* $709\frac{1}{2}$*cu. in.* by $33\frac{1}{3}$.

LONGITUDE AND TIME.

246. *Meridians of Longitude* are direct lines on the globe, from the north pole to the south pole, crossing the equator at right angles.

247. *Longitude* is distance on the globe, east or west of a determined meridian. In the British Isles and on this continent, also generally on the ocean, the meridian of Greenwich Observatory, England is the determined meridian. All parts of the earth on this line are considered to have no longitude.

The highest longitude any place on the earth can have is 180° east, or 180° west from the determined meridian.

248. The **Equator** and parallels of latitude being circles, are divided into 360°, called degrees of *Longitude.*

NOTES.—1. The earth revolves on its axis from west to east once in 24 hours, which constitute a solar day. The middle of this day is 12 noon. When the sun is directly over the meridian of a place, it is noon at that place, and at places west of this meridian the time is before noon; at those east, the time is after noon.

2. The whole circle of the earth = 360° which pass under the sun in 24 hours, and in one hour passes $\frac{1}{24}$ of 360° = 15°. One hour = 60 minutes; hence, in 1 minute passes $\frac{1}{60}$ of $15^0 = \frac{15^0}{60} = \frac{1^0}{4} = 15'$. One minute = 60 seconds; hence, in 1 second passes $\frac{1}{60}$ of $15' = \frac{15'}{60} = \frac{1'}{4} = 15''$. Hence, the following

COMPARISON OF LONGITUDE AND TIME.

15° of longitude	=	1 hour of time.
15′ of longitude	=	1 minute of time.
15″ of longitude	=	1 second of time.

249. RULE.—I. *The difference of longitude between two places, expressed in degrees, minutes, and seconds, divided by* 15 *will give their difference in time expressed in hours, minutes, and seconds.*

II. *The difference of time in two places, expressed in hours, minutes, and seconds, multiplied by* 15 *will give their difference in longitude expressed in degrees, minutes, and seconds.*

NOTES.—1. If one place be in east, and the other in west longitude, the difference of longitude is found by *adding* them; and, if the sum be greater than 180°, it must be subtracted from 360°.

2. Since the sun appears to move from east to west, when it is exactly 12 o'clock at one place, it will be *past* 12 o'clock at all places east, and *before* 12 at all places west. Hence, if the difference of time between two places, be *subtracted* from the time at the *easterly* place, the result will be the time at the westerly place; and, if the difference be *added* to the time at the *westerly* place, the result will be the time at the easterly place.

EXAMPLES FOR PRACTICE.

1. Quebec is in longitude 71° 16' west, and Toronto, 79° 21' west. When it is 12 o'clock at Toronto, what is the time at Quebec?

OPERATION.

79° 21'
71° 16'
15) 8° 5'
0*h.* 32*mi.* 20*sec.*
12
12*h.* 32*mi.* 20*sec.*

ANALYSIS.—The difference of longitude is 8° 5'. Dividing by 15 and changing to time gives 32*mi.* 20*sec.* for the difference of time between the two places; and, as Quebec is east of Toronto, the time is later, and we add the difference of time, which gives 12*h.* 32*mi.* 20*sec.* the time at Quebec.

2. The longitude of Halifax is 63° 35' 30" west, and that of Ottawa is 75° 41' west; when it is 10 o'clock 12*min.* A. M. in Halifax, what time is it at Ottawa?

3. The longitude of Valparaiso is 71° 37' west, and the longitude of Rome is 20° 30' east; when it is 11 o'clock 15*min.* A. M. at Valparaiso, what is the time at Rome? *Ans.* 23*min.* 28*sec.* past 5 P. M.

4. The longitude of New Orleans is 90° 7' west, of Philadelphia, 75° 10' west; what is the time at N. O. when it is 8 o'cl. 20*min.* 40*sec.* at Philadelphia? *Ans.* 7*h.* 20*min.* 52*sec.*

5. When it is noon at St. Paul's, Minnesota, longitude 93° 5' west, it is at Bangor 1*h.* 37*min.* 12*sec.* P. M.; what is the longitude of Bangor, Maine? *Ans.* 68° 47' west.

6. The longitude of Jerusalem is 35° 32' east, and the longitude of Montreal 73° 25' west; when it is 10 o'cl. A. M. at Jerusalem, what time is it at Montreal? *Ans.* 2*h.* 44*min.* 12*sec.* A. M.

7. The longitude of Boston is 71° 4' 9" west, and when it is 10 o'clock A. M. in Boston, it is 8 o'clock 53*min.* 57$\frac{3}{5}$*sec.* in Chicago; what is the longitude of Chicago? *Ans.* 87° 34' 45".

8. The longitude of Constantinople is 28° 48' east, and of Kingston, Canada, 75° 41' west; when it is 3 o'cl. P. M. at the latter place, what time is it at the former? *Ans.* 9*h.* 57*min.* 56*sec.* P. M.

9. A captain at sea finds by his chronometer that it is 3*h.* 40*min.* 30*sec.* P. M., at Greenwich, when it is 1*h.* 10*min.* 45*sec.* by solar time on board his vessel; in what longitude is the vessel?

Ans. 37° 26' 15" west.

DUODECIMALS.

250. Duodecimals are denominate numbers, the denominations of which increase according to the *scale* of 12; or denom-

inate fractions, whose denominators are 1, 12, 144, 1728, etc. In practice, duodecimals are applied to the measurement of extension, the foot being taken as the unit.

TABLE.

12 fourths, marked (''''),	make	1 third,	marked 1'''
12 thirds	"	1 second,	" 1''
12 seconds	"	1 prime, or inch,	" 1'
12 primes, or inches,	"	1 foot,	" *ft.*

The marks ', '', ''', '''', are called *indices*.

251. Duodecimals are added and subtracted in the same manner as compound numbers.

MULTIPLICATION OF DUODECIMALS.

Ex. How many square feet in a floor 9*ft.* 7' long and 7*ft.* 9' wide?

OPERATION.

9*ft.* 7'
7*ft.* 9'
7*ft.* 2' 3''
67*ft.* 1'
74*ft.* 3' 3''

ANALYSIS.—Beginning at the right, 7' × 9' = 63'' = 5' 3''; writing the 3'' one place to the right, we reserve the 5' to be added to the next product. Then, 9*ft.* × 9' + 5' = 86' = 7*ft.* 2', which we write in the places of feet and primes. Next, multiplying by 7*ft.*, we have 7' × 7*ft.* = 49' = 4*ft.* 1'; writing the 1' in the place of primes, we reserve the 4*ft.* to be added to the next product. Then, 9*ft.* × 7*ft.* + 4*ft.* = 67*ft.*, which we write in the place of feet. Adding the partial products, we have 74*ft.* 3' 3'' for the product required. Hence, the

252. RULE.—I. *Write the several terms of the multiplier under the corresponding terms of the multiplicand.*

II. *Multiply each term of the multiplicand by each term of the multiplier separately, beginning with the lowest denomination in the multiplicand, and the highest in the multiplier, and write the first figure of each partial product one place to the right of that of the preceding product, under its corresponding denomination, carrying* 1 *for every* 12.

III. *Finally, add the several partial products; their sum will be the required answer.*

EXAMPLES FOR PRACTICE.

1. How many square feet in a piece of marble 12*ft.* 7' long, and 4*ft.* 3' wide? *Ans.* 53*ft.* 5' 9''.

2. What is the area of a floor, the length of which is 9*ft.* 8' 11'', and width 3*ft.* 7'? *Ans.* 34*ft.* 10' 11'' 5'''.

3. How many square feet in 10 boards, each 18*ft.* 10' long and 1*ft.* 8' wide? *Ans.* 313*ft.* 10' 8''.

4. If a block of granite be 7*ft.* 6' long, 3*ft.* 3' wide, and 1*ft.* 10' thick, what are the solid contents? *Ans.* 44*ft.* 8' 3".

5. How many square feet of boards will it take to inclose a piece of land 80*ft.* 10*in.* long, and 60*ft.* 8*in.* wide, with a close fence 7*ft.* 6*in.* high? *Ans.* 2122*ft.* 6'.

6. What will the plastering of a room cost, at 18 cents a square yard, the length of which is 30*ft.* 10*in.*, width 24*ft.* 6*in.*, and height of ceiling 8*ft.* 4'? *Ans.* $33.55.

DIVISION OF DUODECIMALS.

Ex. There are 8*ft.* 5' 3" in the surface of a marble slab, the length of which is 3*ft.* 9'; what is its width?

OPERATION.

3*ft.* 9') 8*ft.* 5' 3" (2*ft.* 3', *Ans.*
7*ft.* 6'
11' 3"
11' 3"
0

ANALYSIS.—3*ft.* is contained in 8*ft.*, 2 times. Multiplying the whole divisor by 2*ft.* gives 7*ft.* 6' for the product, which we subtract from the corresponding denominations of the dividend, and obtain 11' for a remainder, to which we annex the next denomination of the dividend, and have 11' 3". 3*ft.* is contained in 11', 3 times. The divisor being multiplied by this 3' gives 11' 3", which being subtracted from the last remainder, leaves nothing. Therefore, the marble slab was 2*ft.* 3' in width.

253. RULE.—I. *Divide the highest term of the dividend, by the highest term of the divisor; multiply the divisor by this term of the quotient, and subtract the product from the dividend.*

II. *To the result bring down the next term of the dividend, and divide as before.*

EXAMPLES FOR PRACTICE.

1. Divide 184*ft.* 3' by 40*ft.* 11' 4". *Ans.* 4*ft.* 6'.
2. Divide 41*ft.* 8' 7" 6"' by 7*ft.* 4'. *Ans.* 5*ft.* 8' 3"2"'
3. A table whose length is 6*ft.* 9' 7", has an area of 28*sq. ft.* 3' 11" 2"'; what is its width? *Ans.* 4*ft.* 2'.
4. What is the length of an alley whose area is 792*ft.* 6' 9" 2"', and width, 12*ft.* 7' 8"? *Ans.* 62*ft.* 8' 6".
5. A block of marble contains 64*ft.* 2' 5"; its width is 2*ft.* 6', and its thickness 3*ft.* 7'; what is its length? *Ans.* 7*ft.* 2'.
6. What is the width of a rectangular pond, whose length is 43*ft.* 9' 6", and area, 1075*sq. ft.* 3" 6"' 6"" ? *Ans.* 24*ft.* 6' 7".
7. A stick of timber is 3*ft.* 2' wide, 2*ft.* 11' thick, and contains 135*cu. ft.* 10' 2" 1"'. What is its length?
8. It required 834*sq. ft.* 3' of board to cover the side of a certain building. The height was 17*ft.* 9*in.*; what was the length of the side? *Ans.* 47 feet.

MISCELLANEOUS EXAMPLES.

1. At 1*s.* 6*d.* sterling per yard, how many yards of linen may be bought for £5 6 6? *Ans.* 71*yd.*

2. Reduce 456575 grains to pounds, Apothecaries' weight.

3. If 22¾ gallons of wine be bought for £30 2 10½, what is the cost of each gallon? *Ans.* £1 6 6.

4. What is the value of 15*cwt.* 3*qr.* 14*lb.* of tea, at $950 per cwt.?

5. What would be the expense of making a turnpike 87*mi.* 3*fur.* 15*rd.*, at $578.75 per mile? *Ans.* 50595.41\frac{1}{4}$.

6. What cost 7*hhd.* 47*gal.* of gin, at $87.25 per hogshead?

7. Goliath was 6½ cubits high; what was his height in feet, the cubit being 1*ft.* 7.168*in.*? *Ans.* 10*ft.* 4.592*in.*

8. Reduce 3*cwt.* 1*qr.* 7*lb.* of hay to the decimal of a long ton.

9. A farmer having 17*cwt.* 2*qr.* 19*lb.* of pork, sold 4*cwt.* 3*qr.* 21*lb.* of it, and the remainder he put into 6 barrels; how much did each barrel contain? *Ans.* 2*cwt.* 12$\frac{1}{6}$*lb.*

10. Bought by Avoirdupois weight, 15*lb.* opium, at 4*cts.* a dram, and sold the same by Apothecaries' weight, in doses of 10*gr.* each, at 25 cts. per dose; how much did I gain? *Ans.* $2471.40.

11. How many solid feet in a stick of timber 34*ft.* 9*in.* long, 1*ft.* 3*in.* wide, and 1*ft.* 6*in.* deep? *Ans.* 65.15625*ft.*

12. What is the value of a field 15*ch.* 75*l.* long, and 12*ch.* 50*l.* wide, at $64 per acre? *Ans.* $1260.

13. What part of 4*gal.* 3*qt.* is 2*qt.* 1*pt.* 2*gi.*? *Ans.* $\frac{11}{16}$.

14. Thirty-two men construct 28*mi.* 4*fur.* 32*rd.* of road; after completing ½ of it, ¼ of the number of men left. What distance did each man construct before and after ¼ of the men left?

Ans. 3*fur.* 23*rd.* before, and 4*fur.* 30¾*rd.* after.

15. If it require 3*h.* 20*min.* for a man to cut 1 cord of wood, how many days of 8 hours each will be required to cut 746 cords, 95 feet? *Ans.* 311*da.* 1*h.* 10*min.*

16. A housebreaker, having stolen property to the value of £9 12 9¾, was sentenced to pay at the rate of £9 12 9¾ for every pound stolen. How much was the fine? *Ans.* £92 18 9¾+.

17. Bought 4 barrels of cranberries, each containing 2½*bu.* at $8 per barrel, and retailed the same at 12½ cts. per quart, wine measure. How much was my profit? *Ans.* 14.54\frac{6}{11}$.

18. Andrew received ⅓ of a certain quantity of flour, Edward $\frac{1}{11}$ of it, and Louis the remainder. Now it is found that Andrew has 76*lb.* and 8*oz.* more than Edward. How much did each receive?

Ans. A received 210$\frac{3}{8}$*lb.*, E 133$\frac{7}{8}$*lb.*, and L 1128$\frac{3}{8}$*lb.*

19. A man having a hogshead of sirup, sold $\frac{4}{15}$ of it to F, ⅓ of the remainder to G, and ⅓ of the residue to J. How many gallons remained? *Ans.* 19*gal.* 1*qt.* 1*pt.* $\frac{4}{15}$*gi.*

20. Find the value in Troy weight of 13*lb.* 8*oz.* 11.4*dr.* Avoirdupois weight. *Ans.* 16*lb.* 5*oz.* 10*pwt.* 11.7+*gr.*

21. How much butter, at 18¾ cents a pound, must be given for 12*gal.* 3*qt.* of molasses, at 37½ cents a gallon?

22. The wall of a cellar is 20 feet square on the inside, 8 feet high, and 1½ feet in thickness; how many perches of masonry are there?

23. The total yield of nine copper mines in 1868, was 3942 *T.* 12*cwt.* 1*qr.* 1*lb.*; in 1869, the same mines yielded 4101 *T.* 8*cwt.* 3*qr.* 3*lb.*; if copper was worth 20 cts. per lb., of how much greater value was the amount produced in 1869, than 1868? *Ans.* $63530.40.

24. Sold 15*cwt.* 22*lb.* of rice at $3.75 a cwt., and 7*cwt.* 36*lb.* of pearl barley, at $4.25 a cwt. How much would be gained by selling the whole at $4\frac{1}{2}$ cts. a pound? *Ans.* 13.25\frac{1}{2}$.

25. Bought a lot 25 rods long and 20 rods wide for $10000, and sold the same at 25 cts. per square foot. How much was my gain?

26. Sold 72 yds. carpeting at 1.37\frac{1}{2}$ a yd., and gained $18. How much did it cost me per yard? *Ans.* 1.12\frac{1}{2}$.

27. How many square yards in the walls of a room 40 feet long, 31$\frac{1}{2}$ feet wide and 12 feet high?

28. How many tons of hay, at $0.75 per cwt., must be given for 35 cords of wood, at $0.60 per cord foot? *Ans.* 11$\frac{1}{5}$ tons.

29. Purchased a farm, containing 176*A.* 3*R.* 25*rd.*, at 75.37\frac{1}{2}$ per acre; what did it cost? *Ans.* $13334.308+.

30. What will be the expense of plastering a room 40*ft.* long, 36$\frac{1}{2}$*ft.* wide, and 22$\frac{1}{4}$*ft.* high, at 18 cents a sq. yd., allowing 1375*sq. ft.* for doors, windows, and base board? *Ans.* 69.78\frac{1}{2}$.

31. When it is 11 A. M. at a place 30° east of Greenwich, it is 3*h.* 44*min.* 20*sec.* A. M. at Buffalo, United States; what is the longitude of Buffalo? *Ans.* 78° 55' west.

32. Nineteen lots of equal size contain 159*A.* 2*R.* 17*sq. rd.* 25*sq. yd.* 8*sq. ft.* 130*sq. in.* What is the value of one lot, the land being worth 50 cts. per square foot? *Ans.* 18296\frac{5}{9}$.32.

33. Sold 4 building lots of ground; the first contained $\frac{1}{8}$ of $\frac{2}{5}$ of an acre; the second, 40$\frac{3}{8}$ rods; the third, $\frac{1}{7}$ of an acre; and the fourth, $\frac{2}{7}$ of $\frac{5}{8}$ of an acre. How much land in the four lots? *Ans.* 3*R.* 7$\frac{5}{7}$*per.*

34. How much beef, at 7*d.* per pound, ought I to receive for 27*lb.* 12*oz.* of butter at 1*s.* 9*d.* per lb.? *Ans.* 83$\frac{1}{4}$*lb.*

35. The difference in longitude between London and St. Louis, Mo., is 90° 20'; at a certain time each day it is as much past noon in London as it lacks of noon in St. Louis. What is the time in St. Louis? *Ans.* 8*h.* 59*min.* 20*sec.* A. M.

36. Express in acres and the decimal of an acre the area of 49 square lots, each measuring 5*rd.* 8*ft.* 3*in.* on a side.

37. On an acre of ground there were erected 21 buildings occupying on an average 3*sq. rd.* 112*sq. ft.* 8*sq. in.* How much of the acre remained unoccupied? *Ans.* 88*per.* 97*sq. ft.* 12*sq. in.*

38. Reduce $\frac{3}{7}$ of $\frac{5}{8}$ of 45$\frac{1}{2}$*lb.* to the decimal of a short ton.

39. A person lived in Montreal until he was 18*yr.* 8*mo.* 24*da.* old: in Toronto, $\frac{1}{3}$ as long; in Kingston, $\frac{2}{3}$ as long as in Toronto, and $\frac{1}{2}$ as long in Quebec as in Kingston. What was his age? *A.* 31*yr.* 2*mo.* 20*da.*

40. A farmer owning 195*A.* 3*R.* 38*sq. rd.* of land, divided $\frac{3}{4}$ of it equally among his four sons. How much did each son receive, and how much had the father remaining?

Ans. 36*A.* 2*R.* 39$\frac{3}{4}$*sq. rd.* each, and 48*A.* 3*R.* 39$\frac{1}{2}$*sq. rd.* remaining.

41. A steamer, going from Halifax to Liverpool, traversed 10$\frac{1}{2}$ degrees of longitude daily. What length of time was it from one noon to the next? *Ans.* 23*h.* 18*min.*

42. What cost 0.01975 of a ton of steel at 20 cents per pound?

43. A man having a field 30 rods square, sold 25 square rods to one of his neighbors, and 20 rods square to another. What is the value of the remainder at $175 per acre? *Ans.* $519.53⅛.

44. A man paid $46.50 for a certain pile of wood. Measuring it he found that it contained 5*cd.* 6*cd. ft.* 12*cu. ft.* What did the wood cost him per cord? *Ans.* $7.957+.

45. A grocer lost from ⅞ of a hogshead of molasses, ⅘ of a gallon and ⅝ of a quart. How much of the hogshead, expressed decimally, leaked out, and how much remained?

Ans. 1.008⅓ *gal.* leaked out, and 54.11⅔ *gal.* remained.

46. From a piece of cloth containing 92 yards, 8 coats, each containing 3 yd. were taken; required the value of the remainder at $5.32½ a yard? *Ans.* $362.

47. If a gallon of distilled water weigh 8*lb.* 5*oz.* 6.74*dr.*, what is the weight of 17*gal.* 3*qt.* 1*pt.* 1*gi.*? *Ans.* 149*lb.* 5*oz.* 1.19*dr.*

48. At 3½ cents per foot, what will be the cost of 12 planks, each measuring 56*ft.* 9"? *Ans.* $23.83½.

49. If, when wheat is worth 6*s.* 3*d.* per bushel, a 5-cent loaf weighs 24*oz.*, and allows the baker 1½ cts. a loaf for his labor, what should it weigh when wheat is 8*s.* 4*d.* per bushel, to afford him the same profit on a loaf? *Ans.* 18*oz.*

50. How much will it cost to carpet a room 21*ft.* long, 15*ft.* wide, with carpeting ¾ of yd. wide, at $1.62½ per yd.?

51. What is the value of a pile of wood that is 10*rd.* long, 4*ft.* wide, and 1½*yd.* high, at $5.75 per cord? *Ans.* $133.42 —

52. My garden, which is 180*ft.* long, and 150*ft.* wide, is surrounded by a tight-board fence 5½*ft.* high. How much will it cost to paint the fence on both sides at 12 cts. per sq. yd.? *Ans.* $93.86⅔.

53. A merchant purchased in Manchester 34 bales of cloth for £8 19 5 per bale; he disposed of the cloth at Porto-Rico for 212*cwt.* of sugar, at £1 5 per cwt. Did he lose or gain? and how much?

54. If a person spends in 6 months what he earns in 4½ months; how many dollars can he lay by in a year, supposing he earns $325 in 2½ months? *Ans.* $390.

55. A man has a piece of land 201⅔ rods long and 47¼ rods wide, which he wishes to lay out into square lots of the greatest possible size. How many lots will there be? *Ans.* 396.

56. If a man can pay his creditors only 48 cents on a dollar, how much can he pay on a debt of $52.50?

57. How many bricks, 8*in.* long, 4*in.* wide, and 2½*in.* thick, are required to build the front of a house whose wall is 30*ft.* long, 24*ft.* high, and 2*ft.* thick, allowing the doors and windows to occupy ¼ the surface? *Ans.* 23328 bricks.

58. If I buy 145 *gal.* 2 *qt.* of molasses, at 20 cts. a gal., and use 35 *gal.* of it, how must I sell the remainder per gal., so as to receive as much as the whole cost? *Ans.* $0.263+.

59. If I buy 120 gallons of rum for $75, how much water must be added to it that I may sell it at 60 cents per gallon, and gain $15 in the sale of it? *Ans.* 30 *gal.*

60. Sold 125 equal loads of wood, measuring 115*Cd.* 3*cd.* *ft.* 7*cu.* *ft.* for $492.50. What is the quantity per load, and price per cord?

Ans. 118$\frac{1}{5}$*cu ft.* each load, 4.26\frac{2}{3}$ per cord.

61. How many francs must a merchant in Paris send to Montreal in payment for a debt of $15989.862?

62. If a man fill $\frac{1}{3}$ of a cask with brandy, $\frac{1}{4}$ with wine, and $\frac{1}{5}$ with water, and if it lack 21$\frac{2}{3}$ gallons of being full, how many gallons will that cask contain? *Ans.* 100*gal.*

63. If by selling cloth at 10*s.* 6*d.*, $\frac{1}{3}$ of the price is gain, what part of the cost would be gained by selling it at 13*s.*?

64. A ship's chronometer, set at Greenwich, points to 5*h.* 45*min.* 24*sec.* P. M., when the sun is on the meridian. What is the ship's longitude? *Ans.* 86° 21' E.

65. A grocer bought 15 barrels of salt, of 4 bushels each, at 1\frac{2}{3}$ a barrel, and retailed it at $\frac{2}{3}$ of a cent a pint. How much was his whole gain? *Ans.* $4.60.

66. James owns $\frac{5}{7}$ of a field, and Leo the remainder; $\frac{3}{4}$ of the difference between their shares is 5*A.* 3*R.* 16$\frac{1}{2}$*per.* What is Leo's share? *Ans.* 20*A.* 3*R.* 9$\frac{7}{8}$*per.*

67. A gentleman desirous of giving 1*s.* 6*d.* apiece to some needy boys, found that he had not money enough in his pocket by 5*d.*; he therefore gave them 1*s.* 4*d.*, and had 9*d.* left. Required the number of boys. *Ans.* 7.

68. A liquor agent has 50 gallons of wine of superior quality, worth $7.50 a gallon; he wishes to reduce its quality by the addition of water, so that he may sell it at $5.25 a gallon. How much water must he add? *Ans.* 21$\frac{3}{7}$*gal.*

69. A clothier has 960 soldiers' coats to make, each coat containing 2$\frac{1}{4}$*yd.* of cloth 1$\frac{5}{8}$*yd.* wide, and lined with drilling $\frac{3}{4}$*yd.* wide. How many yards of lining will be required?

70. A ship captain, sailing from London to Portland, found, on taking an observation, that the sun at noon was 3*h.* 25*min.* 40*sec.* earlier than the London time, as shown by his chronometer. How many degrees west had he sailed?

71. My father's garden is 10$\frac{3}{4}$ rods long, and 8$\frac{2}{3}$ rods wide, and surrounded by a fence 7$\frac{2}{5}$ feet high; he has laid out a walk around it, within the fence, 7$\frac{1}{2}$ feet wide on the two sides, and 5$\frac{1}{2}$ feet wide on the ends. How much remains for cultivation? *Ans.* 21296*sq. ft.*

72. A boy having been sent to a store with 5$\frac{1}{2}$ doz. of eggs, was directed to purchase with them equal quantities of sugar, coffee, butter and tea; he disposed of his eggs at the rate of 2 for 5 cents, and paid for the articles purchased 17, 28, 37$\frac{1}{2}$ and 137$\frac{1}{2}$ cents per pound, respectively. What amount of each did he purchase?

RATIO.

254. **Ratio** is that relation between two numbers or quantities, which is expressed by the *quotient* arising from the division of the one by the other. Thus, the ratio of 12 to 4 is $12 \div 4 = 3$.

255. The **Terms** of a ratio are the two numbers compared.

256. A **Couplet** is the two terms of a ratio taken together.

257. The **Antecedent** is the first term, or dividend.

258. The **Consequent** is the second term, or divisor.

259. A ratio may be expressed either by two dots (:) between the terms; or in the form of a fraction, by making the antecedent the numerator and the consequent the denominator. Thus, the ratio of 8 to 4, may be expressed as 8 : 4, or as $\frac{8}{4}$.

260. A ratio is either *direct* or *inverse*.

261. A **Direct Ratio** is the quotient of the antecedent by the consequent. Thus, 8 to 4 is $\frac{8}{4}$ or 2.

262. An **Inverse,** or **Reciprocal Ratio,** is the quotient of the consequent by the antecedent. Thus, 8 to 4 is $\frac{4}{8}$ or $\frac{1}{2}$.

263. A **Simple Ratio** is that having but *one antecedent* and *one consequent;* it may be either direct or inverse. Thus, 6 : 3, or $\frac{1}{6} : \frac{1}{3}$.

264. A **Compound Ratio** is the product of two or more ratios. Thus, the ratio compounded of 6 : 3 and 8 : 4 is $\frac{6}{3} \times \frac{8}{4} = \frac{48}{12} = 4$, or $6 \times 8 : 3 \times 4 = 4$.

265. From the foregoing we deduce the following principles of ratio.

1st. *Multiplying the consequent divides the ratio; dividing the consequent multiplies the ratio.*

2nd. *Multiplying the antecedent multiplies the ratio; dividing the antecedent divides the ratio.*

3rd. *Multiplying or dividing both antecedent and consequent by the same number does not alter the ratio.*

EXAMPLES FOR PRACTICE.

What is the direct ratio of

1. 54 to 6? *Ans.* 9.
2. 108 to 18?
3. 7 to 21? *Ans.* $\frac{1}{3}$.
4. 17 to 68?
5. 60 to 12?
6. 13 to 52? *Ans.* $\frac{1}{4}$.
7. 53 to 212?
8. 72*yd.* to 9*yd.*?
9. 60*mi.* to 4*fur.*? *Ans.* 120.
10. 3*qt.* to 20*gal.*?

Required the inverse ratio of

11. 27 to 81. *Ans.* 3.
12. 72 to 8.
13. 16 to 48.
14. 42 to 6. *Ans.* $\frac{1}{7}$.
15. .02 to 2.503.
16. 256 to 32.

17. Which is the greater, the ratio of 86 to 240, or of 45 to 72?

18. What is the ratio compounded of 35 to 40, 60 to 75, and 21 to 19? *Ans.* $\frac{147}{190}$.

19. If the consequent be 32 and the ratio $4\frac{4}{7}$, what is the antecedent? *Ans.* 7.

20. If the antecedent be $7\frac{1}{2}$ and the ratio $\frac{5}{8}$, what is the consequent? *Ans.* 12.

PROPORTION.

266. **Proportion** is the equality of ratios. It is indicated thus, $6 : 3 :: 8 : 4$; or thus, $6 : 3 = 8 : 4$, and is read 6 is to 3 as 8 is to 4; or the *ratio* of 6 to 3 = the *ratio* of 8 to 4. Hence every proportion has two couplets and four terms.

267. The **Extremes** are the first and fourth terms.

268. The **Means** are the second and third terms.

269. Since in a proportion, the ratio of the first to the second term is equal to the ratio of the third to the fourth term, the proportion, $6 : 3 :: 8 : 4$, becomes $\frac{6}{3} = \frac{8}{4}$, multiplying each member by 3 and 4, we have $6 \times 4 = 8 \times 3$. Hence,

In every proportion, the product of the means is equal to the product of the extremes.

270. From the foregoing principles and illustrations, it follows that, any three terms of a proportion being given, the fourth may readily be found by the following

271. RULE.—I. *Divide the product of the extremes by one of the means, and the quotient will be the other mean. Or,*

II. *Divide the product of the means by one of the extremes, and the quotient will be the other extreme.*

NOTE.—We will denote the required term of a proportion by the letter x.

EXAMPLES FOR PRACTICE.

1. Find the value of x in the proportion,

$$9 : 16 :: 36 : x;\ x = \frac{16 \times 36}{9} = 64,\ \textit{Ans.}$$

What is the value of x in each of the following proportions:

1. $24 : 96 :: 14 : x$? *Ans.* 56.
2. $7 : 42 :: x : 96$? *Ans.* 16.
3. $16 : x :: 10 : 40$? *Ans.* 64.
4. $42 : 70 :: 3 : x$?
5. $x : 15 :: 3 : 9$? *Ans.* 5.
6. $\$7\frac{1}{2}$ $\$10 :: 36bu. : x\,bu.$?
7. $2yd. : 8yd. :: \$3\frac{1}{2} : x$?
8. $7.50 : 18 :: x\,oz. : 7\frac{1}{5}oz.$?

SIMPLE PROPORTION.

272. Simple Proportion is an equality of two simple ratios, and consists of four terms.

NOTE.—Simple Proportion is often called the *Rule of Three*, from the circumstance that three terms being given to find a fourth.

Ex. 1. If 12 yards of cloth cost \$30, what will 42 yards cost at the same rate?

OPERATION.

yd. yd. \$ \$
12 : 42 :: 30 : x

42

60

120

12) 1260

x = \$105, *Ans.*

ELUCIDATION.—To arrange the given numbers in the order of a proportion, or *state the question*, we make \$30 the *third* term, because it is of the same kind as the required *fourth* term; and, as from the nature of the question the latter must be greater than the third term, we make the greater of the other two numbers the *second* term, and the less the *first*; and then, the product of the means divided by the given extreme, gives the required extreme.

THE SAME EXAMPLE BY ANALYSIS.

If 12yd. cost \$30, 1 yard will cost $\frac{1}{12}$ of \$30 = \$2.50; then, if 1 yd. cost \$2.50, 42yd. will cost 42 times \$2.50 = \$105, the answer, as before.

Ex. 2. If 49 soldiers consume a certain quantity of flour in 28 days, how long will it take 70 soldiers to consume it?

OPERATION.

Soldiers. Soldiers. days. days.
70 : 49 :: 28 : x

$\frac{\cancel{49} \times \cancel{28}}{\cancel{70}} = 19\frac{3}{5}$, *Ans.*

$\cancel{10}$

5

ELUCIDATION.—Since the required answer is days, we make the given days the *third* term. Then, as the flour will not last 70 soldiers so long as it will 49 soldiers, we make 49 soldiers, the smaller of the two terms, the *second* term, and 70 soldiers the *first* term; and proceed as in the first example, except that we shorten the work by cancellation.

THE SAME EXAMPLE BY ANALYSIS.

If 49 soldiers consume the flour in 28 days, it will take 1 soldier 49 times 28 days = 1372 days; then, if 1 soldier consume the flour in 1372 days, 70 soldiers will consume it in $\frac{1}{70}$ of 1372 days = $19\frac{3}{5}$ days.

273. RULE.—I. *Write the given number that is of the same name or kind as the required fourth term, or answer, for the third term of the proportion.*

II. *Of the other two numbers, write the larger for the second term, and the less for the first, when the answer should exceed the third term; but write the less for the second term, and the larger for the first, when the answer should be less than the third term.*

III. *Multiply the second and third terms together, and divide their product by the first; or divide the third term by the ratio of the first term to the second.*

NOTES.—1. When the first and second terms are of different denominations, they must be reduced to the same denomination; and when the third term is a compound number, it must be reduced to the lowest denomination mentioned in it. The answer will be of the same denomination as the third term.

2. The pupil should perform these questions by analysis, as well as by proportion, and introduce cancellation when it will abbreviate the operation.

EXAMPLES FOR PRACTICE.

1. Six laborers earn $7.68; how much will 10 laborers earn? 36 laborers? *Ans.* $12.80; $46.08.

2. If 23yd. of cloth cost £25 8 3; how much will 198yd. cost? 126yd.? 137yd.? *Ans.* £218 15 $4\frac{10}{23}$; £139 4 $3\frac{2}{23}$; £151 7 $4\frac{19}{23}$.

3. One-half a bushel of salt costs $45\frac{1}{4}$ cts.; how much will 16 bushels cost? 24 bushels? 72 bushels? $85\frac{1}{2}$ bushels? $90\frac{3}{4}$ bushels? $105\frac{1}{8}$ bushels? *Ans.* $14.56; $30.94; $65.52; etc.

4. 126lb. of butter cost $16.38; how many lb. can be had for $12.61? $25.74? $32.37? $36.10? *Ans.* 97lb.; 198lb.; etc.

5. If a cwt. of tobacco is worth $30.25; what is the value of 1lb.? $7\frac{1}{2}$cwt.? 56lb.? 93lb. 4oz.? $107\frac{3}{4}$lb.? *Ans.* $0.3925; 294.37\frac{1}{2}$; etc.

6. The $\frac{3}{4}$ of a cwt. of sugar cost $6.48; what will be the cost of $\frac{7}{9}$ of a cwt.? $\frac{5}{6}$ cwt.? $\frac{5}{8}$ cwt.? $\frac{4}{5}$ cwt.? *Ans.* $6.72; $7.20; etc. +

7. If 40$\frac{1}{4}$ arpents of land are worth $215.50; what is the value of 6 arpents? 70 perches? 90 toises? 25$\frac{1}{2}$ arpents? 10 per. 4 to. 10 ft.? 110$\frac{1}{2}$ arpents? *Ans.* 31.92\frac{6}{27}$; 3.72\frac{8}{81}$; 0.53\frac{7}{81}$; etc.

8. The $\frac{4}{15}$ of an acre produce 18 cwt. 1 qr. 12 lb. of hay; what quantity will 1 acre produce? 8$\frac{1}{2}$ acres? 36$\frac{1}{4}$ per?

9. At 1s. 8d. per lb., what quantity of coffee can be had for £3 6s.? £9 15 6? £11 7 2$\frac{1}{2}$? £14 0 10$\frac{1}{4}$? *Ans.* 39$\frac{3}{5}$lb.; 117$\frac{3}{10}$lb.; etc.

10. If 19 gallons of oil cost $35.67, how much will 37gal. cost? 42$\frac{1}{2}$gal.? 50$\frac{1}{6}$gal.? 65$\frac{1}{3}$gal.? $\frac{3}{8}$ of a gal.? *Ans.* $71.41; 82.02\frac{1}{2}$; etc.

11. I paid $78.80 for 11 tons of coal; how much must I pay for 15 tons? 3$\frac{3}{4}$ tons? 18$\frac{4}{5}$ tons? *Ans.* 107.45\frac{5}{11}$; $24.67 +; etc.

12. If 3$\frac{3}{4}$lb. of coffee cost 72 cts., how much must be paid for 74$\frac{1}{2}$lb.? 90$\frac{1}{4}$lb.? 109$\frac{1}{2}$lb.? 2$\frac{1}{4}$cwt.? *Ans.* $14.62 +; $18.90; etc.

13. Six cwt. 1qr. 11lb. of beef cost £13 7 6, what quantity can be had for £8 12 3? £1 0 8? £17 12 6?

14. For 17$\frac{3}{5}$ days' work, $25.44 were paid; how much will be paid for 1 day's? 45$\frac{1}{3}$ days'? 89$\frac{1}{3}$ days'? *Ans.* $1.44; $65.52; etc.

15. The rent of a farm containing 12A. 2R. 30per. is $113.75; what is the rent of another containing 5A. 1R.? 16$\frac{1}{4}$A.? 59A. 2R. 20per.? 10$\frac{3}{8}$A.? *Ans.* 47.06\frac{3}{4}$ +; $145.24 +; etc.

16. Seven bushels of rice cost $8.75; how much will 12$\frac{1}{2}$ bushels cost? 18$\frac{7}{11}$ bushels? 26$\frac{3}{8}$ bushels?

17. In paying 11\frac{3}{4}$ for 1128 feet of boards, what quantity can be had for $119.$\frac{7}{22}$? $230.60? $370.95? *Ans.* 1145$\frac{1}{4}$ + ft.; etc.

18. I can get 336 pens for 3s. 6d.; how many can I get for £1 3 4$\frac{1}{2}$? £3 10 1$\frac{1}{2}$? £0 1 10$\frac{1}{2}$? *Ans.* 2244; etc.

19. If 15 yards of cloth are worth 60 yards of linen, how many yards of linen can be procured for 75 yards of cloth? *Ans.* 300.

20. If £100 give £7 interest in 12 months, how much interest will be accumulated in 42 months? in $5\frac{1}{4}$ years? *Ans.* £24 10 0; etc.

21. A workman earned $140 in 20 days; how much would he have earned, had he worked 6 days more? *Ans.* $182.

22. If 5 peaches cost as much as 7 apples, how many apples can be had for 35 peaches? 280 peaches? *Ans.* 49 apples; etc.

23. If 3 men can do a piece of work in 51 days, how many men must be added to this number to do it in 17 days? in 9 days?

24. If £1 14s. is paid for the transportation of a weight of 400lb., 97 leagues; to what distance should it be transported for £6 16s.?

25. A merchant having declared bankruptcy, compromised with his creditors to pay $0.64 on the dollar; how much will one receive on a debt of $2563.50? *Ans.* $1640.64.

26. What will be the price of 21A. 3R. 20per. of land, if 36A. 3R. cost £315? *Ans.* £187 10 0.

27. If 10cwt. 2qr. 14lb. of sugar cost $204, how much should we pay for 3cwt. 1qr. 14lb.?

28. When pomegranates are 40 cts. the hundred, what will a dozen come to? *Ans.* $0.048.

29. If $260.70 will purchase 11lb. 4oz. 10gr. of gold, what weight can be had for $39.50?

30. What is the length of a plane surface of one square foot, of which the breadth is $2\frac{1}{2}$ inches? *Ans.* $57\frac{3}{5}$ inches.

31. A manufacturer having failed owes $900 to B. $1200 to C, $1400 to D, and $1500 to E. The value of his property is $2800; how much will each creditor receive? *Ans.* B $504; C $672; D $784; E $840.

32. What must be paid for 450ft. of split wood, at £4 5s. per M ft.? *Ans.* £1 18 3.

33. If a bowl containing 2 cubic yd. is emptied in 12 minutes; how many hours will be required to empty a cistern 4yd. long, 3yd. wide, and $2\frac{1}{2}$yd. deep? *Ans.* 3 hours.

34. One of two pieces of cloth costs $335, the other, $390; what is the length of each, knowing that the second is 11yd. longer than the first? *Ans.* 1st. 67yd.; 2nd. 78yd.

35. How long will it take to pump 54 barrels of water, if 24 barrels can be pumped in 1h. 45min.?

36. A workman received $264 for 44 days' labor; how much would he have received by working 14 days more? *Ans.* $348.

37. What is the value of $\frac{3}{32}$ of a boat, knowing that the $\frac{7}{16}$ cost £51? *Ans.* £10 18 $6\frac{6}{7}$.

38. If the moon moves 13° 10' 35" in one day, in what time will it perform its revolution? *Ans.* 27da. 7h. 43 + min.

39. I bought 4950 copies on condition that I should receive 6 per cent more; how many shall I receive? *Ans.* 5247.

40. What is the value of 7lb. 11oz. of gold, knowing that 7oz. are worth $120? *Ans.* 1628.57\frac{1}{7}$.

41. The $\frac{4}{5}$ of a bushel of prunes cost $\$\frac{12}{25}$, what part of a bushel can be bought for $\$\frac{7}{20}$?

42. In retailing merchandise for the sum of £5600, I lost \$4.50 on every \$100; how much did I disburse?

43. A pound of cinnamon costs \$1.10; for how much should I retail it to gain at the rate of \$50 on every \$1000? *Ans.* \$1.15½.

44. When metallic pens are 6¼ cts. a dozen, how much will 10¾ gross cost? 16⅙ gross? 25⅔ gross? *Ans.* \$8.06¼; etc.

45. When profits are \$50 on every 100 yards of cloth, how many yards must be sold to raise a profit of \$850? *Ans.* 1700yd.

46. What will be the price of 7 chests of tea, each containing 2¾cwt., if 51lb. cost £8 10s.?

47. One of two numbers is to the other as 5 : 7½, and the smaller is 164.5; what is the greater?

48. Two pieces of cloth are respectively 41 and 36 yards; the first piece costs \$45 more than the second; required the price of each. *Ans.* 1st. \$369; 2nd. \$324.

49. When wheat is sold at 7s. 6d. the bushel, a loaf of bread weighs 9 ounces; what should be the weight if wheat is but 6s. the bushel? *Ans.* 11¼oz.

50. Every soldier in a regiment of 1000 men is to have a watch-coat; each coat will take 3¾yd. of cloth which is 1⅞yd. wide, and is to be lined with flannel, 1¼yd. wide; how many yards of flannel will be required to line the whole? *Ans.* 5625yd.

51. To draw success on my business, I propose to give \$5 to the poor every time I gain \$150; how much will I have gained when my alms amount to \$100? *Ans.* \$3000.

52. John can plough a certain field in 5 days, and Maurice in 6 days; what time will both take, working together, to plough the field? *Ans.* 2$\frac{8}{11}$ days.

53. A father earns 6s. 5d. per day, his son, 3s. 7½d; in what time will they have economised £1 .0 3, if they expend but 5s. per day? *Ans.* 6 days.

54. How much must I pay for paving a yard which is 605ft. long and 44ft. wide, at 1425sq. ft. cost \$34½?

55. Two gangs composed of 20 and 30 men respectively, did 1500 yards of a certain work in 25 days; how much would they have done had their number been augmented by 15? *Ans.* 1950 yards.

56. One hundred degrees of Centigrade are equivalent to 80 degrees of Reaumur; to how many degrees of Reaumur will 23¾ degrees of Centigrade equal? *Ans.* 18$\frac{14}{15}$° of Reaumur.

COMPOUND PROPORTION.

274. Compound Proportion is an expression of equality between a compound and a simple ratio, or between two compound ratios. Thus,

$$\left.\begin{matrix} 12 : 6 \\ 8 : 4 \end{matrix}\right\} :: 24 : 6, \text{ is a compound proportion.}$$

That is, $12 \times 8 : 6 \times 4 :: 24 : 6$; for, $12 \times 8 \times 6 = 6 \times 4 \times 24$.

NOTE.—Compound proportion embraces that class of questions whose solution would require two or more statements in Simple proportion. It is sometimes called *Double Rule of Three.*

Ex. If 6 men can earn $72 in 10 days, by working 8 hours a day, how many dollars can 9 men earn in 5 days, by working 12 hours a day?

STATEMENT.

Men.	$.	Da.	Hr.
6	72	10	8
9	x	5	12

NOTE.—To aid in remembering the question and in forming the ratios, the pupil should write the conditions upon his slate, or blackboard, as in the margin.

METHOD BY PROPORTION.

OPERATION I.

$$\left.\begin{matrix} 6 : 9 \\ 10 : 5 \\ 8 : 12 \end{matrix}\right\} :: 9 : \$72 : x;\ x = \$81.$$

OPERATION II.

$$x = \frac{9 \times 5 \times 12 \times 72}{6 \times 10 \times 8} = \$81.$$

METHOD BY ANALYSIS.

If 6 men in 10 days of 8 hours each earn $72, 1 man in the same time will earn $\frac{1}{6}$ of $72 = $12; and 9 men will earn 9 × $12 = $108. If in 10 days of 8 hours each, 9 men earn $108, in 1 day they will earn $\frac{1}{10}$ of $108 = $10.80; and in 5 days, 5 × $10.80 = $54. If in 5 days by working 8 hours a day, 9 men earn $54, by working 1 hour a day, they will earn $\frac{1}{8}$ of $54 = $6.75; and, by working 12 hours a day, they will earn 12 × $6.75 = $81.

275. RULE.—I. *Make that number which is of the same kind as the answer required, the third term of a proportion.*

II. *Then take the other numbers in pairs, or two of a kind, and arrange them as in simple proportion.*

III. *Finally, multiply the product of the second terms by the third, and divide the result by the product of the first terms. The quotient will be the fourth term, or answer.*

NOTE.—By reference to the above statement, of the question, either method of solution is so plain as to require no rule.

EXAMPLES FOR PRACTICE.

1. Twelve horses can plough 11 acres of land in 5 days; how many horses will it require to plough 33 acres in 18 days? *Ans.* 10.

2. If $240 produce $50 in 9 months, what sum will $450 produce in 5 months? *Ans.* $13.88⅓.

3. Received $21 for 15 days' work of 7 horses, each drawing with an average power of 2250 pounds; how much should be received in proportion for 25 days' work of 5 horses, each drawing with a power of 2430 pounds? *Ans.* $27.

4. By selling 75 otter skins, which cost me $3.60 each, I made a profit of $24; how much would I have gained in proportion on 45 Virginia silvered fox skins, which cost $7.70 each? *Ans.* $30.80.

5. If 144 men, in 6 days, of 12 hours each, build a wall 200ft. long, 3ft. high, and 2ft. thick; in how many days, of 7 hours each, will 30 men build another wall 350ft. long, 6ft. high, and 3ft. thick?

6. If it require 45 tailors to make 300 coats in 36 days, how many will be required to make 200 in 27 days? *Ans.* 40.

7. If 18 men in 24 days, by working 12 hours a day can make 2880 locks; how many men, in 9 days, by working 10 hours a day can make 450 locks? *Ans.* 9.

8. If 6 horses eat 70 bushels of oats in 9 days, how many can be fed with 280 bushels in 27 days? *Ans.* 8.

9. In how many days will 6 persons consume 5 bush. of potatoes, if 3bu. 3pk. suffice for 9 persons during 22 days? *Ans.* 44 days.

10. If 15000 lb. of flour are sufficient to maintain 1500 men during 80 days in a citadel; by how much should this quantity be increased that it may last 2450 men for 232 days? *Ans.* 56050 lb.

11. During 18 days, of 8 hr. each, 14 laborers were employed at a piece of work 136yd. long and 9yd. high; how many yards will 36 laborers do, working 7 hr. per day, during 14 days? *Ans.* 238yd.

12. How many planks $10\frac{2}{3}$ft. long and $1\frac{1}{2}$ inches thick, will be necessary to replace 3000 planks, 12ft. 8in. long and $2\frac{3}{4}$in. thick?

13. The $\frac{3}{4}$ of a wall was constructed by 15 masons in 12 days, after which 7 left; how long did it take the others to finish the work?

14. To perform a piece of work, $46\frac{1}{4}$yd. long, 11 laborers were obliged to work $10\frac{3}{4}$ hours a day; how many men would it require to do $41\frac{5}{8}$yd. of the same labor, working $8\frac{1}{4}$ hr. per day? *Ans.* 12 men.

15. Paid \$12 for the painting of 5 doors, each measuring 8ft. in height by 3ft. 6in. in breadth; how much should be paid for the painting of 7 windows, each 9ft. high by 4ft. wide, reckoning 2 doors for 3 windows? *Ans.* \$14.40.

16. If 300 bushels of wheat at 6s. 3d., liquidate a certain debt, how many bushels at 4s. 6d. will it require to liquidate a debt 3 times larger? *Ans.* 1250 bushels.

17. If the carriage of 5cwt. 3qr. a distance of 150 miles costs \$24.58, what must we pay for the carriage of 7cwt. 3qr. a distance of 64 miles at the same rate? *Ans.* \$14.135+.

18. In a fort there are provisions enough for 1520 soldiers for 5 months. If the garrison be augmented by 100 men, what daily ration can be allowed them, if they remain $1\frac{2}{3}$ mo. longer?

19. If $4\frac{3}{4}$d. is paid for a loaf, weighing $7\frac{1}{4}$oz., when wheat is 4s. 2d. the bushel; what should a 1s. 2d. loaf weigh when wheat is 5s. 6d. the bushel? *Ans.* $16\frac{1}{17}$oz.

20. Knowing that \$500 give \$10 interest in 3 months, what principal should I place at interest to give me \$200 in 1 year? *Ans.* \$2,400.

21. During how many days, of 8 hr. each, must 4 men work, to do as much work as 7 men did in 28 days, of 10 hr. each? *Ans.* 5 da.

22. A piece of cloth 30yd. long, $\frac{4}{5}$ of a yard wide, was woven with 26 lb. of thread; what will be the length of a piece $\frac{3}{4}$ of a yard wide, and which requires 32 lb. of thread? *Ans.* $39\frac{3}{13}$ yards.

PERCENTAGE.

276. **Per Cent.**, or **Rate per Cent.**, also written %, signifies *by the hundred.* Thus, 6% means 6 of every hundred, and may signify 6 cents of every 100 cents, 6 dollars of every 100 dollars, etc.

277. The **Base** is the number on which the percentage is computed.

278. **Percentage** is the required number of hundredths of the base. Thus, the percentage of \$200, at 5% is $\frac{5}{100}$ of \$200 = \$10.

279. The **Amount** or **Difference** is the sum or difference of the base and percentage. Hence,

The Amount = the Base + the Percentage.

The Difference = the Base — the Percentage.

The Base = the Amount — the Percentage, or the Difference + the Percentage.

The Percentage = the Amount — the Base, or the Base — the Difference.

280. The rate per cent. may be expressed either by a *decimal* or a *common fraction*, as shown in the following

TABLE.

Symbols.		Decimals.		Common fractions.		
1%	of a number is	.01	of it =	$\frac{1}{100}$	=	$\frac{1}{100}$
2%	" " " "	.02	" " =	$\frac{2}{100}$	=	$\frac{1}{50}$
4%	" " " "	.04	" " =	$\frac{4}{100}$	=	$\frac{1}{25}$
5%	" " " "	.05	" " =	$\frac{5}{100}$	=	$\frac{1}{20}$
6%	" " " "	.06	" " =	$\frac{6}{100}$	=	$\frac{3}{50}$
8%	" " " "	.08	" " =	$\frac{8}{100}$	=	$\frac{2}{25}$
10%	" " " "	.10	" " =	$\frac{10}{100}$	=	$\frac{1}{10}$
18%	" " " "	.18	" " =	$\frac{18}{100}$	=	$\frac{9}{50}$
75%	" " " "	.75	" " =	$\frac{75}{100}$	=	$\frac{3}{4}$
100%	" " " "	1.00	" " =	$\frac{100}{100}$	=	1
125%	" " " "	1.25	" " =	$\frac{125}{100}$	=	$\frac{5}{4}$
$\frac{1}{2}$%	" " " "	.005	" " =	$\frac{5}{1000}$	=	$\frac{1}{200}$
$\frac{3}{4}$%	" " " "	.0075	" " =	$\frac{75}{10000}$	=	$\frac{3}{400}$
$7\frac{1}{2}$%	" " " "	.075	" " =	$\frac{75}{1000}$	=	$\frac{3}{40}$

281. CASE I.—*Given, the base and rate, to find the percentage.*

Ex. What is 6% of 512 yards of cloth?

OPERATION.

512
.06
30.72yd. *Ans.*

ANALYSIS.—6% = .06. Therefore, 6% of 512yd. is .06 of 512 = 30.72yd.

Or, 6% = $\frac{3}{50}$. Therefore, 6% of 512yd. is $\frac{3}{50}$ of 512 yards = 30.72yd.

Or, $\frac{3}{50} \times \frac{512}{1} = 30.72$yd. *Ans.*

Or, 100% = 512yd.
1% = 5.12yd.
6% = 30.72yd. *Ans.*

Or, if 100% = 512yd., 1% = $\frac{1}{100}$ of 512yd. = 5.12yd., and 6% = 6 times 5.12yd. = 30.72yd. Hence the following

282. RULE.—*Multiply the base by the rate% expressed decimally, and point off as in decimals.* Or,

Find that part of the base which the rate % is of 100.

EXAMPLES FOR PRACTICE.

1. What is 5% of $462? 4% of 1550? 8% of $630.25? 7% of $846? *Ans.* $23.10; 62; $50.42; etc.

2. What is 9% of $75.37½? 7% of 580? 12½% of 1260lb.? 11% of $111? *Ans.* $6.78⅜; 40.6; etc.

3. What is 32% of $760.60? 4½% of 48? 7⅜% of 345? 6¾% of £125 12 6? *Ans.* $243.392, etc.

4. What is 20% of 90cwt.? ¼% of $850? ⅜% of £320 8? 9⅛% of 550gal.? *Ans.* 18cwt.; $4.25; etc.

5. What is 15% of ⅞? ¼% of $80? 2½% of 8¾? 3⅛% of £20 15 8?

6. A merchant having $3456 in the National Bank, wishes to withdraw 18%; how much will remain?

7. A man having $3947, gave 15% of it for flour; 25% for tea; 45% for cloth and linen; and the remaining 15%, for sugar. How much did he spend for each?

Ans. For F. $592.05; T. $986.75; C. $1776.15; etc.

8. A merchant bought 475 barrels of molasses for $7125; and sold 40% of it at $21 a barrel; 30% at $18 a barrel; and the remainder for what it cost. How much did he gain? *Ans.* $1567.50.

283. CASE II.—*Given, the base and percentage, to find the rate %.*

Ex. What per cent. of $450 is 27?

OPERATION.

100
27
450) 2700 (6%, *Ans.*
000

Or, $\frac{3}{50} \times 100\% = 6\%$, *Ans.*

Or, $450 = 100\%$
$1 = \frac{2}{9}\%$
$27 = 6\%$, *Ans.*

ANALYSIS.—$450 is 100% of itself. $27 is $\frac{27}{450}$ of $450; therefore, $27 is $\frac{27}{450}$ of 100%, or $\frac{1}{450}$ of 27 times 100% = 6% of $450.

Or, $27 is $\frac{27}{450} = \frac{3}{50}$ of $450; therefore, it is $\frac{3}{50}$ of 100% = 6% of $450.

Or, $450 is 100% of itself; therefore, $1 is $\frac{1}{450}$ of 100% = $\frac{2}{9}$%, and $27 is 27 times $\frac{2}{9}\% = 6\%$ of $450. Hence the

284. RULE.—*Multiply* 100% *by the percentage and divide by the base.* Or,

Find that part of 100 *per cent. which the percentage is of the base.*

EXAMPLES FOR PRACTICE.

1. At what rate per cent. must we place \$20 to have \$2? \$5 to have \$0.25? \$1440 to have \$21.60? £160 5 to have £12 16 $4\frac{4}{5}$? \$4 to have \$0.30? *Ans.* 10%; 5%; etc.

2. What per cent. of 40 is 15? of 480 perches is 24per.? of $3\frac{1}{3}$ is $\frac{1}{12}$? of $\frac{1}{3}$ is $\frac{1}{2}$? of 92gal. is 11gal. 2qt.? *Ans.* $37\frac{1}{2}$%; 5%; etc.

3. What per cent. of 148 is $24\frac{3}{8}$? of 30lb. Avoirdupois is 11lb. 4oz.? of 720lb. is 60lb.? of 620yd. is $46\frac{1}{2}$yd.? of 140lb. is 77lb.? *Ans.* $16\frac{3}{8}$%; $37\frac{1}{2}$%; etc.

4. What per cent of \$578 is \$26.01? of \$250 is \$80? of $\frac{2}{3}$ is $\frac{3}{50}$? of £3 15 is 3s. 9d.? *Ans.* $4\frac{1}{2}$%; etc.

5. What per cent. of \$300 will give 25% of \$72? *Ans.* 6%.

6. Bought a horse for \$840, and sold him for \$560; how much did I lose per cent.? *Ans.* $33\frac{1}{3}$%.

7. A number increased by 2 equals 14; required the increase per cent.

285. CASE III.—*Given, the rate per cent. and percentage, to find the base.*

Ex. I lost \$27, which is 6% of the money I had; how much had I at first?

OPERATION.

\$27 ÷ .06 = \$450, *Ans.*

Or. \$27 ÷ $\frac{3}{50}$ = \$450, *Ans.*

Or, 6% = \$ 27.
1% = \$ 4.5.
100% = \$450, *Ans.*

ANALYSIS.—If 6%, or .06 of some number is \$27, that number must be \$27 ÷ .06, or $\frac{3}{50}$, = \$450.

Or, 6% of some number is \$27, 1% of it is $\frac{1}{6}$ of \$27 = \$4.5, and 100%, or the whole number, is 100 times \$4.5 = \$450. Hence the

286. RULE.—*Divide the percentage by the rate % expressed decimally, or in the form of a common fraction.* Or,
Divide the percentage by the rate %, and multiply by 100.

EXAMPLES FOR PRACTICE.

1. 35 is 10% of what number? 84 is 7% of what number? \$3.60 is 15% of what number? \$55.50 is $4\frac{1}{2}$% of what number? 240 is $12\frac{1}{2}$% of what number? *Ans.* 350; 1200; etc.

2. \$66 is $5\frac{1}{2}$% of what sum? $5\frac{1}{3}$ is $1\frac{1}{2}$% of what sum? $\frac{5}{8}$ is 30% of what sum? *Ans.* \$1200; etc.

3. £32 8 3 is $7\frac{1}{2}$% of how much? 207 is 60% of how much? \$1.82$\frac{1}{2}$ is $12\frac{1}{2}$% of how much? *Ans.* £432 3 4; etc.

4. \$2.81$\frac{1}{4}$ is $12\frac{1}{2}$% of how much? 3mi. 1fur. 1per. is $6\frac{1}{4}$% of how much? $16\frac{1}{2}$ is $2\frac{3}{4}$% of how much? *Ans.* \$22.50; etc.

5. If the percentage be \$37.50, and the rate $2\frac{1}{2}$%; what is the base? *Ans.* \$1500.

6. A farmer saved annually \$145.50, which was $33\frac{1}{3}$% of his annual income; required his income?

287. CASE IV.—*Given, the rate per cent. and amount or difference, to find the base.*

Ex. What number increased by 6% of itself is equal to 477?

OPERATION.

$1 + .06 = 1.06$
$477 \div 1.06 = 450$, *Ans.*

Or, $\frac{53}{50} = 477$
$\frac{1}{50} = 9$
$\frac{50}{50} = 450$, *Ans.*

ANALYSIS.—A number increased by 6% of itself, equals 106%, or 1.06 of itself, which, by the condition of the question, is 477; hence, once the number equals $477 \div 1.06 = 450$.

Or, 6% of a number is $\frac{6}{100} = \frac{3}{50}$ of the number, which being increased by $\frac{50}{50}$, the number, equals $\frac{53}{50}$ of the number, $= 477$. If $\frac{53}{50}$ of the number $= 477$, $\frac{1}{50}$ of the number $= \frac{1}{53}$ of $477 = 9$, and $\frac{50}{50}$, the number, equals 50 times $9 = 450$. Hence the

288. RULE—*Divide the amount by* 1 *plus the rate %, expressed decimally, or as a common fraction; and the difference by* 1 *minus the rate %, expressed decimally, or as a common fraction.*

EXAMPLES FOR PRACTICE.

1. What is that number, which, diminished by 5% of itself, gives 429.40? *Ans.* 452.

2. What number increased by 5% of itself, gives £7 1 9?

3. I have \$407.55, or $4\frac{1}{2}\%$ more than my neighbor; what sum does my neighbor possess? *Ans.* \$390.

4. The difference is $\$9.48\frac{1}{2}$, and the rate, $12\frac{1}{2}\%$; what is the base? *Ans.* \$10.84.

5. Andrew has £189 9 8, which is 7% less than that of Louis; what sum has the latter? *Ans.* £203 14 $10\frac{3}{4}$ +.

6. Find the number which, augmented by the $\frac{5}{8}\%$ of itself, gives $\$52.32\frac{1}{2}$. *Ans.* \$52.

7. A teacher spends 45% of his income, and saves \$858; what was his income?

8. After taking 12% of a pile of wheat, there remain 44 bushels; how many bushels were in the pile? *Ans.* 50bu.

9. Having increased my capital by $15\frac{1}{2}\%$ of itself, I find I possess \$5682.60; how much had I at first?

10. A shepherd lost, by disease 12% of his flock; how many sheep composed his primitive flock, knowing that there remain 1100?

11. A clerk spends 20% of $66\frac{2}{3}\%$ more than $\frac{1}{2}$ of his income; what is his income, if he saves \$533?

12. A gentleman sold two horses at \$420 each; for one he received 25% more, and for the other 25% less than his value; what is his loss?

13. A man, wishing to sell a horse, asked 25% more than it cost; he finally sold it for 15% less than his asking price, and gained \$7.50. How much did the horse cost him, and what was his asking price?

Ans. cost, \$120; asking price, \$150.

MISCELLANEOUS EXAMPLES IN PERCENTAGE.

1. Find $\frac{1}{5}\%$ of 70cwt. 1qr. 12lb. *Ans.* 14.074 lb.

2. $\$1\frac{3}{5}$ is $\frac{1}{4}\%$ of what number? *Ans.* $180.

3. Find a number which, diminished by 10% of itself, gives £48.

4. A merchant owes $4500; his property is valued at $2295; what rate per cent. can he pay? *Ans.* 51%.

5. A superior officer, having 1500 men under his command, lost 9% of them in a battle, and 40% of the remainder by sickness; how many remain? *Ans.* 819 men.

6. I sold cloth at £1 10 3 a yard, which is but 65% of the cost; how much did it cost a yard? *Ans.* £2 6 $6\frac{1}{4}$ +

7. A man expends $18, which is $33\frac{1}{3}\%$ more than his weekly wages; what are his wages? *Ans.* $13.50.

8. After paying $42\frac{1}{2}\%$ of my debt, I find that $2650 will settle the balance; how much did I owe? *Ans.* $4608.69 +.

9. What per cent. of £40 will give 20% of £7 15? *Ans.* $37\frac{1}{2}\%$.

10. A little boy laid out 40% of his money in play things, 35% in sugar-plums, and has 12 cents remaining; what did his purse contain? *Ans.* 48 cts.

11. What per cent. of a number gives $22\frac{1}{2}\%$ of the $\frac{5}{8}$ of this number? *Ans.* $18\frac{3}{4}\%$.

12. A cargo of barley having been damaged, the owner was obliged to sell the whole for $1999.20 which was at a loss of 32%; how much did the cargo cost him? *Ans.* $2940.

13. A merchant having $2150 deposited in a bank, desires to draw 15% of it; how much will remain? *Ans.* $1827.50.

14. There remains $25\frac{1}{2}$yd. of a piece of linen, after having sold 16% of it; what was the length of the piece? *Ans.* 30 yards.

15. The number of deaths in a certain town, during the year, was 1950, which is $3\frac{1}{4}\%$ of the population; what is the number of its inhabitants? *Ans.* 60000.

16. A fish-monger had 720bbl. of fish, and sold 288bbl.; what per cent. remained unsold? *Ans.* 60%.

17. 18lb. 15oz. is $12\frac{1}{2}\%$ of how many lb.? *Ans.* 151lb. 8oz.

18. Gave to a Benevolent Society 29bush. of wheat, which was $14\frac{1}{2}\%$ of my entire crop; how many bushels had I remaining? *Ans.* 171.

19. What per cent. of $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{5}$ gives $\frac{1}{4}$? *Ans.* 25%.

20. Joseph having received a legacy, deposited 75% of it in a bank. A short time after, he drew forth 30% of his deposit, and there still remained £1280 17 6; what was the legacy? *Ans.* £2439 15 $2\frac{2}{7}$.

21. In a certain coin there are 21 parts copper and 4 parts nickel; what per cent. is the copper and nickel? *A.* copper 84%, nickel 16%.

22. A gentleman has an annual income of $2700; if he expends 20% for nourishment, 8% for clothing, $3\frac{1}{2}\%$ in alms, 5% in books, and 14% in casual expenses, what are his annual expenses? *A.* $1363.50.

23. In an engagement, 5% of the army were killed on the field of battle, and 6% of the remainder died of their wounds in the hospitals. The difference between the number of the dead and the number of the wounded was 154; how many men composed the army? *Ans.* 22000.

24. Edward lives $3\frac{1}{4}$ miles from the city, which distance is $9\frac{1}{2}$ % of Leo's residence nearer the city; how far from the city does Leo live?

25. An army, having been twice decimated in battle, is reduced to 19440 men; what was the strength of the army before the combat?

26. The sales of a mercantile establishment amount to $131000 yearly; the $\frac{2}{5}$ of these sales were made at a profit of 28 %; the $\frac{4}{15}$ of the remainder, at a profit of 40 %; and the remainder, at a profit of $17\frac{1}{2}$ %; how much did the merchandise cost? *Ans.* $104113.18+

27. The proprietor of the $\frac{5}{8}$ of a factory, sells 24 % of his share to C., and the remainder to D., at the same rate, for $15800; what is the value of the factory? *Ans.* $24947.368+

28. After cutting a certain number of yards from a piece of cloth, there remains 12yd. $2\frac{1}{2}$qr., which is 70% less than the quantity cut; what was the length of the piece? *Ans.* 54yd. $1\frac{3}{5}$qr.

29. If £36 17 6 is $14\frac{1}{3}$% of Paul's money, and if $5\frac{1}{2}$% of Paul's money is 12% of Leo's, how much has Paul more than Leo?

30. My crop of potatoes this year is 9% greater than that of last year, and I have gathered in the two years 6479 bushels; of how many bushels did my last crop consist? *Ans.* 3379bu.

31. If the population of the Dominion of Canada, in 1869, was 4260000 inhabitants, what ought it to be in 1879, supposing it to increase at the rate of $27\frac{1}{2}$%? *Ans.* 5431500.

32. The net gains of a nursery in two years was £2178; the gains of the second year were 6% greater than those of the first year; what were the gains of each year?

Ans. £1057 5 $7\frac{59}{103}$, gains of the 1st. year; £1120 14 $4\frac{44}{103}$, 2nd. yr.

33. I had $15000 in a bank; I drew out at first 22%, then, 34% of the remainder, and finally deposited 12% of what I had drawn; how much remains in the bank? *Ans.* $8595.36.

34. A person having a revenue of $560, makes the following expenses: board, $130; tailor, $145; shoemaker, $28; sundries, $36; what per cent. of his revenue is each article, and what per cent. remains? *Ans.* $23\frac{3}{14}$%; $25\frac{25}{28}$%; 5%; $6\frac{3}{7}$%; $39\frac{13}{28}$%

35. If a number be augmented by 11% of itself, and this amount by 1%, it will become $7.75; what is the number? *Ans* $6.40\frac{6640}{12098}$.

36. A merchant expended the same sum in the purchase of wine, whiskey and coffee. In selling, he gained 8% on the wine and 5% on the whiskey; but he lost 14% on the coffee; he received from his entire sales £630 10; how much did he pay for each sort of merchandise? *Ans.* £210 17 $4\frac{208}{239}$.

37. Edmund and Charles have respectively 6% and 4% more money than Maurice, and the three have together $22320; how much has Maurice? *Ans.* $7200.

38. A young man commences business on the 1st. of February, with a capital of $2700. At the end of 10 months, I read in his Ledger as follows: Feb., 2% gain; March, $3\frac{1}{4}$% gain; April, $1\frac{1}{2}$% loss; May, $1\frac{1}{4}$% gain; June, $2\frac{1}{2}$% gain; July, $1\frac{3}{4}$% loss; August, $\frac{3}{8}$% gain; Sept. 1% loss; Oct. $2\frac{3}{4}$% gain; Nov. $4\frac{1}{2}$% gain; what were the net profits of his business during the 10 months?

SIMPLE INTEREST.

289. **Interest** is the compensation made by the borrower to the lender for the use of money.

290. The **Principal** is the sum lent.

291. The **Rate per cent.** is the interest paid for the loan of \$100, £100, etc., during any time whatever, which is ordinarily a year.

NOTE.—The rate per cent. is commonly expressed decimally as hundredths.

292. The **Amount** is the sum of the principal and interest.

293. **Simple Interest** is the sum paid for the use of the principal only, during the time of the loan.

294. **Legal Interest** is the rate per cent. established by law. It varies in different countries.

NOTE.—When no rate is mentioned, the rate established by the laws of the country in which the transaction takes place, is always understood to be the one intended by the parties.

295. **Usury** is a higher rate % than is allowed by law.

NOTE.—The law prohibits usury and makes it subject to a penalty.

296. *To find the interest on any sum, at any rate %, for any number of years and months.*

Ex. What is the interest of \$780, for 5 years and 3 months ($5\frac{1}{4}$ years), at 7%? What is the amount?

OPERATION.

\$780 Prin.
.07 Rate.
\$54.60 Int. 1yr.
$5\frac{1}{4}$
\$273.00 " 5yr.
13.65 " 3mo.
\$ 286.65, " $5\frac{1}{4}$yr.
780.00 Prin. added.
\$1066.65, Amount.

ANALYSIS.—The interest of \$1 for 1 year, at 7% is \$0.07, and of \$780 it is 780 times \$0.07 = \$54.60. If the interest of \$780 for 1 year, at 7% is 54.60, for $5\frac{1}{4}$ years it is $5\frac{1}{4}$ times \$54.60 = \$286.65.

Or, $\frac{7}{100}$ of the principal = the interest for 1 year at 7%. The amount is found by adding the principal and interest together.

297. RULE.—I. *Multiply the principal by the rate % expressed decimally, and the product will give the interest for one year.*

II. *Multiply this product by the number of years, and the months as a fraction of a year, for the interest required.*

The amount is found by adding the principal and interest together.

NOTE.—When part of the time for interest is given in months or days, one month is considered as $\frac{1}{12}$ of a year, and one day as $\frac{1}{30}$ of a month.

EXAMPLES FOR PRACTICE.

What is the interest of

1. $450 for 3 years, at 4%? *Ans.* $54.
2. $16 for 7 years, at 8%? *Ans.* $8.96.
3. $656 for 2 years, at 7%?
4. $1728 for 1 year 6 months, at 6%? *Ans.* $155.52.
5. $1740 for 3 years, at $8\frac{1}{3}$%? *Ans.* $435.
6. $878.25 for 2 years, at $9\frac{1}{2}$%? *Ans.* $166.86\frac{3}{4}$.
7. $118.15 for 2 years 6 months, at 6%?
8. $300 for 3 years 10 months, at 7%?
9. $125.75 for 4 years 6 months, at 8%? *Ans.* $45.27.
10. $97.16 for 1 year 5 months, at 6%? *Ans.* $8.258.
11. $58.60 for 2 years 9 months, at 8%? *Ans.* $12.892.
12. $76.50 for 2 years 2 months, at 5%? *Ans.* $8.28+.
13. $444.44 for 5 years, at $6\frac{2}{3}$%?
14. $960.18 for 1 year 2 months, at 7%? *Ans.* $78.414.
15. $4501.80 for 2 years 4 months, at $6\frac{1}{4}$%? *Ans.* $656.5125.
16. $1671.32 for 14 months, at 6%? *Ans.* $116.99.

What is the amount of

17. $53.68 for 2 years 6 months, at 5%? *Ans.* $60.39.
18. $978.18 for 2 years 3 months, at 6%? *Ans.* $1110.234.
19. $3050 for 4 years 8 months, at $5\frac{1}{4}$%? *Ans.* $3797.25
20. $81.81 for 8 years 4 months, at 6%? *Ans.* $122.715.
21. $95 for 1 year and 6 months, at 5%?
22. $65256 for $4\frac{1}{2}$ months, at 7%?
23. $894 for 20 months, at 6%?
24. $760 for 5 years 7 months, at $5\frac{1}{2}$%?

298. *To find the interest on any sum, for any time, at any rate %.*

SIX PER CENT. METHOD.

To find the interest of $1 for any time, at 6%; also, at any other rate %.

ANALYSIS.—At 6% per annum the interest on $1.

For 12 months		is $.06	
" 2 months ($\frac{2}{12}=\frac{1}{6}$ of 12 mo.)		" .01	
" 1 month, or 30 days ($\frac{1}{12}$ of 12 mo.)		" .00$\frac{1}{2}$ = $.005 ($\frac{1}{12}$ of $.06)	
" 6 days ($\frac{1}{5}$ of 30 days)		" .001	
" 1 " ($\frac{1}{6}$ of 6 da. $=\frac{1}{30}$ of 30 da.)		" .000$\frac{1}{6}$.	Hence,

1st. The interest on $1 is $.005 per month, or $.01 for every 2 months; 2nd. the interest on $1 is $.000$\frac{1}{6}$ per day, or $001 for every 6 days. Hence the

299. RULE.—I. To find the rate:—*Call every year* $.06, *every* 2 *months* $.01, *every* 6 *days* $.001, *and any less number of days sixths of* 1 *mill.*

II. To find the interest:—*Multiply the principal by the rate.*

Ex. 1. What is the interest of $660, at 6%, for 3 years 7 months 27 days?

OPERATION.

Int. of $1 for 3yr. = $0.18
" " " " 7mo. = 0.035
" " " " 27 days. = 0.004½
" " " " 3yr. 7mo. 27da. = $0.219½

Principal, $660
.219½
5940
660
1320
330
Interest, $144.870

ANALYSIS.—The interest of the given principal is 660 times the interest of $1 for 3 years 7 months 27 days. As the int. of $1 for 1yr. is $.06, for 3yr. it will be $.18; and since the interest for 2 months is $.01, for 7 months it will be as many times $.01, as 2 is contained in 7, or 3½cts.; again, since the interest for 6 days is $.001, for 27 days, it will be as many times $.001, as 6 is contained in 27, or 4½ mills. Adding these three results together, we have $0.219½ which equals the interest of $1 at 6% for the given time. Multiplying $660 by $0.219½, we obtain $144.87, the int. required.

Ex. 2. Required the interest on $750 for 8 years 8 months 9 days, at 7%.

OPERATION.

$0.48 = Int. on $1 for 8yr.
0.04 = " " " " 8mo.
0.001½ = " " " " 9 days.
$0.521½ = " " " " 8yr. 8mo. 9da.
$0.086 1/12 = " " " " " " " at 1%.
7
$0.608 5/12 = " " " " " " " at 7%.
$ 750
$.608 5/12
6000
45000
312½
$456.312½ = Int. required.

ANALYSIS.—After finding the interest of $1 for the given time, at 6%, by the method laid down in the preceding example, we divide the result by 6, and then find the interest at 1%; we then multiply by the given rate, 7, and obtain the interest on $1 for the given time, at 7%. Multiplying the principal, $750, by the rate, $.608 5/12, we obtain $456.31¼, which is the interest required. Hence the

300. RULE.—I. When the rate is greater or less than 6%: —*Find the interest on* $1, *at* 6%, *for the given time, as in the preceding example.*

II. *Then divide by* 6, *and multiply the quotient by the given per cent. This result multiplied by the given principal, will give the interest required.*

NOTE.—The interest of the given principal, at 6%, for the given time, could be obtained at first; then, this result divided by 6 and multiplied by the given per cent. will give the same answer; or, add or subtract from this interest such a fractional part of itself as the given rate exceeds or falls short of 6% per annum. Thus, if the rate be 9%, the interest at 6% should be increased $\frac{3}{6}$ or $\frac{1}{2}$ of itself, because 3, the excess of 9 over 6, is $\frac{1}{2}$ of 6, and so on.

When the time is short, business men use the following

301. RULE.—*Multiply the principal by the number of days, divide the product by* 6, *and remove the decimal point three places toward the left.* (The result is the interest at 6%.) *Then proceed as in the above rule.*

METHOD BY ALIQUOT PARTS.

Ex. What is the interest of \$421.50 for 3yr. 8mo. and 15da, at 9%?

OPERATION.

Principal,	\$421.50
Rate %,	.09
Interest for 1 year,	\$37.9350
	3
Int. for 3 years,	\$113.8050
Int. for 6mo. $= \frac{1}{2}$ of 1yr.	18.9675
Int. for 2mo. $= \frac{1}{3}$ of 6mo.	6.3225
Int. for 15da. $= \frac{1}{4}$ of 2mo.	1.5806$\frac{1}{4}$
Int. for 3yr. 8mo. 15da.	\$140.6756$\frac{1}{4}$, *Ans.*

ANALYSIS.—Having found the interest for 1yr. and then for 3yr., the int. for 8mo. is obtained by first taking $\frac{1}{2}$ of 1 year's int., for 6mo., and then $\frac{1}{3}$ of this last int. for 2mo. And since 15 days are $\frac{1}{2}$ of 1mo., or $\frac{1}{4}$ of 2mo., we take $\frac{1}{4}$ of 2mo.'s int. for 15 days. The int. as found for the several parts of the whole time, added together, gives the interest required.

NOTE.—Whenever the number of mills is 5 and upwards, in business transactions, we add 1 cent and drop the mills. Hence, the interest in the above example is called \$140.68.

302. RULE.—I. *First find the interest for one year by multiplying the principal by the rate %, decimally expressed, and this product by the number of years.*

II. *Find the interest for the months and days by aliquot parts. The sum of the partial interests will be the interest required.*

METHOD BY MONTHS.

Ex. What is the interest of \$24.20 for 4yr. 7mo. and 15da., at 6%?

OPERATION.

$$\frac{6.05 \quad 18.5}{24.20 \times .06 \times 55.5} = \$6.7155.$$
$$3 \times 4$$

NOTES.—1. 4yr. 7mo. 15da. = 55.5mo.

2. The above is the product of the principal, rate per cent., decimally expressed in months and decimals of a month, divided by $12 = 3 \times 4$.

OPERATION.

\$24.20	
.06	
12) 1.4520	= Int. for 1 yr.
.1210	= Int. for 1 mo.
55.5	
6050	
6050	
6050	
\$6.71550	= Int. for 55.5 mo.

303. RULE.—I. *Reduce the time to months and decimals of a month.*

II. *Find the interest for* 1 *year, and divide it by* 12; *the quotient will be* 1 *month's interest.*

III. *Multiply this interest by the time expressed in months, and the product will be the interest required.*

METHOD BY PROPORTION.

Ex. What is the interest of $52.50, at 6%, for 4 years 5 months and 10 days?

Sol. 100 : 6 × 4yr. 5mo. 10da. :: $52.50 ; x; whence the

304. RULE.—100 *is to the* per cent. *multiplied* by *the* time, *as the* principal *is to the* interest.

EXAMPLES FOR PRACTICE

TO BE SOLVED BY ANY OF THE ABOVE METHODS.

NOTE.—If the principal be given in old currency, reduce the shillings, pence and farthings, to the decimal of a £; then proceed as in decimal currency.

What is the interest on

1. $500 for 1yr. 10mo. and 15da., at 6%?	*Ans.* $56.25.
2. $9862.12½ for 3yr. 5mo., at 4%?	*Ans.* $1347.82+.
3. £26 10 0 for 2yr. 4mo., at 6%?	*Ans.* $14.84.
4. $972.40 for 1yr. 7mo. 18da., at 7%?	*Ans.* $111.177+.
5. $143 for 2yr. and 9mo., at 8%?	*Ans.* $31.46.
6. $47.25 for 1yr. and 6mo., at 6%?	*Ans.* $4.25¼.
7. £42 18 0 for 3yr. 4mo. 25da., at 6%?	*Ans.* £8 15 $2\frac{1}{10}$.
8. $147.90 for 8mo. 4da., at 5%?	*Ans.* $5.01+.
9. $145.50 for 1yr. 9mo., 24da., at 6%?	*Ans.* $15.85+.
10. $579.75 for 1yr. 3mo. 2da., at 5%?	*Ans.* $36.395+.
11. £94 12 6 for 4yr. 6mo. 7da., at 8%?	*Ans.* $136.848+.
12. $123.75 for 2yr. 8mo. 12da., at 6%?	*Ans.* $20.04¾.
13. $50.40 for 1yr. and 10mo., at 7%?	*Ans.* $6.468.
14. $475 for 2yr. 7mo. 20da., at 6%?	*Ans.* $75.208⅓.
15. £6 11 3 for 2yr. 4mo., at 7%?	*Ans.* £1 1 5¼.
16. $336 for 5mo. 15da., at 5%?	*Ans.* $7.70.
17. $1265.60 for 5yr. 2mo. 9da., at 7%?	*Ans.* $459.94+.
18. $72.12½ for 6yr. and 5mo., at 4%?	*Ans.* $18.51+.
19. $497.36 for 1yr. 6mo. 1da., at 5%?	*Ans.* $37.37+.
20. £191 5 4 for 2yr. 9mo., at 1%?	*Ans.* $21.039+.
21. $7671.09 for 3yr. 8mo. 5da., at 8%?	*Ans.* $2258.70+.
22. $49.80 for 2yr. and 11mo., at 7%?	*Ans.* $10.16⅔.
23. $350.80 for 15mo. and 8da., at 10%?	

24. $1040 for 5yr. 11mo. 29da., at 7 %? *Ans.* $436.596.
25. £24 18 8 for 10mo. and 20da., at 7%? *Ans.* £1 11 0¼ +.
26. $51.17 for 10mo. and 29da., at 4 %?
27. $548.12 for 6yr. 1mo. 3da., at 7 %? *Ans.* $233.72 +.
28. $500 for 2yr. 5mo. 12da., at 6 %?
29. $909.50 for 5yr. 5mo. 4da., at 6 %? *Ans.* $296.19 +.
30. £92 12 0 for 2yr. 10mo., at 6½ %?
31. $680 for 4yr. 1mo. 15da., at 6 %? *Ans.* $168.30.
32. $2000 for 1yr. 3mo. 10da., at 9 %?
33. $471.11 for 4yr. and 8mo., at 7½ %? *Ans.* $164.888 +.
34. $190.016 for 3mo. 24da., at 4½ %?
35. £427 8 8 for 1yr. 5mo., at 5¾ %? *Ans.* £34 16 4 +.
36. $708.20 for 2yr. 2mo. 12da., at 4¾ %?
37. $640.70 for 8mo. and 26da., at 5½ %? *Ans.* $26.037 +.
38. $730.50 for 18mo. and 23da., at 6¼ %?
39. $950 for 4yr. 7mo. 9da., at 8¼ %? *Ans.* $361.178 +.
40. £81 10 0 for 2yr. and 5mo., at 4¾ %?
41. $150.80 for 7mo. and 20da., at 7¼ %? *Ans.* $6.98 +.
42. $1072.40 for 5 yr. 10mo. 5da., at 6⅓ %?
43. $601.20 for 4yr. 2mo. 3da., at 8½ %? *Ans.* $213.35 +.
44. $1425.20 for 1yr. and 16da., at 4½ %?
45. £319 10 9 for 1yr. 10mo., at 4⅜ %? *Ans.* $102.518 +.
46. $742.30 for 4yr. 9mo. 19da., at 6¾ %?
47. $1370.40 for 3yr. 4mo. 27da., at 7½ %? *Ans.* $350.30 +.
48. $160.75 for 2yr. 11mo. 4da., at 5¾ %?
49. $1463.60 for 7yr. 7mo. 22da., at 6½ %? *Ans.* $727.24 +.
50. £184 18 8 for 1yr. 9mo. 6da., at 3⅛ %?

What is the amount of

51. $0.145 for 9yr. 9mo. and 9da., at 6 %? *Ans.* $0.23 +.
52. $1051.50 for 2yr. 10mo., at 7 %? *Ans.* $1260.045 +.
53. $168.13 for 8yr. 5mo. 3da., at 6 %? *Ans.* $253.119.
54. $100.25 for 2mo. and 29da., at 4 %? *Ans.* $101.241.
55. $1.011 for 10yr. 10mo. 10da., at 6 %?
56. $1000 for 3yr. 3mo. 29da., at 5½ %? *Ans.* $1183.18.
57. $168.50 for 1yr. 5mo. and 10da., at 6¼ %?
58. $2000 for 1mo. 5da., at 6¾ %? *Ans.* $2013.12½.
59. $0.05 for 20yr. 10mo. 15da., at 8 %?
60. $325.25 for 2yr. 9mo. 12da., at 6½ %? *Ans.* $384.09 +.
61. $495.95 for 5yr. 5mo. 5da., at 6¾ %?
62. £109 3 9 for 7yr. 9mo. 18da., at 3⅝ %?
63. $2560.75 for 4yr. 3mo. 25da., at 6½ %?
64. What is the interest of $1560 from April 9, to November 10, at 5½ %? *Ans.* $50.28⅝.
65. What is the amount of $175.08 from May 7, 1861, to September 25, 1863, at 7 %? *Ans.* $204.289.
66. What is the interest of $176.89½ from January 6, 1868, to July 22, 1869, at 6½ %?
67. What is the amount of $1756.75 from June 29, 1860, to February 12, 1863, at 7 %?

68. What is the interest of £43 2 6 from March 17, to December 7, at $7\frac{1}{4}\%$? *Ans.* £2 5 $1\frac{3}{4}+$.

69. What is the interest of $1530.50 from February 10, 1868, to January 25, 1869, at $\frac{1}{2}\%$? *Ans.* $7.33+.

70. What is the amount of $158.30 from February 17, 1868, to December 30, 1871, at $7\frac{1}{2}\%$?

71. What is the interest of $1728.19, from May 7, 1868, to July 17, 1869, at $\frac{1}{4}\%$? *Ans.* $5.16+.

72. What is the interest of £32 8 9 from September 25, 1867, to July 9, 1869, at $\frac{3}{4}\%$?

73. What is the amount of $89.96 from June 19, 1870, to December 9, 1871, at $8\frac{1}{4}\%$? *Ans.* $100.886.

74. What is the interest of $990.75 from October 5, 1868, to January 15, 1869, at $1\frac{3}{4}\%$?

75. What is the interest of $1030.10 from November 8, 1867, to March 3, 1869, at $8\frac{7}{8}\%$? *Ans.* $120.625+.

76. What is the interest of £45 10 4 from December 10, 1866, to May 5, 1869, at $\frac{5}{8}\%$?

EXACT METHOD OF COMPUTING INTEREST.

305. In the preceding methods of computing interest, which are in general use, we have reckoned 30 days to the month, and 12 months to the year, which allows to each year 360 instead of 365 days. Hence, the results obtained in these calculations are not strictly correct.

The following exact method is used by business men in computing interest when the time is short.

NOTE.—The exact time, when it is less than a year, is found by the table on page 124.

306. RULE.—*Multiply the interest of the principal for* 1 *year by the exact number of days it has been on interest, and divide the product by* 365, *the quotient will be the interest required.*

1. What is the interest of $345.60, from February 5, 1869, to Aug. 20, 1871, at 7 %? *Ans.* $61.374+.

2. What is the interest, at $5\frac{1}{2}\%$, of $425.50, from January 8th., until November 20th.? *Ans.* $20.26+.

3. What is the interest, at $6\frac{1}{2}\%$, of $140.40, from Aug. 29th., 1870, to Nov. 29th., 1871? *Ans.* $11.426+.

4. What is the interest, at 8 %, of $4560, from May 18th., 1868, to Sept. 25th., 1871?

5. What is the interest, at $7\frac{1}{4}\%$, of $3790.45, from July 20th., 1869, to Sept. 12th., 1871?

6. What is the interest, at $4\frac{1}{4}\%$ of £48 16 3. from Sept. 12th., 1868, to Aug. 28th., 1871?

PARTIAL PAYMENTS.

307. Partial Payments are payments of part of a note, bond, or other moneyed obligation, made at different times.

The payments are acknowledged by receipts written by the creditor on the back of the note or obligation, which are called **Indorsements.**

308. RULE.—I. If the interest be paid by days:—*Multiply the principal by the number of days which have elapsed before any payment was made. Subtract the first payment, and multiply the remainder by the number of days which passed between the first and second payments. Subtract the second payment, and multiply this remainder by the number of days which passed between the second and third payments. Subtract the third payment, etc.*

II. *Add all the products together, and find the interest of their sum for one day.*

III. If the interest is to be paid by the week or month:—*Substitute weeks or months for days, in the above rule.*

Ex. 1. How much principal and interest have I to pay on the following note, due Dec. 29, 1871?

$ 420. Quebec, Sept. 8, 1868.

For value received, I promise to pay James Carroll, or order, four hundred and twenty dollars, with interest, at 7 %? Thomas Brown.

On this note were indorsed the following payments:—

Oct. 1, 1869, received,.....$22.28.
Nov. 20, 1869, " ..... 50.00.
May 8, 1871, " .....247.87.

OPERATION.

From Sept. 8, 1868, to Oct. 1, 1869, there are 388 days.
" Oct. 1, 1869, to Nov. 20, 1869, " " 50 "
" Nov. 20, 1869, to May 8, 1871, " " 534 "
" May. 8, 1871, to Dec. 29, 1871, " " 235 "

Whole principal $420.00 for 388 days = $162960.00 for 1 day.
First indorsement 22.28

Balance $397.72 for 50 days = $ 19886.00 for 1 day.
Second indorsement 50.00

Balance $347.72 for 534 days = $185682.48 for 1 day.
Third indorsement 247.87

Balance $ 99.85 for 235 days = $ 23464.75 for 1 day.

Whole interest = that of $391993.23 for 1 day.

Interest on \$391993.23 at 7 % for 1yr. = \$27439.5261.
Hence, the int. for 1 day = \$27439.5261 ÷ 365 = \$75.1767 + .

Then interest due	= \$75.1767 + .
Balance on note	= 99.8500.
Principal and interest due	\$175.0267 + .

\$ 450. Montreal, January 13, 1869.

2. Nine months after date, I promise to pay Louis Merrill, or order, four hundred and fifty dollars, with interest, at 6 %, for value received. A. N. Moreau.

Indorsed as follows: Oct. 7, 1869, \$125.10; Aug. 25, 1870, \$225.35. How much remained due Sept. 19, 1871? *Ans.* \$142.8802+.

\$ $325\frac{25}{100}$. Kingston, July 26, 1866.

3. Four years after date, we promise to pay Lawrence Boyce, or order, three hundred twenty-five and $\frac{25}{100}$ dollars, with interest, at 7 %. Value received. L. R. Whelan & Co.

Indorsed as follows: Jan. 20, 1867, \$121.18; March 14, 1868, \$72.45; July 26, 1869, \$133.65. How much remained due Sept. 8, 1870? *Ans.* \$41.01+.

\$ $1737\frac{50}{100}$. Toronto, March 6, 1868.

4. On demand, we promise to pay Fisher & Howe, or order, one thousand seven hundred thirty-seven and $\frac{50}{100}$ dollars, for value received, with interest, at 6 %? T. Johnson & Bro.

Indorsed as follows: June 1, 1868, \$623.80; Sept. 10, 1868, \$700. How much was due Jan. 31, 1869? *Ans.* \$466.763+.

\$ 1240. Ottawa, Aug. 18, 1869.

5. For value received, I promise to pay R. N. Kelly, or order, twelve hundred and forty dollars, on demand, with interest, at 6 %. Joseph Rogers.

Indorsed as follows: Sept. 25, 1869, \$95; Oct. 28, 1869, \$217.86; Dec. 12, 1869, \$432.36; April 6, 1870, \$120.20; July 3, 1870, \$366.50. How much remained due Sept. 10, 1870? *Ans.* \$43.758+.

£ 304 6 6. Halifax, June 2, 1868.

6. For value received, I promise to pay N. J. Webster, or order, on demand, three hundred and four pounds six shillings and six pence, with interest, at 6 %. A. C. Murphy.

Indorsed as follows: July 17, 1868, £51 19 0; Oct. 6, 1868, £52 8 0; Dec. 11, 1868, £80 4 6; March 29, 1869, £105 9 $1\frac{1}{5}$. How much was due Oct. 7, 1869? *Ans.* £24 13 $0\frac{21}{125}$.

$ 14696.50. St. John, June 17, 1866.

7. For value received, we jointly and severally promise to pay Edward Hammond, or order, on demand, fourteen thousand six hundred and ninety-six $\frac{50}{100}$ dollars, with interest, at 8 %. J. P. Rooney.
S. E. Hamilton.

Indorsed as follows: Sept. 5, 1866, $4927.60; Dec. 7, 1866, $784.40; June 11, 1867, $1964.40; Feb. 7, 1868, $5685.80; Dec. 19, 1868, $634.46. How much remained due May 1, 1869? *Ans.* $2006.266+.

8. A farmer gave a mortgage on his farm for $4875, dated June 1, 1867, to be paid in 4 years, with 7½ % interest. Six months from date he paid $223.25; Oct. 20, 1869, $1250; July 3, 1870, $750; Jan. 1, 1871, $250; how much was due at the expiration of the given time?
Ans. $3595.31+.

PROBLEMS IN INTEREST.

309. It will be observed that there are *five parts* or *terms* connected with each of the preceding questions in interest, viz.: the Principal, the Rate %, the Time, the Interest, and the Amount. The investigation of these involves five cases: I. To find the interest; II. To find the amount; III. To find the principal; IV. To find the rate %; V. To find the time.

The Cases I. and II. have already been solved (296, 298).

310. CASE III.—*The interest, time, and rate %, being given, to find the* PRINCIPAL.

Ex. What principal in 3 years, at 6%, will gain $47.70 interest?

OPERATION.

.06 int. of $1 for 1yr.
3
.18) $47.70 ($265, *Ans.*

By proportion.

$100 : x :: $6 × 3 : $47.70.

ANALYSIS.—We find the interest of $1 for 3 years. Since it requires 3 years from a principal of $1 to gain 18 cents, it will require a principal of as many dollars to gain $47.70 as $0.18 is contained times in $47.70; dividing, we obtain $265, the required principal. Hence

311. RULE.—*Divide the given interest or amount by the interest or amount of* $1 *for the given time and rate, and the quotient will be the principal.*

EXAMPLES FOR PRACTICE.

What principal will in

1. 6yr. 3mo., at 6 %, give $56.25? *Ans.* $150.
2. 1yr. 6mo., at 6 %, give $1.2924 int.? *Ans.* $14.36.

3. 4mo. 18da., at 4 %, give $27.60 int.? *Ans.* $1800.
4. 1yr. 4mo., at $8\frac{1}{4}$ %, give $13.20 int.? *Ans.* $120.
5. 3yr. 8mo. 15da., at 6 %, give $76.095 int.? *Ans.* $342.
6. 4yr. 9mo. 18da., at 9 %, give $65.016 interest?
7. 8yr. 8mo. 12da., at 5 %, gain $147.9435? *Ans.* $340.10.
8. 10yr. 10mo. 20da., at $6\frac{4}{5}$ %, gain $1.7770\frac{2}{3}$?
9. If the interest on a sum borrowed at 2 % a month, is $24 for 90 days, what is the sum? *Ans.* $400.
10. What sum of money loaned for 183 days, at $7\frac{1}{4}$ %, is sufficient to produce $619.15?

312. CASE IV.—*The principal, time and interest being given, to find the* RATE %.

Ex. The interest of $750 for 4 years is $180, what is the rate %?

OPERATION.

$750
.04
$30.00) $180.00 (6%, *Ans.*
180 00

By proportion.

$100 : $750 :: $x \times 4$: $180.

ANALYSIS.—We find the interest on the principal for 4 years at 1 %. Since the interest of $1 at 1 % for 4 years is 4 cts., the interest of $750 will be 750 times as much, or $30. Now, if $30 is 1 %, $180 will be as many % as $30 is contained times in $180; dividing, we obtain 6, the required rate %. Hence the

313. RULE.—*Divide the given interest by the interest of the principal for the given time, at 1 %, and the quotient will be the rate % required.*

EXAMPLES FOR PRACTICE.

Required the rate per cent. if the interest of

1. $500 for 1yr. 3mo. is $56.25. *Ans.* 9 %.
2. $40 for 2yr. 9mo. 12da. is $13.36. *Ans.* 12 %.
3. $540 for 1yr. 2mo. 6da. is $38.34. *Ans.* 6 %.
4. £37 15 0 for 1yr. 4mo. is £3 10 $5\frac{3}{5}$. *Ans.* 7 %.
5. $125 for 3yr. 6mo. is 32.37\frac{1}{2}$. *Ans.* $7\frac{2}{5}$ %.
6. $1500 for 3yr. 3mo. 29da. is $274.77.
7. $124 for 4yr. 3mo. 10da. is 29.17\frac{4}{5}$. *Ans.* $5\frac{1}{2}$ %.
8. $36 for 3yr. 8mo. 19da. is $8.034.
9. At what rate % must $1, or any other sum, be on interest to double itself in $14\frac{2}{7}$ years? *Ans.* 7 %.
10. A man invested $4500 in the Montreal Bank, and received a semi-annual dividend of $167.50; what % was the dividend?

314. CASE V.—*The principal, interest, and rate % being given, to find the* TIME.

Ex. In what time will \$450 gain \$54 interest, at 6 %?

OPERATION.

\$ 450
.06
\$27.00) \$54.00 (2yr. *Ans.*
54 00

By proportion.

\$100 : \$450 :: 6 × x : \$54.

ANALYSIS.—We find the interest on the given principal for 1 year. Since the interest of \$1 for 1 year is 6 cents, the interest of \$450 will be 450 times as much, or \$27. Now, if it require 1 year for the given principal to gain \$27, it will require as many years to gain \$54 as \$27 is contained times in \$54; dividing, we obtain 2 years, the required time. Hence the

315. RULE.—*Divide the given interest by the interest on the principal for* 1 *year, and the quotient will be the time required in years and decimals.*

NOTE.—The decimal part of the quotient, if any, may be reduced to months and days (by 210).

EXAMPLES FOR PRACTICE.

In what time will

1. \$26, at 6 %. give \$1.95 interest? *Ans.* 1yr. 3mo.
2. \$280, at 6 %. give \$84 interest? *Ans.* 5 years.
3. \$45.25, at 6 %. give \$1.81 interest? *Ans.* 8mo.
4. \$98, at 8 %. gain \$25.48?
5. \$240, at 6 %. amt. to \$280? *Ans.* 2yr. 9mo. 10da.
6. \$70.50, at 9 %, give \$31.72½ interest?
7. \$408, at 7 %. amt. to \$434.18? *Ans.* 11 mo.
8. £120, at 4½ %, amt. to £140 8 0?
9. \$1, or any other sum, double itself, at 5 % int.? *Ans.* 20yr.
10. \$2365.24 double itself, at 7 %?

PROMISCUOUS EXAMPLES IN SIMPLE INTEREST.

What principal will in

1. 5yr. 4mo., at 4 %, give \$2048 int.? *Ans.* \$9600.
2. 5mo. 6da., at 6 %, give £136 3 6 int.? *Ans.* £5237 10.
3. 1yr. 8mo., at 6½ %, give \$97.50 int.? *Ans.* \$900.
4. 9mo. 21da., at 5 %, give £15 15 0 int.? *Ans.* £389 13 9+
5. 3yr. 5mo. 18da., at 5¼ %, give \$288 int.? *Ans.* \$1582.42.
6. 11mo. 9da., at 5½ %, give £466 2 6 int.? *Ans.* £9000.
7. 4yr. 5mo. 14da., at 5 %, give \$150.37½ int.? *Ans.* \$675.
8. 3yr. 5mo. 17da., at 5¾ %, give \$1451.52 int.? *Ans.* \$7287.71.

In what time will

9. \$625, at 6 %, give \$262.50 int.? *Ans.* 7yr.
10. £67 10 0, at 4 %, give £24 6 0 int.? *Ans.* 9yr.
11. \$1779, at 5 %, give \$296.50 int.? *Ans.* 3yr. 4mo.

12. $242, at 4¾ %, give $55 int.? *Ans.* 4yr. 9mo. 12da.
13. £460, at 5¼ %, give £50 int.? *Ans.* 2yr. 25da.
14. $2178, at 4⅛ %, give $635.25 int.? *Ans.* 7yr.
15. £405, at 6 %, give £151 17 6 int.? *Ans.* 6yr. 3mo.
16. $481.25, at 5 %, give $192.50 int.? *Ans.* 8yr.

Required the rate %, if the interest of

17. $978.20 for 1yr. is $48.91. *Ans.* 5 %.
18. £110 12 6 for 50da. is £1 16 10½. *Ans.* 12 %.
19. $1290 for 124da. is $19.99½. *Ans.* 4½ %.
20. $4340 for 3yr. is $585.90. *Ans.* 4½ %.
21. $675 for 44mo. is $142.31¼. *Ans.* 5¾ %.
22. $7500 for 48da. is $60. *Ans.* 6 %.
23. $11004.75 for 1yr. is $550.23¾. *Ans.* 5 %.
24. £120 for 6mo. 15da. is £32 10 0. *Ans.* 50 %.

25. The annual sales of a starch manufacturer amount to £2737 10; supposing that his profits are 5 % per year, in how many years will they reach £323 18 9? *Ans.* 2yr. 4mo. 12da.

26. An individual disposed of the ⅘ of his funds at 4 % and ⅕ at 5 %; every year he draws as much as will pay the harnessing of a horse which harness is worth $117.60; what is the amount of his funds? *Ans.* $2800.

27. What is the interest of $17.18, from July 29th., 1864, to Sept. 1st., 1868, at 6 %? *Ans.* $4.214+.

28. What will be the amount of £19 15 9, at 7½ %, from Feb. 17th., 1864, to Dec. 30th., 1867? *Ans.* £25 10 7+.

29. If $1756.75 is placed on interest, June 29th., 1866, what will it amount to Feb. 12th., 1869, at 7 %? *Ans.* $2078.869+.

30. What principal, at 5 %, during 1yr. 8mo. 12da. will amount to £231 12 11⅘? *Ans.* £213 10 0.

31. On Aug. 15th., 1860, I lent $5259, at 6 %; what amount will be due me on May 1st., 1868? *Ans.* $7692.164.

32. An individual buys 65⅘ acres of land at the rate of $509.72 per 100 acres; if he pays only at the end of 3yr. 1mo. 15da., the int. will equal to ⅛ of the principal; what is the rate? *Ans.* 4 %.

33. A person placed a certain sum on interest at 4 %, which produced £427 10, in 3 years; what is the sum? *Ans.* £3562 10.

34. What is the interest on a bill of $257.81, dated March 1st., 1865, and payable July 16th., 1867, at 7 %? *Ans.* $42.86+.

35. Find the amount of $17041.20, at 4¾ %, for 1yr. 7mo. 28da.

36. What sum is that which will give an interest of $900, in 10yr., at 4½ %? *Ans.* $2000.

37. A principal of £112 10 was put on interest, and at the end of 8yr. amounted to £144; at what rate was the principal placed?

38. A boy has accumulated a sum of money by his savings, and wishes to obtain an annual revenue of $140; if the rate is 5 %, what principal must he have? *Ans.* $2800.

39. A merchant borrows the sum of £938 12 3, which is owned by a minor aged 15yr. 3mo. 20da. He keeps it until the owner is 21 years old; what sum will be then due, at 6 % simple interest?

40. What will be the interest of $325, from June 5th., 1866, to July 4th., 1868, at 7¼ %? *Ans.* $49.02 +.

41. A merchant says that his gain, during the nine years he carried on business, equals the price of 3659 yards of cloth at $2.08 a yard; what was his annual revenue, supposing he placed his gain on interest at 5 %? *Ans.* $380.536.

42. From 1857 to 1867, the population of Syracuse augmented 24¾%; knowing the last year's number of inhabitants to be 102295, tell us what was the population in 1857? *Ans.* 82000 inhab.

43. What sum must be placed on interest, at 4 %, to amount to £627 18 6 in 2yr. 10mo 16da.? *Ans.* £563 2 1½.

44. A man assures me that if he places on interest a sum equivalent to 968 yd. of cloth at $3.18 a yard, he will secure an annual revenue of $153.91½; what must be the rate? *Ans.* 5 %.

45. From an investment of $35680 in commercial concerns, I withdraw a gain of $223 per month; what is the annual rate of the interest? *Ans.* 7½ %.

46. A property was sold for £2830; the conditions were £800 in cash, £875 in 6 months, £625 in 10 months, and the remainder in 1yr. 3mo., with interest at 7 %; what was the amount paid?

47. A merchant having raised, during the 6 years of his business, a capital of $2965.10, desires to know in what time he will receive $889.53 as interest at 5 %? *Ans.* 6yr.

48. An individual borrowed £3750 at 7 %, and then lent it at 6 %; what will he lose in 146 days, if the year, for the first transaction, consists of 360 days, and that of the second, 365 days?

49. During what time must a certain sum be on interest at 4½ % to produce ⅘ of it? *Ans.* 17yr. 9mo. 10da.

50. In selling merchandise at 12s. the yard, I make a profit of 6¼ %; what is the price per yard? *Ans.* 11s. 3½ + d.

51. The ⅔ of a sum of money is lent at 4 %, and the ⅔, at 5 %; what is the sum, knowing that the annual interest is $28.82? *Ans.* $655.

52. An apparatus for astronomical purposes cost £49; but, as this sum could not be paid before 3yr. 9mo., the price was augmented $\frac{5}{26}$ of its primitive value; what was the rate? *Ans.* 4 %.

53. A man placed on interest, at 4 %, a certain sum of money which produced in 5 years the funds requisite for the purchase of 368 lbs. of preserved tamarinds, at 46½ cts. a lb.; what was the sum?

54. A merchant has invested in business a capital of $21840 which produces him 12½ % annually; but, for sanitary reasons, he retires from mercantile affairs, and loans his money at 7¾ %; how much will he lose in 2yr. 5mo. 10da. by the change? *Ans.* $2535.86⅔.

55. What is that principal the ⅘ of which at 6 %, and the remainder at 7 %, will give $4340 interest? *Ans.* $70000.00.

56. A speculator desires to purchase a tract of land, containing 450 acres, at £6 17 6 per acre, and, for this purpose, borrows money at 5½ %. At the expiration of 4yr. 11mo. 20da., he sells the ⅖ of the land at £8 10 an acre, and the remainder, at £8 2 9 the acre; how much does he lose by the transaction?

COMPOUND INTEREST.

316. Compound Interest is interest on both principal and interest, when the latter is not paid when due.

NOTE.—The simple interest may be added to the principal annually, semi-annually, quarterly, or monthly, according to agreement. A creditor may receive compound interest without being liable to the charge of usury, but cannot *legally* demand it.

Ex. What is the compound interest of $390 for 3 years, at 5 %?

OPERATION.

	$390.00	Principal for 1st. year.
$390.00 × .05 =	19.50	Interest for 1st. year.
	$409.50	Principal for 2nd. year.
$409.50 × .05 =	20.475	Interest for 2nd. year.
	$429.975	Principal for 3rd. year.
$429.975 × .05 =	21.49875	Interest for 3rd. year.
	$451.47375	Amount for 3 years.
	$390.00000	Given principal.
	$ 61.47375	Compound interest.

317. RULE.—I. *Find the amount of the given principal at the given rate for one year, and make it the principal for the second year.*

II. *Find the amount of this new principal, and make it the principal for the third year, and so continue to do for the given number of years.*

III. *Subtract the given principal from the last amount, and the remainder will be the compound interest.*

NOTES.—1. When the time contains years, months, and days, find the amount for the years, upon which compute the interest for the months and days, and add it to the last amount, before subtracting.

2. When the interest is payable semi-annually or quarterly, find the amount of the given principal for the first interval, and make it the principal for the second interval, proceeding in all respects as when the interest is payable yearly.

EXAMPLES FOR PRACTICE.

1. What is the compound interest of $970 for 2 years 9months and 24 days, at 6 %? *Ans.* $173.295.

2. What is the compound interest of $520 for 3 years, at 5 %?

3. What is the amount of $128 for 3 years 5 months and 18 days, at 6 %. compound interest? *Ans.* $156.717.

4. What is the compound interest of $340 for 2yr., interest being payable semi-annually, at 6 %? *Ans.* $42.67 + .

5. What is the compound interest of $737.75 for 2½ years, payable semi-annually, at 7 %?

6. What will $900 amount to in 1 year, at 7 %, compound interest, payable quarterly? *Ans.* $964.67 +

7. What is the amount of $500 for 1yr., interest payable every 3 months, compound interest, at 8 %?

8. Find the compound interest of $948 for 3 years 4 months and 18 days, at 6 %? *Ans.* $207.051.

318. Compound interest may be computed more expeditiously by the use of the following

TABLE

Showing the amount of $1, *or* £1, *at* 3, 4, 5, 6, 7, *and* 8 *per cent., compound interest, for any number of years from* 1 *to* 34.

Years	3 per cent.	4 per cent.	5 per cent.	6 per cent.	7 per cent.	8 per cent.
1	1.030000	1.040000	1.050000	1.060000	1.070000	1.080000
2	1.060900	1.081600	1.102500	1.123600	1.144900	1.166400
3	1.092727	1.124864	1.157625	1.191016	1.225043	1.259712
4	1.125509	1.169859	1.215506	1.262477	1.310796	1.360489
5	1.159274	1.216653	1.276282	1.338226	1.402552	1.469328
6	1.194052	1.265319	1.340096	1.418519	1.500730	1.586874
7	1.229874	1.315932	1.407100	1.503630	1.605782	1.713824
8	1.266770	1.368569	1.477455	1.593848	1.718186	1.850930
9	1.304773	1.423312	1.551328	1.689479	1.838459	1.999005
10	1.343916	1.480244	1.628895	1.790848	1.967151	2.158925
11	1.384234	1.539454	1.710339	1.898299	2.104852	2.331639
12	1.425761	1.601032	1.795856	2.012197	2.252192	2.518170
13	1.468534	1.665074	1.885649	2.132928	2.409845	2.719624
14	1.512590	1.731676	1.979932	2.260904	2.578534	2.937194
15	1.557967	1.800944	2.078928	2.396558	2.759032	3.172169
16	1.604706	1.872981	2.182875	2.540352	2.952164	3.425943
17	1.652848	1.947901	2.292018	2.692773	3.158815	3.700018
18	1.702433	2.025817	2.406619	2.854339	3.379932	3.996020
19	1.753506	2.106849	2.526950	3.025600	3.616528	4.315701
20	1.806111	2.191123	2.653298	3.207136	3.869685	4.660957
21	1.860295	2.278768	2.785963	3.399564	4.140562	5.033834
22	1.916103	2.369919	2.925261	3.603537	4.430402	5.436540
23	1.973587	2.464716	3.071524	3.819750	4.740530	5.871464
24	2.032794	2.563304	3.225100	4.048935	5.072367	6.341181
25	2.093778	2.665836	3.386355	4.291871	5.427433	6.848475
26	2.156591	2.772470	3.555673	4.549383	5.807353	7.396353
27	2.221289	2.883369	3.733456	4.822346	6.213868	7.988062
28	2.287928	2.998703	3.920129	5.111687	6.648838	8.627106
29	2.356566	3.118651	4.116136	5.418388	7.114257	9.317275
30	2.427262	3.243398	4.321942	5.743491	7.612255	10.062657
31	2.500080	3.373133	4.538040	6.088101	8.145113	10.867669
32	2.575083	3.508059	4.764942	6.453387	8.715271	11.737083
33	2.652335	3.648381	5.003189	6.840590	9.325340	12.676050
34	2.731905	3.794316	5.253348	7.251025	9.978114	13.690134

NOTE.—The compound interest of $1, or £1, is $1, or £1, less than the amounts in the above table.

Ex. What is the compound interest of $90 for 7 years and 6 months, at 7%?

OPERATION.

Amt. of $1 for 7yr.,	$1.605782
Principal,	90
Amt. $90 for 7yr.,	144.520380
Interest of $1 for 6mo.,	.03½
	4.3356114
	.7226019
Int. of amt. for 6mo.,	5.0582133
Amt. added,	144.520380
Amt. for 7yr. 6mo.,	149.5785933
Principal subtracted,	90.
Comp. int. for giv. time,	$59.57 +, *Ans.*

ANALYSIS. — We find the amount of $1 for 7 years in the table, and multiplying it by the given principal, obtain the amount of the $90 for 7 years. We then find on this amount the interest for the 6 months, and add it to its principal. From the last amount subtracting the original principal, we have left the compound interest required. Hence the

319. RULE.—*Multiply the amount of* $1 *for the given rate and time, as found in the table, by the given principal, and the product will be the amount. Subtract the principal from the amount, and the remainder will be the compound interest.*

EXAMPLES FOR PRACTICE.

1. What is the compound interest of $60 for 8 years and 6 months, at 7%? *Ans.* $46.69 +.
2. What is the amount of $25.50 for 20 years 2 months and 12 days, at 7%, compound interest? *Ans.* $100.058.
3. What is the compound int. of $3000 for 2yr. 6mo. 18da., at 6%?
4. What is the amount of $12 for 6 months, the interest to be compounded monthly, at 6%? *Ans.* $12.364 +.
5. What is the compound interest of $600 for 10 years 7 months and 15 days, at 6%?
6. To what sum will $75, deposited in a savings bank, amount, at compound interest, for 17 years, at 6%, payable semi-annually?

PROMISSORY NOTES.

320. A **Promissory Note** is a written or printed engagement to pay a certain sum either on demand or at a specified time.

321. The **Maker** or **Drawer** of a note is the person who signs it and thus becomes responsible for its payment when due.

322. The **Payee** of a note is the person to whom or to whose order it is made payable.

323. The **Indorser** of a note is the person who signs his name on the back of it, and by so doing guarantees its payment, unless he writes "Without Recourse" over his name at the time.

324. A **Negotiable Note** is a promissory note which is made payable to bearer or the order of some person (*see Notes, Forms*, 2, 3, 4).

NOTES.—1. If a note is payable to the bearer, it may be negotiated without indorsement.

2. A note should contain the words "value received", and the sum for which it is given should be written out in words.

325. A note may be made payable on demand, as in *Form No.* 1, or at the expiration of a certain time after its date, as in *Forms No.* 2, 3, and 4. A note may be made payable to a particular person, as in *Form No.* 1; or to any person who is the bearer or holder of it, as in *Form No.* 2; or to the order of a person named in it, as in *Form No.* 3; and may be made payable at a particular place, as in *Form No.* 4.

The Note, *Form No.* 1, is due when the payee demands its payment from the maker of it.

REMARK.—If no time is fixed, in a note, for payment, it is payable on demand.

The Note, *Form No.* 2, is payable to the holder of it at the expiration of six calendar months from its date.

The Note, *Form No.* 3, is due at the time specified in it, to the payee who indorses it. Jos. A. Walter may indorse this note in blank, that is to say, only write his name, and thus make any person lawfully holding the note, the payee; or, he may indorse it payable to the *order* of a particular person, in which case such person can make another person the payee, as Jos. A. Walter could, by indorsing the note in blank or otherwise.

The Note, *Form No.* 4, is payable only at the Bank named in it.

326. The **Face** of a note is the sum named in it.

327. **Days of Grace** are the three days usually allowed by law for the payment of a note after the expiration of the time specified in the note.

328. The **Maturity** of a note is the expiration of the days of grace; a note is *due* at maturity.

REMARK.—1. When a note promises interest, as *Forms* 1, 2, and 3, the interest begins at the date of the note, and continues until the note is paid. If the time expressed in a note for its maturity be stated in months, calendar months are understood; and if a note promises interest without stating the rate %, it bears the legal interest of the country in which it is dated; also, a note which does not promise interest, if not paid when due, bears the same legal rate % of interest from the time it matures until paid.

2. If a note be not paid by its maker when it matures, it may on the same day be protested for non-payment, and the indorsers may be required to pay it if they are at once notified of the *protest*.

3. If a note matures on Sunday or a legal holiday, it must be paid on the day previous.

329. A **Business Note** is a note given for a valuable consideration. It renders the maker liable for the amount to the payee, or to any subsequent *bona fide* holder.

330. An **Accommodation Note** is a note given for no valuable consideration. It does not render the maker liable to the payee, but makes him liable to any *bona fide* holder after it has been negotiated for value.

NOTE.—Accommodation notes are usually given to enable the payee to borrow money on the credit of the makers of the notes.

331. A **Joint Note** is a note signed by two or more persons, who are jointly liable for its payment. A *Joint and Several Note* is a note signed by two or more persons, who may be held, either jointly or singly, for its payment.

332. A **Produce Note** is a written promise to deliver goods to a specified amount.

333. A **Due Bill** is an acknowledgment of a debt due in money, or its equivalent in goods.

334. A **Bond** is a written obligation, authenticated by a seal, to secure the payment of a sum of money for the performance or non-performance of certain acts.

335. A **Mortgage** or **Mortgage Deed** is a conveyance of property given to secure the payment of a bond or debt, on condition that when payment is made, the conveyance is void and the mortgage is discharged.

FORMS OF NOTES.

Form No. 1.—DEMAND NOTE.

$64\frac{36}{100}$. *Quebec, January 15th, 1870.*

On demand, I promise to pay R. N. Harris, sixty-four and $\frac{36}{100}$ dollars, with interest, at 5%. Value Received.

(STAMP.) *Louis Garneau.*

Form No. 2.—NOTE PAYABLE TO BEARER (NEGOTIABLE.)

265\frac{75}{100}$. *Montreal, February 3, 1870.*

Six months after date, I promise to pay W. S. Colton, or bearer, two hundred sixty-five and $\frac{75}{100}$ *dollars, with interest at 6%. Value received.*

(STAMP.) *T. U. Power.*

Form No. 3.—NOTE PAYABLE TO ORDER (NEGOTIABLE).

$ 99$\frac{47}{100}$. *Kingston, March 10, 1870.*

One year after date, I promise to pay to the order of T. H. Samson, ninety-nine and $\frac{47}{100}$ *dollars, with interest, at 7%. Value received.*

(STAMP.) *E. F. Byrne.*

Form No. 4.—NOTE PAYABLE AT A BANK (NEGOTIABLE.)

$ 87$\frac{80}{100}$. *Toronto, April 12, 1870.*

Forty days after date, I promise to pay to the order of C. D. Nichols, at the Ontario Bank, eighty-seven and $\frac{80}{100}$ *dollars. Value received.*

(STAMP.) *John Douglas.*

Form of Produce Note.

$ 58$\frac{35}{100}$. *Halifax, N. S., May 14, 1870.*

For value received, we promise to pay to J. H. Martel, on demand, fifty-eight and $\frac{35}{100}$ *dollars in goods, at our store.*

(STAMP.) *S. Richard & Co.*

Form of Due Bill.

$103. *Ottawa, June 9, 1870.*

Due Edward G. Larkin, for value received, one hundred and three dollars, with interest.

(STAMP.) *Philip Alleyn.*

PROFIT AND LOSS.

336. Profit and **Loss** are commercial terms, used to express the gain or loss in business transactions.

337. There are four terms or quantities to be considered in Profit and Loss, viz.:—

1st. The *Cost*, or original number, which is the **Base.**
2nd. The *Rate % of gain* or *loss*, which is the **Rate %.**
3rd. The *Gain*, or *Loss*, which is the **Percentage.**
4th. The *Selling Price*, which is the **Amount**, or **Difference.**

The questions follow the same rules as in Percentage.

Selling Price = Cost + Gain, or *Cost — Loss.*
Cost = Selling Price — Gain, or *Selling Price + Loss.*
Gain = Selling Price — Cost.
Loss = Cost — Selling Price.

EXAMPLES FOR PRACTICE.

1. I bought cloth, at \$2.50 per yard, and sold it so as to gain 25 %; for how much did I sell it a yd.? *Ans.* \$3.12½.

To solve this Example, *see* Case I., 282, RULE.

2. A farm was bought for \$4500, and sold so as to gain \$900; how much was the gain %? *Ans.* 20 %.

To solve this Example, *see* Case II., 284, RULE.

3. By selling a building lot, a man gained \$175, which was 12 % of the cost; what was the cost? *Ans.* \$1458.33⅓.

To solve this Example, *see* Case III., 286, RULE.

4. A gentleman sold a horse for \$180, and thereby gained 20 %: what was the cost of the horse? *Ans.* \$150.

To solve this Example, *see* Case IV., 288, RULE.

5. A merchant lost 15 % on his old stock of goods; how much did he lose on those that cost 12½ cts., \$6⅜, 38⅛ cts., 33⅓ cts., and \$18⅘? *Ans.* 1⅞ cts.; \$1; 5¾ cts.; etc.

6. Bought sugar, at 12 cts. a pound, and sold it so as to gain 1½ cts. a pound; required the gain %.

7. Sold butter at ⅝ of a dollar a pound, which was at a gain of 25 %; required the cost per pound. *Ans.* 66⅔ cts.

8. A market woman sold oranges so as to gain ⅔ of a cent on each orange, which was at a gain of 33⅓ %; what was the cost of an orange? *Ans.* 2 cents.

9. Sold a horse at 33⅓ % gain, and with the money bought another horse, which I sold for \$120, and lost 25 %. Did I gain or lose by my trading? and how much?

10. If I make a profit of 15 5/11 % by selling paper for \$0.85 above the cost per ream, how much must be added to the selling price to realize a profit of 32½ %? *Ans.* 93¾ cts.

11. What should I sell a barrel of flour for, that cost me £1 2 6, to gain 16⅔ %? *Ans.* £1 6 3.

12. A neighbor offers his house, which cost him \$6900, for 20 % less than cost; what is his price? *Ans.* \$5520.

13. A merchant sells cloth for \$5 a yard, which cost him \$3.75 a yard; what is his gain per cent.? *Ans.* 33⅓ %.

14. I bought 640 yards calico at 15 cts. per yd., and sold it at a reduced price of 2½ %; what did I lose? *Ans.* \$2.40.

15. A grocer sells coffee at 7½d. a lb. which cost him 9d.; what is his loss per cent.? *Ans.* 17⅓ %.

16. A merchant buys at auction \$9562.50 worth of goods; if he sell them at an advance of 20 % on the cost, what will be his net profits, deducting \$600 for expenses? *Ans.* \$1312.50.

17. How much should I sell different qualities of sugar which cost me £1 15, £2 1 3, and £2 12 6 the cwt., to gain 12½ %?

18. Bought 45 bbl. of apples at $3.50 per bbl., and sent them by railroad, to be sold on commission at 5 %; knowing that I paid for freight and other expenses $5.38, what will be my total loss if the selling price is 10 % below the buying price? *Ans.* $28.2175.

19. Bought a horse for $130, paid $6 for his nourishment during 5 weeks, and then sold him for $120; what was my loss per cent on the whole cost? *Ans.* $0.11 $\frac{43}{47}$

20. Bought codfish at $4.25 the cwt., and sold it at $4.93; what was my gain per cent.? *Ans.* 16 %.

21. A grocer sold tea which cost 3s. 1½d. for 3s. 9d. per lb.; sugar which cost 5¾d. for 7½d.; flour which cost £1 6 0 for £1 8 9; what was his gain per cent. on each article? *Ans.* 20 % on the tea; etc.

22. Bought 9cwt. 72lb. of sugar for $65; paid $5.15 for freight and drayage; at how much per pound should it be sold to gain 25 % on the buying price? *Ans.* $.08$\frac{3}{8}$.

23. A dealer in furs made a profit of $156 in selling a certain quantity at 12 % advance; what was the amount sold? *Ans.* $1300.

24. A merchant bought a hogshead of wine for $189; a part having been lost by leakage, he sold the remainder at $3.99 per gallon, and found that his loss was 5 % on the cost; how many gallons did he lose by leakage? *Ans.* 18 gallons.

25. Sold a cargo of corn for £4000, at 25 % profit; what did the cargo cost? *Ans.* £3200.

26. In selling tea at 90 cts. a lb., I gained 20 %; how much would I have gained had I sold it at $1 a pound? *Ans.* 33⅓ %.

27. By selling cloth at $4 the yard, I lose 20 %; what was the cost?

28. What will I gain per cent. by selling silks at $5 which cost $4.25? *Ans.* 17$\frac{17}{17}$ %.

29. By selling lard at £1 15s. per cwt. I gain 75 %; how much % would I gain or lose by selling it for 18s.? *Ans.* Lose 10 %.

30. Sold wheat at $1.25 the bushel thereby losing 15 %; how much per cent. would I have gained had I sold it at 1.64\frac{1}{17}$ the bushel?

31. Lost 15 %, by selling a lot of paper for $480; for how much should I have sold it to gain 12 %? *Ans.* $632.4705 + .

32. Sold a field containing 106A. 3R. 30rd., at $120 an acre, thereby making a profit of 18 % on the cost; what did the field cost?

33. Tea, sold at 25 % loss, is $1.25 a lb.; what would be the gain or loss per cent. in selling it at $1.60 a lb.? *Ans.* 4 % loss.

34. A lumber merchant sold 36840 feet of wood at £5 5 7$\frac{2}{3}$ per M., and gained 28 %; how much would he have gained or lost by selling the wood at £4 5 per M.?

35. The retail prices of my goods are 40 % above the cost. I supply my customers wholesale at a reduction of 12 % on the retail price; what is my profit on the goods sold by wholesale? *Ans.* 23$\frac{1}{5}$ %.

36. An engineer sold an engine for $8812.50 and lost 6 % on the cost; what should it have been sold for to gain 12½ %?

37. I sold a horse at an advance of 30 %, and with this money bought another which I sold for £45 10, losing 12½ %; what did each horse cost me? *Ans.* 1st. horse £40; 2nd. horse £52.

38. A speculator sold the goods of a store at a reduction of 7$\frac{2}{5}$ %, and realized a profit of 5 %; at what rate of reduction were the goods bought? *Ans.* 12 %.

39. My retail price for grey cloth is \$4.75 per yd., by which I make a profit of $33\frac{1}{3}$ %. I sell 100yd. by wholesale at 30 % reduction on the retail price. What is my gain or loss per cent., and how much do I receive a yard? *Ans.* $6\frac{2}{3}$ % loss; \$3.32½ a yd.

40. A merchant sells linen $2\frac{1}{4}$ cts. more than the cost and realizes a profit of 8 %; what is the cost of a yard? *Ans.* $31\frac{1}{4}$ cts.

41. A grocer demanded for a certain quantity of prunes a price 22 % above the cost; but being a little musty, he sold them at 10 % less than his first demand, and thus gained \$98 by the sale; what was his first demand? *Ans.* \$1220.

42. At what price should I sell codfish which costs 16s. $5\frac{4}{5}$d. per cwt. to realize a profit of $12\frac{1}{2}$ % on the cost, after deducting $12\frac{1}{2}$ % of the price? *Ans.* £1 1 $2\frac{11}{35}$.

43. Bought a quantity of cheese at 12 cts. a pound. Supposing the weight to be 5 % less than that calculated, and 10 % of the sales to be in bad debts, for how much must it be sold a pound to make a net profit of 14 % on the cost? *Ans.* 16 cents a pound.

44. J. Moran & Co. bought dry goods for the amount of \$6840; they sold $\frac{1}{4}$ at 15 % profit, $\frac{1}{3}$ at $18\frac{3}{4}$ %, $\frac{1}{5}$ at 20 %, and the remainder at $33\frac{1}{3}$ % profit; what was their total profit? *Ans.* \$1482.00.

COMMISSION AND BROKERAGE.

338. Commission and **Brokerage** are the percentages paid an agent, or broker, for the transaction of business, and is estimated at a certain rate per cent. on the amount of the sale, purchase, collection, etc., effected.

339. An **Agent, Factor, Broker, Collector,** or **Commission Merchant,** is a person who transacts business for another.

NOTES.—1. An agent may be a *Special Agent*,—that is, authorized to transact only such business as is specified,—or a *General Agent*, who, as such, can transact any business of the person who employs him.

2. Merchandise and Produce sent to a person for sale or superintendence, are said to be *consigned*. The person sending them is termed a *Consignor*; the person to whom they are sent, is termed a *Consignee*.

3. A consignee whose business office is remote from a consignor, is sometimes termed a *Correspondent*, and usually acts as agent of the firm consigning him the goods.

4. Brokers are classified according to the nature of the sales and contracts they effect. Thus, a *Bill Broker* is one who negotiates the discount on bills of exchange, etc.; a *Real-Estate Broker* is one who negotiates the sale of houses and lands; *Insurance Broker*, *Ship Broker*, *Stock Broker*, *Pawn Broker*, etc.

5. A collector may have the business of settling accounts between individuals, or he may be an officer of the government, as a *Collector of the Port*, whose business is to collect duties; a *Collector of Taxes*, etc.

340. The **Net Proceeds** is the amount received from a sale or collection, less the commission and other charges.

Questions on Commission and Brokerage follow the same rules as those in Percentage.

EXAMPLES FOR PRACTICE.

1. A broker sold $15800 worth of stock for C; required his brokerage at $\frac{1}{4}\%$? *Ans.* $39.50.

To solve this Example, *see* Case I., 282, RULE.

2. An agent received $1600 for selling a house and lot for $25600; what was his rate of commission? *Ans.* $6\frac{1}{4}\%$.

To solve this Example, *see* Case II., 284, RULE.

3. A commission merchant receives $84 for selling wood, at $5\frac{1}{3}\%$; what is the amount sold? *Ans.* $1575.

To solve this Example, *see* Case III., 286, RULE.

4. An agent receives $3105 to be invested in dry goods; after retaining his commission, $3\frac{1}{2}\%$, how much was invested? *Ans.* $3000.

To solve this Example, *see* Case IV., 288, RULE.

5. What is the commission on $874, at $2\frac{1}{4}\%$? on $71.50, at $3\frac{1}{2}\%$? on $1580.70, at $4\frac{3}{4}\%$? on $309.10, at $5\frac{1}{2}\%$; on $4705.26, at 6%? *Ans.* $19.66\frac{1}{2}$; $2.50\frac{1}{4}$; etc.

6. What is the commission on £15 9 10, at 3%? on £170 10 6, at $4\frac{1}{2}\%$? on £630 9 0, at $6\frac{1}{4}\%$? on £96 12 3, at $5\frac{1}{4}\%$? on £918 7 0, at $6\frac{1}{2}\%$? *Ans.* £0 9 $3\frac{1}{4}+$; £7 13 $5\frac{1}{2}+$; etc.

7. How much will I pay for the brokerage of $750, at $\frac{1}{4}\%$? of $1540.40, at $\frac{1}{2}\%$? of $3610.80, at $1\frac{1}{2}\%$? of $823.50, at $\frac{3}{4}\%$? of $1560.70, at $1\frac{1}{4}\%$? *Ans.* $1.87\frac{1}{2}$; 7.702; etc.

8. Sold merchandise as follows: 1st. for £942 16 0, at $4\frac{1}{2}\%$ commission; 2nd. for £15 11 6, at 5%; 3rd. for £310 5 7, at 6%; 4th. for £530 0 5, at $3\frac{1}{2}\%$; what is the total com.? *Ans.* £80 7 $5\frac{1}{4}+$.

9. What amount of brokerage must I pay for exchanging greenbacks, as follows: $590, at 26%; $745.30, at 28%; $1615.72, at 30%; $4532.09, at 32%; $87.30, at 29%? *Ans.* $2322.385+$.

10. A farmer paid a broker $\frac{7}{8}\%$ for investing $11730, in Ontario bonds; what is the brokerage? *Ans.* $102.637+$.

11. A broker received $465 for buying stocks, at $\frac{5}{8}\%$ brokerage; how much stock did he buy? *Ans.* $74400.

12. A flour merchant remits to his agent in Toronto $4740 for the purchase of grain, after deducting the commission at 2%; how much will the agent expend for his employer, and what will be his commission? *Ans.* $4647.06—, for grain; $92.94+ for commis.

13. An agent sold real estate on 4% commission, and remitted $100.36 to the owner as the net proceeds; for what price did he sell the property, and what was his commission?

14. An agent receives $4920 to expend in purchasing cows at $32 a head; after reserving his commission, $2\frac{1}{2}\%$, how many cows did he purchase? *Ans.* 150.

15. A merchant having on hand 4700 barrels of sugar, gave an agent $3\frac{1}{3}\%$ for selling it; what are the net proceeds, if sold at $16 a bbl.?

16. I purchased 6000 bushels of wheat in Buffalo, at $1.37½, and shipped the same to my agent in Kingston, who sold it at $1.62½. How much did I make, after paying $543 for expenses and a commission of 2⅔%? *Ans.* $723.

17. A broker charged me 5¼% for the exchange of £681 4 10 in greenbacks; what was his brokerage? *Ans.* £35 15 3½.

18. A commission merchant sold a consignment of oats for $12686. He charged $66 for storage, and 6¼% commission; what were the net proceeds? *Ans.* $11827.12½.

19. An architect charges ⅜% for his plan and survey of a building which cost $24000, and 1½% for superintending the work; how much did he receive? *Ans.* $450.

20. I sent to my correspondent in Bordeaux £2097 10, with advice to invest in the purchase of wines, after deducting his commission of 3½%; what was the sum invested and what was his commission?

Ans. £2026 11 4⅘, wines; £70 18 7⅕, commission.

21. An agent having a debt of $1570 to collect, compromises for 90%; what was his commission at 5½%? *Ans.* $77.71½.

22. Paid Folger Brothers $5.46 for exchanging $364 in United States' money; what was the rate of brokerage? *Ans* 1½%.

23. A consignee in Glasgow informs his constituent of the purchase of Dry Goods to the amount of £395 15 5; what is his commission at 2¼%? *Ans.* £8 18 1+.

24. Bought at Halifax a cargo of wheat, 9500 bushels, at $1.20 per bushel, and sent it to my agent in Portland who sold it at $1.50 per bushel; what did I realize on the whole after paying $320 for expenses, and commission at 3½%? *Ans.* $2031.25.

25. My correspondent at Bordeaux charges $74.20 for purchasing 264 cwt. of honey at $10.50 per cwt.; what was the rate of commission? *Ans.* 2⅔⅞%.

26. A broker receives £2085 7 6 comprising the sum to be invested in Railroad stock at £20 15 a share, and his brokerage at ½%; how many shares can he buy, and what is his brokerage?

27. A certain piece of land was sold for $3925, but the owner received $3866.12½ as the net proceeds; what was the rate of commission? *Ans.* 1½%.

28. I remitted $5500 to my broker with advice to invest in Bank stock, after deducting his brokerage at ⅞%; what was the investment?

29. The net proceeds of a sale were £1408 15, and the commission, £28 15; what was the rate of commission? *Ans.* 2%.

30. In charging 1½% for the investment of a certain sum, a broker realized $285; what was the amount of the investment? *A.* $19000.

31. My agent in Cincinnati gives me information of the purchase of 4000 bushels of indian meal at 80 cts. per bushel, and desires me to remit a check on New York which he can sell to a broker at ¾% premium; what should the amount of the check be, his commission being 3%? *Ans.* $3271.464.

32. A factor received £5 12 for the sale of grain at 4% commission; what was the amount sold? *Ans.* £140.

33. Received from A $700 in specie; paid 3½% for changing it to

gold; and, after deducting the commission at 2 %, employed the balance in the purchase of fruit; what was paid for the fruit; and what was the commission? *Ans.* $661.99, fruit; $13.51 commission.

34. Remitted to my correspondent at Rouen £255, for the purchase of calico at 9d. per yard, after deducting his commission at 2 %; how many yards will I receive? *Ans.* 6666⅔yd.

35. A speculator receives $4113.50 as the net proceeds of a sale, allowing 5 % commission; what was the value of the property?

36. A commission merchant who charges 5 % commission on sales and investments, receives 260 cwt. of cheese, at 6d. per lb., and £748 10 6, in cash, with advice to purchase a cargo of cotton for the whole amount; what will be his total commission? *Ans.* £971 0 11⅗

37. A Halifax agent buys 34 boxes of chocolate; he pays $7.50 for freight and cartage, and his commission is 1½ % on the amount of the purchase. He sends me a bill of $740.83¾ for the whole; what was his commission; and, allowing 25 lb. per box, how much did I pay per lb. for the chocolate? *Ans.* $10.83¾ com.; $0.08½ per lb.

38. A commission merchant receives 125 barrels of flour from A, 150 bbl. from B, 225 bbl. from C; he finds on inspection that A's is 10 % better than B's, and C's is 5 5/11 % better than A's; he sells the whole lot at $7 per barrel, and charges 4 % commission. How much must be remit to each? *Ans.* A, $842.30; B, $918.87; C, $1598.83.

INSURANCE.

341. Insurance is a contract of indemnity, by which one party engages, for a stipulated sum, to insure another against a risk or loss to which he is exposed.

342. It is of two kinds: insurance on property, and insurance on life (1).

343. The **Insurer** or **Underwriter** is the party taking the risk; and the **Insured** or **Assured,** the party protected.

344. The **Policy** is the written obligation or contract.

345. Premium is the sum paid for insurance. It is always reckoned at a certain per cent. on the value of the property insured, varying according to the degree or nature of the risk assumed.

FIRE AND MARINE INSURANCE.

346. Insurance on property is of two kinds: *Fire Insurance,* and *Marine Insurance.*

347. Fire Insurance is an indemnification of damage and loss caused by *fire* or *lightning.*

(1) Life insurance will be treated of later.

348. Marine Insurance is an indemnification of damage and loss caused by the perils peculiar to navigation.

349. In insurance, the calculations are based on the following principles:

I. Premium is *percentage.* (278)

II. The sum insured is the *base* of premium.

III. The sum covered by insurance is *difference.*

EXAMPLES FOR PRACTICE.

1. What premium must be paid for insuring goods to the amount of \$4500, at $2\frac{1}{2}\%$? *Ans.* \$112.50.

To solve this Example, *see* Case I., 282, RULE.

2. A man paid \$64.80 for the insurance of \$8640 on his house; what was the rate of insurance? *Ans.* $\frac{3}{4}\%$.

To solve this Example, *see* Case II., 284, RULE.

3. The premium for insuring a tannery for $\frac{3}{4}$ of its value, at $1\frac{2}{3}\%$, was \$145.60; required the value of the tannery. *Ans.* \$11648.

To solve this Example, *see* Case III., 286, RULE.

4. What must be paid for an insurance of \$5728 at $1\frac{3}{4}\%$?

5. What premium must be paid for the insurance of a vessel and cargo valued at £3649 8, at $3\frac{1}{4}\%$? *Ans.* £118 12 $1\frac{1}{4}+$.

6. A schooner, insured for \$5000, at $2\frac{1}{4}\%$, was completely wrecked; how much of the loss was covered by the insurance? *A.* \$4887.50.

7. A hotel valued at £3750 is insured for $\frac{2}{3}$ of its value, at $\frac{3}{8}\%$. The policy and survey of the premises are charged 7s. 6d.; what is the insurance? *Ans.* £9 15 0.

8. A store and its stock are worth \$6370; what sum must be insured, at 2%, to cover both property and premium? *Ans.* \$6500.

9. What is the premium of insuring £695 11 8, at £5 13 9%?

10. I pay annually \$45 insurance for my library, and this sum is 3% of the amount for which I am insured; what is the amount?

11. What is the premium for an insurance of £1486 13 9, at £3 16 8%? *Ans.* £56 19 $9\frac{1}{2}+$.

12. A business man, having \$12000 worth of goods, gets them insured for $\frac{4}{5}$ of their value, at $\frac{3}{4}\%$; if, in a conflagration, he saves but \$2000 of the stock, what real loss will he sustain? *Ans.* \$472.

13. For what sum must a house, valued at \$8274, be insured, at $1\frac{1}{2}\%$, to cover the entire loss, in case it is destroyed by fire? *A.* \$8400.

14. My goods are worth £1563 12. For what sum must I insure them to cover, in case of loss, both premium and property, at £2 5 6%? *Ans.* £1600.

15. The premium of a school-house, insured at $1\frac{1}{4}\%$, is \$50; for what sum was it insured? *Ans.* \$4000.

16. An Insurance Company, after having insured a block of buildings for \$36000, at $2\frac{1}{2}\%$, re-insure the $\frac{1}{2}$ at 3%; what is the balance of premium? *Ans.* \$360.

17. For what sum must goods worth £1938 12 6 be insured to cover both premium and goods in case of loss, the rate being $5\frac{3}{4}\%$?

18. A brig estimated at \$40000 is insured for $\frac{3}{4}$ of its value at $1\frac{1}{2}\%$, and its cargo, worth \$36000, at $\frac{4}{5}\%$; what is the insurance?

19. A merchant paid \$1450 for premium of insurance on a cargo of cotton coming from Havana, the rate of insurance being $2\frac{1}{2}\%$; what was the value of the cargo? *Ans.* \$58000.

20. I paid \$18 for an insurance of \$1200; what is the rate of the premium? *Ans.* $1\frac{1}{2}\%$.

21. To £579 16 10, add $7\frac{1}{2}\%$ commission, and find the insurance of the sum, at $4\frac{3}{8}\%$? *Ans.* £27 5 $4\frac{3}{4}$+.

22. A merchant, having a cargo of 500bbl. flour, has it insured for 80% of its value at $3\frac{1}{4}\%$, and paid \$107.25 for premium; what was the price per bbl.? *Ans.* \$8.25.

23. A ship-owner has two of his vessels insured for \$30000 in the Royal Insurance Co., at $\frac{3}{4}\%$, and for \$45000 in the Colonial Insurance Co., at $\frac{1}{2}\%$; what is the rate of premium for the whole insurance?

24. A house estimated at £300 was insured for $\frac{2}{3}$ of its value, during 3 years, at 1 % per annum. Towards the end of the third year, it was destroyed by fire; what is the actual loss of the proprietor without any allowance of interest? *Ans* £106.

25. My house was insured for \$45000 during 5 years. The first year I paid \$1.50 for the policy and plans, and $\frac{5}{8}\%$ premium; every succeeding year, I paid $\frac{1}{2}\%$ premium. The house having been destroyed the fifth year, what was the loss of the insurance, no interest having been allowed? *Ans.* \$43817.25.

26. I paid \$46.75 for insuring a store for the $\frac{1}{2}$ of its value, at $1\frac{3}{8}\%$; what is the store worth? *Ans* \$6800.

27. I took a policy of £3011 5 for the the value of both property and premium; what is the worth of the insured property, the rate being $\frac{3}{8}\%$? *Ans.* £3000.

28. A shipment of wheat was insured at $2\frac{3}{5}\%$, to cover $\frac{3}{4}$ of its value; the premium paid was \$44.07; the wheat being worth 80 cts. per bushel, how many bushels were shipped? *Ans.* 2825 bu.

ASSESSMENT OF TAXES.

350. A **Tax** is a sum of money assessed on the person or property of an individual, for public purposes.

351. When a tax is assessed on *property*, it is apportioned at a certain *per cent.* on the estimated value. When assessed on the *person*, it is apportioned *equally* among the male citizens liable to assessment, and is called a *poll tax.*

352. Property is of two kinds, viz.: *real estate*, and *personal property.*

353. **Real Estate** is *fixed* or *immovable* property, such as lands, houses, etc.

354. Personal Property is *movable* property, such as money, stocks, furniture, cattle, etc.

355. An **Inventory** is a written list of articles of property, with their value.

356. A **Schedule** is a list of taxable property with its owners' names and its value as estimated by assessors.

357. Assessors are officers appointed to make out a schedule of taxable property, and apportion taxes thereon.

Ex. A tax of $840.75 is to be raised in a town containing 65 polls; the taxable property of the town amounts to $48000, and each poll tax is 75 cts.; what will be the tax on a dollar, and how much will be C's tax, whose property is valued at $5600, and who pays for 2 polls?

OPERATION.

$0.75 × 65 = $48.75, amount assessed on the polls.
$840.75 − $48.75 = $792, amt. to be assessed on the property.
$792 ÷ $48000 = $0.0165, tax on $1.
$5600 × $0.0165 = $92.40, C's tax on property.
$0.75 × 2 = $1.50, C's tax on 2 polls.
$92.40 + $1.50 = $93.90, amount of C's tax. Hence the

358. RULE.—I. *Find the amount of poll tax, if any, and subtract it from the whole tax to be raised; the remainder will be the property tax.*

II. *Divide the property tax by the whole amount of taxable property; the quotient will be the per cent., or the tax on* $1.

III. *Multiply each man's taxable property by the tax on* $1, *and to the product add his poll tax, if any; the result will be the whole amount of his tax.*

EXAMPLES FOR PRACTICE.

1. The tax assessed on a certain town is $1485; its property, both personal and real, is valued at $42000, and it contains 300 polls, which are assessed 75 cts. a piece. What per cent. is the tax; that is, how much is the tax on a dollar; and how much is A's tax who pays for 3 polls, and whose property is valued at $2250?

Ans. 3 cts. on $1; $69.75, A's tax.

2. What is the tax of a non-resident, having property in the same town, worth $7900? *Ans.* $

3. How much will B's tax be, in the same town, who pays for 3 polls, and whose real estate is valued at $32000, and his personal property, at $18880? *Ans.* $1528.65.

4. What sum must be assessed in order to raise a net amount of $11123, and pay the commission for collecting at 2½%?

5. The expense for repairs of a public building was $2521.06, which was defrayed by a tax upon the property of the town. The rate of taxation was 3¼ mills on one dollar, and the collector's commission was 3½%; what was the valuation of the property? *Ans.* $803843.69 +

CUSTOM-HOUSE BUSINESS.

359. Duties, or **Customs,** are taxes levied on imported goods, for the support of government and the protection of home industry.

360. All goods coming into the Dominion of Canada from Foreign countries are required by law to be landed at certain places or ports called *Ports of Entry.* Every Port of Entry has a *Custom-House.*

361. A **Custom-House** is an office established by government for the transaction of business relating to duties. The officers attached to it are called *Custom-House Officers.* Their business is to inspect the cargoes of all vessels entering at any of these ports; to inspect the invoice of goods, collect the duties, etc.

NOTES.—1.—Besides the duties on merchandise, all vessels engaged in commerce are required to pay certain charges for the privilege of entering the port, etc.; these charges are called harbor dues.

2. To carry on foreign commerce secretly, without paying the duties imposed by law, is *smuggling.*

362. Duties are of two kinds —*Ad Valorem* and *Specific.*

363. Ad Valorem Duty is a certain per cent. on the cost of goods, as stated in the invoice.

364. Specific Duty is a tax computed on the weight or measure of the goods, without regard to their cost; hence, allowances are made before computing the duty.

365. An **Invoice** is a statement of goods, from the seller to the buyer, or importer, showing the quantity and prices of the articles.

366. In the United States Custom-Houses, certain legal allowances are made for draft, tare, leakage, etc., before specific duties are imposed. In Canada, however, these are not known, the tare being found by actually weighing one or more of the boxes, etc., containing the goods, and the leakage by gauging the cask.

NOTE.—At present, the various kinds of spirits are the only articles upon which specific duties are charged by the Canadian Tariff.

367.—To compute *ad valorem* duties.

Ex. What is the ad valorem duty, at 18 %, on an invoice of merino which cost $256.50?

OPERATION.

$256.50 × .18 = $46.17, *Ans.*

ANALYSIS. — According to Case I, (282), we multiply the invoice, $256.50, which is the *base* of the duty, by the given *rate,* and obtain the duty, $46.17. Hence the

368. RULE.—*Find the percentage on the invoiced value of the goods, at the given rate of tariff, and the result will be the ad valorem duty.*

369. To compute *specific* duties.

Ex. What is the duty on 4 hogsheads of sugar, each weighing 1280 lb., gross weight, at $2\frac{3}{4}$ cts. a pound; tare 14 %?

OPERATION.

1280 × 4 = 5120lb., gross weight.
5120 × .14 = 716.8lb., tare.
5120 — 716.8 = 4403.2lb., net value.
4403.2 × .02$\frac{3}{4}$ = $121.088, duty.

ANALYSIS.—We first find the whole weight of the invoice which is 5120lb. From this amount we deduct the allowance for tare, 716.8lb., and compute the duty on the remainder. Hence the following

370. RULE.—*Deduct allowances, if necessary, and compute the duty, at the given rate, on the net value.*

EXAMPLES FOR PRACTICE.

1. What is the ad valorem duty, at 19 %, on 15780 lb. of cordage, invoiced at 15 cts. per lb.? *Ans.* $449.73.

2. At 7 cts. a pound, what is the specific duty on 346 kegs of tobacco, each weighing 130 lb., allowing $6\frac{1}{4}$ lb. per keg for tare?

3. At 30 cts. per gallon, what is the specific duty on 40 hhd. of wine, each gauging $58\frac{1}{2}$ gallons?

4. What is the duty at 33 %, on a bale of Holland linens which cost $1593.50? *Ans.* 525.85\frac{1}{2}$.

5. What is the duty, at 20 %, on an invoice of broadcloth which cost in Liverpool £657 1 0, the pound sterling being valued at 4.86\frac{2}{3}$?

6. What is the specific duty, at 10 cts. per lb., on 25 chests of tea, each weighing 120 lb.; tare 10 %?

7. What was the rate % of duty on whose invoice value was $2250, and for which $337.50 duty was paid? *Ans.* 15 %.

8. A merchant imported 64 casks of wine, each containing 42 gal. net, the duty at 30 % amounting to $1036.80; at what price per gal. was the wine invoiced?

9. A merchant in Montreal makes an importation of goods invoiced at $16448. On goods invoiced at $2400, the duties were at the rate of 4 %; on goods invoiced at $3360, the duties were at the rate of 15 %; goods invoiced at $4800, were free of duty; and on the remainder, the duties were at the rate of 30 %; what was the whole amount of the duties? *Ans.* $2366.40.

10. What is the duty at 18 % on 60 kegs of prunes, each weighing 1 cwt., invoiced at $7\frac{3}{4}$ cts. per lb.; tare at $3\frac{1}{2}$ %?

11. A. Hamel & Bro., of Quebec, import from Manchester 15 pieces of Belgian carpeting, 40 yd. each, purchased at 5s. per yd,, duty 24 %; 300 yd. of merino, at 4s. per yd., duty 19 %; 150 yd. Irish linen, at

2s. 6d., duty 15 %; and leather to the cost of £90, duty 4 %. What is the whole amount of duty, allowing the value of the pound sterling to be $4.86\frac{2}{3}$? *Ans.* $261.88+.

12. S. R. Wilson & Co., of Toronto, imported from Amsterdam 48 pieces of linen of 32 yd. each, on which they paid for the duties, at 24 %, $184.32, and other charges to the amount of $61.44. What was the invoice value per yd., and the cost per yd. after duties and charges were paid?

DISCOUNT AND PRESENT WORTH.

371. Discount is an allowance or deduction made for the payment of a debt before it is due.

372. The **Present Worth** of a note or debt, payable at a future time, without interest, is such a sum as, being placed at legal interest, will amount to the given debt when it becomes due.

Ex. What is the present worth and discount of $25.44, at 6 %, payable in 1 year?

OPERATION.

$ 1.06, amount of $1.
25.44 ÷ 1.06 = $24.
25.44, given sum.
24.00, present worth.
$ 1.44, discount.

ANALYSIS.—Since $1 is the present worth of $1.06, it is evident that the present worth of $25.44 will be as many dollars as 1.06 is contained in 25.44, or $24. We find $24 to be the present worth which, subtracted from the given sum, gives $1.44 discount. Hence the following

373. RULE.—I. *Divide the given sum by the amount of* $1 *for the given time and rate, and the quotient will be the* PRESENT WORTH.

II. *Subtract the present worth from the given sum, and the remainder will be the* DISCOUNT.

By proportion.

I. To determine the present worth:—

$100 + (6 \times 1) : 100 :: 25.44 : x = \$24$; whence the following formula:

One hundred plus the rate multiplied by the time, is to one hundred as the given sum is to x, *or the present worth of this sum.*

II. To determine the discount:—

$100 + (6 \times 1) : 6 \times 1 :: 25.44 : x = \$1.44$; whence the following formula:

One hundred plus the rate multiplied by the time, is to the rate multiplied by the time, as the given sum is to X, *or the discount of this sum.*

NOTES.—1. The terms *present worth*, *discount*, and *debt*, are equivalent to *principal*, *interest*, and *amount*. Hence, when the time, rate, % and amount are given, the principal may be found by Case III., (311); and the interest by subtracting the principal from the amount.

2. When payments are to be made at different times without interest, find the present worth of each payment separately. Their sum will be the present worth of the several payments, and this sum subtracted from the sum of the several payments will leave the total discount.

EXAMPLES FOR PRACTICE.

What is the present worth of the following notes:(1)

1. Dated Feb. 3rd., amounting to \$104.60, on 5 months' credit, discounted June 6th., at 5 %? *Ans.* \$104.20 + .
2. Dated March 4th., amounting to £58 10 5, on 7 months' credit, discounted Aug. 10th., at 4 %? *Ans.* £58 3 5 + .
3. Dated April 2nd., amounting to \$206.15, on 4 months' credit, discounted May 30th., at 4½ %? *Ans.* \$204.564 + .
4. Dated May 15th., amounting to £135 9 0, on 8 months' credit, discounted Nov. 15th., at 6 %? *Ans.* £134 2 2 + .
5. Dated Aug. 7th., amounting to \$8000.00, on 6 months' credit, discounted Dec. 5th., at 5 %? *Ans.* \$7931.699 + .
6. Dated Jan. 3rd. amounting to £90 3 6, on 9 months' credit, discounted Sept. 20th., at 7 %? *Ans.* £89 18 11¼.
7. Dated June 14th., amounting to \$1560.90, on 3 months' credit, discounted Aug. 2nd., at 6 %? *Ans.* \$1550.049 + .
8. Dated Sept. 8th., amounting to \$795.10, on 10 months' credit, discounted Feb. 12th., at 5 %? *Ans.* \$779.297 + .
9. Dated Nov. 25th., amounting to £875 6 8, on 7 months' credit, discounted May 11th., at 6 %? *Ans.* £868 19 2½ + .
10. Dated Dec. 6th., amounting to \$630.50, on 11 months' credit, discounted Sept. 18th., at 5 %? *Ans.* \$626.324 + .
11. Dated Oct. 9th., amounting to £95 15 0, on 9 months' credit, discounted June 7th., at 6¼ %? *Ans.* £95 4 5 + .
12. Dated July 16th., amounting to \$208.95, on 5 months' credit, discounted Oct. 12th., at 4¾ %? *Ans.* \$207.20 + .
13. Dated March 2nd., amounting to £140 16 4, on 8 mos.' credit, discounted Sept. 28th., at 6½ %? *Ans.* £139 19 1¾.
14. Dated Jan. 7th., amounting to \$780.50, on 11 months' credit, discounted Nov. 3rd., at 7¼ %? *Ans.* \$775.19 + .
15. Dated April 10th., amounting to £780 5 3, on 10 mos.' credit, discounted Dec. 4th., at 4¾ %? *Ans.* £773 10 6¼.
16. Dated May 17th., amounting to \$436.75, on 3 months' credit, discounted June 22nd., at 5½ %? *Ans.* \$433.110 + .
17. Dated March 14th., amounting to \$600.00, on 7 months' credit, discounted Sept. 7th., at 7 %? *Ans.* \$595.714.

(1) We reckon only 30 days to the month for all the notes in true discount.

18. Dated Feb. 9th., amounting to £850 18 0, on 5 months' credit, discounted April 13th., at 7½ %? *Ans.* £835 18 5½ +.

19. Dated Nov. 11th., amounting to $175.30, on 7 months' credit, discounted May 4th., at 6 %? *Ans.* $174.225 +.

20. Dated March 6th., amounting to £701 9 6, on 4 mos.' credit, discounted June 9th., at 7½ %? *Ans.* £697 11 0¼ +.

21. What is the present worth of $117.60, payable in 1 year, at 12 %? *Ans.* $105.

22. What is the present worth of a debt of £96 6 6¾, due 5mo. 15da. hence, at 6 %? *Ans.* £93 15 0.

23. What should be the discount on $373.75, paid 11mo. before the term of maturity, at 6½ %? *Ans.* $21.01 +.

24. What is the discount on £200 12 6, at 7¼ %, payable in 1yr.?

25. A note of $139.94 is payable in 9 months; what is the present worth, discount being 5 %? *Ans.* $134.881 +.

26. Discounted a note of £75, payable in 4 years, at 5½ %; what sum shall I receive? *Ans.* £61 9 6½.

27. What is the actual discount of a note of $429.98⅜, due in 1yr. 6mo. 1da., at 5½ %? *Ans.* $32.82 +.

28. The sum of $195.10 is payable in 13 months; what will be the discount, at 4 %, by immediate payment? *Ans.* $8.10 +.

29. What is the present worth of £169 13 9, payable in 3yr. and 7mo., at 7⅛ % discount?

30. Bought cloth, on 21 months' credit, for £140 0 7$\frac{7}{25}$; how much ready money will acquit me of the debt, if ⅔ % discount per month, is allowed? *Ans.* £129 3 7¼.

31. I sold a house, which cost me $2964.12 ready money, for $3665.20 payable in 1yr. 6mo.; what will be my gain, in ready money, by discounting at 8 %? *Ans.* $308.38.

32. I bought silks for $43713.60, on 15 months' credit; but, by paying before the time due, I will obtain 5 % discount; at what epoch should I pay the debt, so as to disburse but $41632? *Ans.* In 3mo.

33. A flour-mill was offered for $25000 cash, or for $12000 payable in 6mo., and $15000 payable in 15mo. Accepting the latter condition, I would like to know whether I gained or lost, and how much, money being worth 10 %?

34. Louis bought goods to the amount of £82 0 6$\frac{8}{25}$, on 20 mos.' credit; at what time did he pay, knowing that he obtained ⅔ % discount per month, and that he disbursed but £75 19? *Ans.* 8mo.

35. A merchant gave out two notes: the first, of $243.36, payable May 6th., 1867; the second, of $178.64, payable Sept. 25th. 1867; what sum is required to pay the two notes Oct. 11th., 1866, discount at 7 %?

36. What quantity of produce must be bought at 5s. per lb., on 22 months' credit, in order to pay but £50 19 10⅕, after deducting the discount at 7 %?

37. On 9 months' credit, I bought 120 bales of cotton, each bale weighing 488lb., at 5$\frac{3}{10}$d. the lb. Selling it immediately for £1610 8 cash, I paid my own debt, and received 8 % discount; how much did I gain? *Ans.* £390 8.

38. I paid \$320 for a sum I owed; what was this sum, knowing that $5\frac{1}{4}\%$ discount was allowed? *Ans.* \$336.80.

39. Paid £23 15 for 50yd. of cloth; having received 5 % discount, how much did it cost me per yard? *Ans.* 9s. $11\frac{7}{10}$d.

40. Is it more advantageous to purchase flour at \$6.25 per bbl. on 6 months' credit, or at \$6.50 on 9 months' credit, discount being 8 %? *Ans.* Flour at \$6.25 is the more advantageous.

BANK DISCOUNT.

374. A **Bank** is a corporation, legally established for the purpose of receiving and loaning money, and of furnishing a paper circulation.

375. Bank Notes, or **Bank Bills,** are the notes made and issued by banks to circulate as money. They are payable in specie at the banks.

OBS.—A bank which issues notes to circulate as money, is called a *bank of issue;* one which lends money, a *bank of discount;* and one which takes charge of money belonging to other parties, a *bank of deposit.* Some banks perform two and some all these duties.

376. The **Capital** of a bank is the money paid in by its stockholders, as the basis of business.

377. The affairs of a bank are usually managed by a *board of directors* chosen by the stockholders, and the *principal officers* are a *president*, a *cashier*, and one or more *tellers*.

OBS.—The president and cashier sign the notes issued; the cashier superintends the bank accounts; and the tellers receive and pay out money. A *bank check* is an order, payable to bearer and drawn on the cashier for money.

378. Bank Discount is the simple interest of a note, draft, or bill of exchange, deducted from it in advance, or before it becomes due. Thus, the *bank discount* on a note of \$106, payable in 1 year, at 6 %, is \$6.36; while the *true discount* is but \$6.

The interest is computed not only for the specified time, but for three days additional called *days of grace.*

OBS.—1. The difference between *bank discount* and *true discount* is the same as the difference between interest and true discount.

2. The legal rate of discount is ordinarily the same as the legal rate of interest.

379. The **Proceeds, Avails,** or **Cash Value** of a note is its face or amount minus the discount.

380. CASE I.—*The face of a note being given, to find the discount and proceeds.*

Ex. What is the bank discount, and what are the proceeds of a note of $500, payable in 30 days, at 6 %?

OPERATION.

Sum discounted,	$500.00
Int. for 30 days, or $\frac{1}{12}$ of a year,	2.50
" " 3 " " $\frac{1}{10}$ of a mo.,	.25
Bank discount,	2.75
Proceeds, or present worth,	$497.25

ANALYSIS.—We find the interest on the sum discounted according to 297, and this int. is the bank discount; we then subtract the discount from the sum, and obtain the present worth, $497.25. Hence the

381. RULE.—I. *Compute the interest on the face of the note for three days more than the specified time; the result will be the discount.*

II. *Subtract the discount from the face of the note; the remainder will be the proceeds.*

By proportion.

$100 : 500 :: 6 \times \frac{33}{360} : x$, or the discount;
or, $100 : 500 :: 100 - (6 \times \frac{33}{360}) : x$, or the proceeds.

NOTE.—We take calendar months for the reckoning of time on all the notes in bank discount, and compute interest as if the year contained only 360 days, instead of 365, then the result is too large by $\frac{5}{365}$, or $\frac{1}{73}$ of itself. Hence, if greater accuracy is required, the interest for the days, when obtained by the rule, must be diminished by $\frac{1}{73}$ of itself; or, the method of computing interest, page 183, must be followed.

EXAMPLES FOR PRACTICE.

1. What is the discount and what are the proceeds of a note of $1000, due in 60 days, at 6 %? *Ans.* Dis. $10.50; pro. $989.50.

2. What is the present worth of a note of £2000, payable in 60 days and discounted at the Quebec Bank? *Ans.* £1979.

3. Desiring to loan £250 of a Montreal Bank which discounts at 8 %. I gave my note for £243 15 payable in 60 days; how much must I add to complete the amount I require? *Ans.* £9 13 3.

4. A man sold his farm, containing 195A. 2R. 25p., at $27.50 an acre, and received a note payable in 4 mo. 15 da., at 7 % interest. Being in immediate need of money, he discounts the note at a bank; how much did he receive? *Ans.* $5236.168 + .

5. Find the day of maturity, the time of discount, and present value of the following notes:—

£40 2. Quebec, Dec. 3rd., 1868.

Six months after date, for value received, I promise to pay Daniel Lee & Co., or order, forty pounds and two shillings currency, at the Bank of Quebec. A. T. HERMANN.

Discounted, April 3rd., 1869, at 6 %.
Ans. Due June 3 | 6 1869: term of disc. 64da.; pro., £39 13 $5\frac{1}{4}$ + .

$1066\frac{75}{100}$. Montreal, April 19th., 1869.

Ninety days after date, we promise to pay C. Simson, one thousand sixty-six and $\frac{75}{100}$ dollars, at the Union Bank, for value received.

RAPPE, WEBBER, & Co.

Discounted May 8th., at 7 %.

Ans. Due July 18 | 21; term of disc., 74da.; proceeds, $1051.40 +

6. What is the difference between the true discount and bank discount of $950, for 3mo., at 7 %?

7. What is the difference between the true discount and the bank discount of £2000 0 9, for 6 months, at 3 %?

382. CASE II.—*The proceeds of a note being given, to find the face.*

Ex. What is the amount of a bill, payable in 60 days, which discounted at a bank, at 6 %, gives $989.50 for the proceeds?

OPERATION.

	$1.0000
Int. of $1 for 63 days	.0105
Proceeds of $1	$0.9895

989.50 ÷ 0.9895 = $1000, *Ans.*

ANALYSIS.—Since $0.9895 is the proceeds of $1, the note of which $989.50 is the proceeds, must be as many dollars as $0.9895 is contained in $989.50. Hence the

383. RULE.—*Divide the proceeds of the note, by the proceeds of* $1, *for the time and at the rate mentioned; the quotient will be the face of the note.*

By proportion.

$$100 - (6 \times \tfrac{63}{360}) : 989.50 :: 100 : x = \text{the face.}$$

EXAMPLES FOR PRACTICE.

1. What sum, payable in 90 days, and discounted at 7 %, at a bank, will give £170? *Ans.* £173 2 $7\frac{1}{5}$.

2. A merchant desires to draw $5000 from a bank, and for this purpose discounts his bill, payable in 90 days, at 6 %; what should be the amount of it? *Ans.* $5078.72 +.

3. The proceeds of a note, due in 4 months, and discounted at the bank, at 6 %, are £407 18; what is the face of the note?

4. Bought goods at Toronto for the sum of $1486.90, and gave in payment my note at 4 months, at $7\frac{1}{2}$ % discount; what should be the amount of the note? *Ans.* $1526 +.

5. A merchant wishes to borrow $750 in a bank; what should be the face of his note, payable in 30da., allowing 1 % discount per mo.?

6. I gave my note at 60 days for a debt of £163 16; if discount is $1\frac{1}{2}$ % monthly, what was the face of the note?

384. CASE III.—*The rate of bank discount being given, to find the corresponding rate of interest.*

Ex. What is the rate of interest of a note payable in 90 days and discounted at 6 %?

OPERATION.

$\$0.06 \div 0.9845 = 0.06\frac{186}{1969}$, *Ans.*

ANALYSIS.—Every $1 discounted for the given time and rate yields as its proceeds $0.9845. Then, if $1 in the given time yield a certain interest at 6 per cent., $0.9845 in the same time will yield the same interest, at as many per cent. as the given rate, .06, contains .9845.

385. RULE.—*Divide the given rate per cent., expressed decimally, by the number denoting the proceeds of $1 for the given time and rate. The quotient will be the rate of interest required.*

By proportion.

$$100 - (6 \times \tfrac{93}{360}) : 100 :: 6 : x = 6\tfrac{186}{1969}\ \%.$$

EXAMPLES FOR PRACTICE.

1. What rate of interest is paid when a note payable in 30 days is discounted at 6 %? *Ans.* $6\frac{22}{663}$ %.

2. A note payable in 2 months was discounted at 2 % per month; at what rate was the interest? *Ans.* $25\frac{25}{47}$ % annually.

3. A note, payable in 1 year, was discounted at 6 %, without regard to days of grace; to what rate % of interest does the bank discount correspond? *Ans.* $6\frac{18}{47}$ %.

4. When a note, payable in 90 days, is discounted at $1\frac{1}{2}$ % per mo., at what rate was the interest paid? *Ans.* $18\frac{1874}{1907}$ %.

5. What was the rate per cent. of a note payable in 60 days, and discounted at $\frac{3}{4}$, 1, 2, $2\frac{1}{2}$, 3 % monthly? *Ans.* $9\frac{567}{3937}$ %, $12\frac{252}{787}$ %; etc.

6. What is the rate of interest corresponding to 5, 6, 7, 10, 12 % discount on a bill due in 10 months, without days of grace? *Ans.* $5\frac{5}{23}$ %, $6\frac{6}{19}$ %, etc.

386. CASE IV.—*The rate of interest being given, to find the corresponding rate of bank discount.*

Ex. A man buys notes payable in 90 days, at a discount such that his money brings him 2 % per month; what is the rate of dis.?

OPERATION.

90 days + 3 days = 93 days.

Base,	$100.00
Int. for 93 days,	6.20
Amt. " "	$106.20

$\$6.20 \div 0.278775 = 22\frac{198}{177}$ %, *Ans.*

ANALYSIS.—If we assume $100 for the proceeds of a note, the int. for 93 days, at 24 %, will be $6.20, and the face of the note $106.20. We have then, the face of the note, $106.20, the interest, $6.20, and the time, 93 days, to find the rate per cent., which is done according to the preceding case. Hence the

387. RULE.—I. *Find the interest and the amount of* \$1 *or* \$100 *for the time the note has to run.*

II. *Divide the interest by the interest of the amount at* 1 % *for the same time.*

By proportion.

$$100 + (24 \times \tfrac{93}{360}) : 100 :: 24 : x = 22\tfrac{106}{177}\,\%, \textit{ Ans.}$$

EXAMPLES FOR PRACTICE.

1. At what rate of bank discount must a note, payable in 60 days, be discounted to obtain 6 % interest? *Ans.* $5\frac{1895}{2021}$ %.
2. At what rate must a note, due in 30 days, be discounted to obtain 6 % interest? *Ans.* $5\frac{1945}{2011}$ %.
3. At what rate must a note, payable in 120 days, be discounted to obtain 8 % interest? *Ans.* $7\frac{808669}{1027383}$ %.
4. What rates of bank discount, of notes payable in 30 days, correspond to 5, 6, 7, 10 % interest? *Ans.* $4\frac{23356}{24111}$ %, $5\frac{1945}{2011}$ %, etc.
5. What will be the rate of bank discount, on a note payable 8yr. and 4mo. hence, without grace, corresponding to 5 % interest?
6. At what rates must notes, payable at 60 days, be discounted, to pay a broker 1, $1\frac{1}{2}$, 2, $2\frac{1}{2}$ % per month? *Ans.* $11\frac{669}{1021}$ %, etc.

PROMISCUOUS EXAMPLES IN DISCOUNT.

What was the present worth, at true discount, of the following notes, when discounted:—

1. Dated Feb. 3rd., discounted June 6th., amounting to \$313.80, payable in 5 months, at 5 %? *Ans.* \$312.62 + .
2. Dated March 4th., discounted Aug. 10th., amt'g to £175 11 3, payable in 7 mo., at 4 %? *Ans.* £174 10 3 + .
3. Dated April 2nd., discounted May 30th., amounting to \$618.45, payable in 4 mo., at $4\frac{1}{2}$ %? *Ans.* \$613.55 + .
4. Dated May 15th., discounted Nov. 15th., amt'g to £406 7 0, payable in 8 mo., at 6 %? *Ans.* £402 6 $6\frac{1}{4}$ + .
5. Dated Aug. 7th., discounted Dec. 5th., amounting to \$8000.00, payable in 6 mo., at 5 %? *Ans.* \$7931.69 + .
6. Dated Jan. 3rd., discounted Sept. 20th., amt'g to £270 10 6, payable in 9 mo., at 7 %? *Ans.* £269 16 $10\frac{1}{4}$ + .
7. Dated June 14th., discounted Aug. 2nd., amounting to \$4682.70, payable in 3 mo., at 6 %? *Ans.* \$4650.14 + .
8. Dated Sept. 8th., discounted Feb. 12th,, amounting to \$2385.30, payable in 10 mo., at 5 %? *Ans.* \$2337.89 + .
9. Dated Nov. 25th., discounted May 11th., amt'g to £2626 5 3, payable in 7 mo., at 6 %? *Ans.* £2607 2 $10\frac{1}{4}$ + .
10. Dated Dec. 6th., discounted Sept. 18th., amounting to \$1891.50, payable in 11 mo., at 5 %? *Ans.* \$1878.97 + .

What were the proceeds, at bank discount, of the following notes, when discounted:—

11. Dated Oct. 9th., discounted June 7th., amounting to £287 5 0, payable in 9 mo., at $6\frac{1}{4}$ %? *Ans.* £285 10 1+.

12. Dated July 16th., discounted Oct. 12th., amt'g to $626.85, payable in 5 mo., at $4\frac{3}{4}$ %? *Ans.* $621.225+.

13. Dated March 2nd., discounted Sept. 28th., amt'g to £422 9 0, payable in 8 mo., at $6\frac{1}{2}$ %? *Ans.* £419 11 $0\frac{1}{4}$+.

14. Dated Jan. 7th., discounted Nov. 3rd., amounting to $2341.50, payable in 11 mo., at $7\frac{1}{2}$ %? *Ans.* $2324.052+.

15. Dated April 10th., discounted Dec. 4th., amt'g to £2340 15 6, payable in 10 mo., at $4\frac{3}{4}$ %? *Ans.* £2318 16 11+.

16. Dated May 17th., discounted June 22nd., amt'g to $1310.25, payable in 3 mo., at $5\frac{1}{2}$ %? *Ans.* $1298.439+.

17. Dated March 14th., discounted Sept. 7th., amounting to $1800, payable in 7 mo., at 7 %? *Ans.* $1786.

18. Dated Feb. 9th., discounted April 13th., amt'g to £2552 14 0, payable in 5 mo., at $7\frac{1}{2}$ %? *Ans.* £2504 16 $8\frac{3}{4}$+.

19. Dated Nov. 11th., discounted May 4th., amounting to $525.90, payable in 7 mo., at 6 %? *Ans.* $522.306+.

20. Dated March 6th., discounted June 9th., amt'g to £2104 8 6, payable in 4 mo., at $7\frac{1}{2}$ %? *Ans.* £2091 5 $5\frac{1}{4}$+.

21. On March 12th., discounted at a bank, at 6 %, a note of $705.60, payable June 28th.; what sum did I receive? *Ans.* $692.546+.

22. A bill on 4 months' credit having been discounted at 5 %, bank discount, was reduced to £37 5 $4\frac{1}{5}$; what was the amt. of the bill?

23. The contract for a public school was given to a builder on the deduction of 12 % of his tender. The building being finished, he was ordered to do extra work for $1529. Required the amount of the extras, so that the contractor may receive the $1529, after deducting the 12 %? *Ans.* $1737.50.

24. The proceeds of a note, payable Aug. 2nd., and discounted May 9th., at the bank, are £39 0 $9\frac{9}{25}$; what is the face of the note, discount being 6 % yearly? *Ans.* £39 12 $4\frac{3}{4}$+.

25. I owe the sum of $514.22 as follows: $208.32 payable in 10 mo., $123.20 in 18 mo., and the remainder in 22 mo.; if I can obtain true discount at 4 %, how much must I pay? *Ans.* $488.043+.

26. A bill amounts to £300 7, and the discount allowed is $2\frac{1}{2}$ %: to what sum is the amount of the bill reduced? *Ans.* £292 16 $9\frac{3}{4}$.

27. What is the present worth of $769.60, due 3 years and 5 months hence, at 6 %? *Ans.* $638.67+.

28. Paul invested in business the sum of £1441 10 payable in 3 years, and is at liberty to advance the payment at the rate of $\frac{3}{4}$ %, bank discount, per month, without days of grace. At the end of 15 months he gave £716 2 6; in what time did he balance the remainder, knowing that he disbursed but £532 7? *Ans.* After 22 mo. 20 da.

29. The sum of $1720 is payable in 1 year, and $10900, in 18 months; but by paying immediately, 5 % true discount, on the first sum, and $5\frac{1}{2}$ % on the second, can be obtained; what is the diminution?

30. For what sum must a note, to run 4mo. 15da., at 6 %, be given that the bank proceeds may be \$1954? *Ans.* \$2000.

31. A person owes £2250 0 4½, payable in 6 months; if he pays ready money at 2 % discount for the 6 months, how much will he pay? *Ans.* £2205 0 4½.

32. Had I bought goods for £875, I would have obtained £120 discount; but as I bought them for £620, the discount amounted to only £98; did I obtain more diminution in proportion to my purchases, and at what % does the surplus amount to? *Ans.* $2\frac{20}{217}$ %.

33. A merchant bought \$4612.80 worth of oil, on 3 years' credit, and has the liberty of advancing the payment, at a discount of ¾ %. After 15 months he gave \$2291.60; at what time did he settle the remainder, knowing that he disbursed but \$1703.52?

Ans. 22mo. 20da. after the purchase.

34. What sum discounted for 7mo. 9da., at 6¼ % per annum, can produce a discount with which may be purchased the makings of 8 covered benches, using 1¾yd. for each, at \$1.80 per yd.? *A.* \$662.79+

35. Having bought two clocks for \$505, on 16mo.'s credit, and having paid them before the term of maturity, I obtained \$18.05 discount, at 5 % per annum; at what epoch did I acquit the debt? *A.* 7mo. 3da. aft.

36. In a new building, two iron floors were laid, each floor being 15.36yd. long and 8.25yd. wide. The weight of the iron is 70lb. per yard of superficie, and after being laid costs \$5 per 100lb. I ask, 1st. the total price of the two floors; 2nd. the discount that can be obtained by paying 68 days before the time, at ½ % discount per month.

STOCKS.

388. Stocks is a general name given to government bonds, and to money capital invested in corporations.

389. A **Corporation** is a body formed and authorized by law to act as a single person.

390. The legal act of incorporation which defines the rights and powers of the corporation is called a **Charter**.

391. The **Capital Stock** of a corporation is the money contributed and employed to carry on the business of the company.

NOTES.—1. When the capital stock has been all paid in, money may be raised, if necessary, by *loans*, secured by mortgage upon the property. The bonds issued for these loans entitle the holders to a fixed rate of interest. Thus, bonds drawing 6 % annually are called 6 per cent. stock, or 6's; &c.

2. To the bonds are attached what are called *coupons*, each of which is a due bill for the interest on the bond to which it is attached, representing the amount of the periodical dividend or interest, and the time of payment, which coupons are severally *cut off* and presented for payment as they become due.

3. *Consols* is a term abbreviated from the expression "consolidated annuities." The British government having at various times borrowed money at different

rates of interest, and payable at different times, consolidated the stock or bonds thus issued, by issuing new stock drawing interest at 3% per annum, payable semi-annually, and redeemable only at the option of the government, becoming practically *perpetual annuities*. With the proceeds of this the old stock was redeemed. The quotations of these 3 % perpetual annuities or *consols*, indicate pretty accurately the state of the money market, as they form a staple credit and become a standard for reference.

392. **Stockholders** are the owners of stock, either by original title or by subsequent purchase.

393. A **Share** is one of the equal parts into which capital stock is divided. The value of a share in the original contribution of capital varies in different companies; in bank, insurance, and railroad companies of recent organization, it is usually $100.

394. Stocks are **At Par** when they sell for their original value.

395. **Above Par**, at a premium or advance, when they sell for more than their original value.

396. **Below Par**, or at a discount, when they sell for less than their original value.

397. An **Installment** is a portion of the capital stock required of the stockholders, as a payment on their subscription.

398. An **Assessment** is a sum required of stockholders, to meet the losses or the business expenses of the company.

399. A **Dividend** is a sum paid to the stockholders from the profits of the business.

400. A person who buys and sells stocks, either for himself, or as the agent of another, is called a **Stock Broker** or **Stock-jobber**.

EXAMPLES FOR PRACTICE.

Ex. 1. What is the cost of 27 shares of Grand Trunk Railroad stock at $4\frac{1}{2}$ % premium?

NOTE.—In all these examples, $100 will be considered as the value of a share, unless otherwise mentioned.

OPERATION.

$2700 × .045 = $121.50, premium.
$2700 + $121.50 = $2821.50, *Ans.*

Or, $2700 × $1.045 = $2821.50, *Ans.*

ANALYSIS. — We calculate firstly the premium on the par value, which we find to be $121.50; we add this to $2700, and obtain $2821.50 which is the cost. Or, since $1 of the stock costs $1 plus the premium, or $1.045, $2700 will cost $2700 × $1·045 = $2821.50, *Ans.*

By proportion. $100 : 100 + 4.5 :: 27 × 100 : x$.

Ex. 2. A broker sold for me 64 shares of the Ocean Steamers Co. stock, at 15% discount, for which he charged $\frac{1}{4}$ % brokerage; how much did I receive?

OPERATION.

$\$0.15 + .0025 = 0.1525$.
$\$1.00 - \$0.1525 = \$0.8475$ proceeds of $1 of stock.
$6400 \times \$0.8475 = \$5424$, *Ans.*

ANALYSIS.—Adding the rate of brokerage to the rate of discount, we have .1525; hence $1 will bring $1 — $0.1525 = $0.8475, and 64 shares or $6400 will bring 6400 × .8475 = $5424.

By proportion. $100 : 100 - (15 + 0.25) :: 64 \times 100 : x$.

Ex. 3. I put $17700 into the hands of a broker to be invested in Ontario Province Bonds when their market value is 12% below par; how many shares will I receive if the broker charges $\frac{1}{2}$% for his services?

OPERATION.

$\$1.00 - \$0.12 = \$0.88$, market value of $1.
$\$0.88 + \$0.00\frac{1}{2} = \$0.885$, cost of $1.
$\$17700 \div \$0.885 = \$20000 = 200$ shares, *Ans.*

ANALYSIS.—Since the stock is 12% below par, the market value of $1 will be $0.88; adding the rate of brokerage, we find that every dollar of the stock will cost $0.885. Hence, for $17700, the broker can purchase $17700 ÷ .885 = $20000 or 200 shares.

By proportion. $100 - (12 + .5) : 100 :: 17700 : x \div 100$.

Ex. 4. The Richelieu Company declares a dividend of $15\frac{1}{2}$%; what will I receive for 24 shares?

OPERATION.

$\$2400 \times .15\frac{1}{2} = \$372$.

ANALYSIS.—According to 282, we multiply the base, $2400, by the rate, $.15\frac{1}{2}$, and obtain the dividend, $372.

By proportion. $100 : 15\frac{1}{2} :: 24 \times 100 : x$.

Ex. 5. What income can we obtain by investing $10260 in Quebec Province 6% bonds, purchased at 95%?

OPERATION.

$\$10260 \div .95 = \$10800$, stock purchased.
$\$10800 \times .06 = \$648$, annual income.

ANALYSIS.—We divide the investment, $10260, by the cost of $1, and obtain $10800, the stock which the investment will purchase, (288). And since the stock bears 6% interest, we have $10800 × .06 = $648, the annual income.

By proportion. $95 : 100 :: 10260 : x \times .06$.

Ex. 6. A person desires to secure \$450 annual revenue; what capital must he invest in 5 % bonds, when stock is purchased at 80 %?

OPERATION.

\$450 ÷ .05 = \$9000, stock required.
\$9000 × .80 = \$7200, cost, or investment.

ANALYSIS. — Since \$1 of the stock will secure \$0.05 income, to obtain \$450 will require \$450 ÷ .05 = \$9000, (Ex. 5). Multiplying the par value of the stock by the market price of \$1, we have \$9000 × .80 = \$7200, the cost of the required stock, or the sum to be invested.

By proportion. 5 : 100 :: 450 : x × .80.

Ex. 7. What per cent. of my investment shall I secure, by purchasing Montreal 7 per cents., at 105 %?

OPERATION.

.07 ÷ 1.05 = $6\frac{2}{3}$ %.

ANALYSIS.—Since \$1 of stock will *cost* \$1.05, and *pay* .07, the income is $\frac{7}{105}$ = $6\frac{2}{3}$ % of the investment.

By proportion. 105 : 100 :: 7 : x.

Ex. 8. A man invested in a Steamboat Company, and received a dividend of 9 %, which was $8\frac{1}{3}$ % on his investment; at what price did he purchase?

OPERATION.

\$0.09 ÷ \$0.08$\frac{1}{3}$ = \$108, *Ans.*

ANALYSIS.—Since \$0.09, the income of \$1 of the stock, is $8\frac{1}{3}$ % of the sum paid for it, we have, \$0.09 ÷ \$0.08$\frac{1}{3}$ = \$108, the purchase price.

By proportion. $8\frac{1}{3}$: 100 :: 9 : x.

9. A person buys 25 shares of the Marine Bank, of \$100 each, at 12 % discount; how much must he pay? *Ans.* \$2200.

10. What will I receive for 20 shares of the Central Railroad stock, at 135 %, brokerage being $1\frac{3}{4}$ %? *Ans.* \$2665.

11. At $7\frac{1}{2}$ % premium, and $\frac{1}{4}$ % brokerage, what will be the cost of 36 shares of the Bank of Commerce? *Ans.* \$3879.

12. A canal cost £400000; all expenses defrayed it brings in £15000 annually. Suppose it to have been constructed by means of shares of £50 each, and that an individual took 25 shares, what dividend will he receive annually? *Ans.* £46 17 6.

13. If 300 shares of the Ottawa Bank sell for \$30112.50, what is the premium, each share being \$100? *Ans.* $\frac{3}{8}$ % premium.

14. When the nominal value of stock is £12 10. and the discount $3\frac{1}{2}$ %, how much must I pay for 30 shares? *Ans.* £361 17 6.

15. The steamboat company of the Saguenay declares a dividend of 15 %; what shall I receive for 65 shares the nominal value of which is \$100 per share? *Ans.* \$975.

16. Bought stock at par, and sold it at 3 % premium, gaining £187 10 0; how many shares did I purchase? *Ans.* $62\frac{1}{2}$ shares.

17. An individual bought, at the rate of \$168.75, a number of shares in the Pictou coal-mine company, the annual income of which is \$10 per share. With the income he purchases \$260 worth of goods; what was his investment, the brokerage being $\frac{1}{4}\%$? *Ans.* \$4398.46$\frac{7}{8}$.

18. A merchant retires from business with a sum of \$34520.50, and buys with this capital government 6's, at the rate of \$70.45; what will be his annual income? *Ans.* \$2940.

19. Ontario 4½'s are sold at the rate of £94 17; what income will I obtain for £3794? *Ans.* £180.

20. Sold \$16400 worth of North Bank Stock at 13% premium; what shall I receive? *Ans.* \$18532.

21. A person, having £2250, invests this sum in Ocean Telegraph Company Stock which sells at 17% discount; what amount of capital does he purchase? *Ans.* £2710 16 $10\frac{1}{4}+$.

22. Bought 36 shares of the Western Copper Mine Company, the par value of each being \$500, at 2% premium, and sold it at 28% discount; what is my loss? *Ans.* \$5400.

23. I have an investment of \$15000 in a transatlantic steamship company; how many shares shall I own after a dividend of 8% is declared and payable in capital stock? *Ans.* 162 shares of \$100 each.

24. What should be the rent of a farm, which cost \$16992.10, in order that the purchase capital may produce the same revenue as would be produced by the same sum, employed in the purchase of $6\frac{1}{2}\%$ bonds, at $91\frac{3}{4}\%$? *Ans.* \$1203.80.

25. A farmer invests £36, the price of three oxen, in the purchase of 5% bonds sold at the rate of £78 10; at what real rate was his money placed? *Ans.* $6\frac{58}{157}\%$.

26. An exchange agent having \$45000 invested in bonds of the Canadian Transatlantic Steamship Company, exchanged them at 88%, for capital stock in the same company valued at $62\frac{1}{2}\%$. The bonds brought 7% annually, while the shareholders received two dividends during the year, the first of 3%, and the second of $3\frac{1}{2}\%$; how much did the agent gain annually by the exchange? *Ans.* \$968.40.

27. An agent receives \$25000, with instructions to deduct his brokerage at $1\frac{3}{4}\%$, and then purchase bank stock for the balance; if the stock is selling at 3% discount, what will be the amount of his capital stock? *Ans.* \$25329.92+

28. An individual desires to invest \$11158 in 5% bonds. The market value being but \$67.35, he waits a few days, when it rises to \$69.10. Find, now, what income did he lose, and what income he would have gained had the market value lowered to \$66.25, brokerage being $\frac{1}{8}\%$? *Ans.* Lost \$20.90+ income, would have gained \$13.73+

29. I have \$60500 to invest in bonds. I can purchase $4\frac{1}{2}\%$ bonds at the rate of \$95.30, and 3% bonds at the rate of \$69.25; which would be the more profitable of the two? *Ans.* The $4\frac{1}{2}\%$ bonds.

30. How much more advantageous is it to invest \$1128 in $4\frac{1}{2}\%$ bonds, at $91\frac{3}{4}\%$, than \$1128 in 3% bonds, at $69\frac{9}{10}\%$, brokerage being at $\frac{1}{8}\%$? *Ans.* \$6.923+.

31. A banker owns 150 shares in the Quebec Insurance Company.

I order my agent to buy them when they will rate at $5\frac{1}{2}\%$ premium: how much will the 150 shares cost me, knowing that the agent will charge me $\frac{7}{8}\%$ brokerage? *Ans.* $15956.25.

32. A farmer sold corn for the amount of $4134.40. With this sum he buys three $4\frac{1}{2}\%$ bonds which produce an annual income of $18, at $90\frac{3}{4}\%$, and one 3% bonds, producing annually $20, at $64\frac{7}{20}\%$. With the remainder diminished by $1.95, he buys 3% bonds at $68\frac{1}{4}\%$; at what average rate should he purchase $4\frac{1}{2}\%$ bonds, to have, for the price of the corn sold, the same quantity of revenue? *Ans.* $98.43 +

33. In buying stock in the Labrador Company for the value of $10425, (at 500 per share, and producing $36 for interest and dividend,) a farmer secured a revenue of $540. Required the market value of the stock per share, and at what rate he let out his money?

Ans. 1st. $695; 2nd. $5.\frac{25}{139}\%$.

34. In January 1848, the total amount of British consols was £378019855; what was the amount of interest paid on them semi-annually? *Ans.* £5670297$\frac{3}{10}$.

35. A person desires to sell $3500 of Montreal 7's; the market value being at $95\frac{1}{3}\%$, he waits a few days longer when stock rises to $95\frac{17}{20}\%$; what profits did he realize? What loss would he have sustained had the market value lowered to $94\frac{7}{10}\%$, brokerage, in both cases, being $\frac{1}{8}\%$? *Ans.* $22.75 gain, and $17.50 loss.

36. A mason built 965 sq. yd. of a wall at $21.80 per sq. yd. He desires to invest this sum in insurance company stock. In the Phœnix Insurance Co., the shares are $5000 each; they produce $200 as interest and dividend, and are negotiated at 40% premium. In the Providence Co. the shares are $2500 each; they produce $50 as interest and dividend, and are negotiated at 45% premium. Which are the most advantageous, and by how much %? How many shares can he purchase in taking the most advantageous, and what revenue could he secure? *Ans.* The first are the more advantageous by 1.478%; 3 shares; and $600 of revenue.

PARTNERSHIP.

401. A **Partnership** is an association of two, or more persons in business, each of whom is called a *Partner*. Such an association is called a *Company*, *Firm*, or *House*.

NOTE.—The terms *Capital* or *Stock*, *Dividend*, and *Assessment*, have the same signification in Partnership as in Stocks.

402. CASE I.—*To find each partner's share of the profit or loss, when there is no regard to time.*

Ex. Three merchants, A, B, and C, associate together in business; A puts in $275, B $475, and C $500. They gained $150; what part of the profit must be given to each?

OPERATION.

A's	stock, $275	$275 × 0.12 = $ 33, A's profit.
B's	" 475	475 × 0.12 = 57, B's profit.
C's	" 500	500 × 0.12 = 60, C's profit.
Whole	" $1250	Proof $150, whole profit.

$150.00 ÷ 1250 = $0.12, profit on $1.

ANALYSIS.—Since the whole stock is $1250, and the whole profit, $150, the profit on every $1 of stock will be as many dollars as 150 contains times 1250, or $0.12 on every $1 of stock. Then, each merchant's stock multiplied by .12 gives his part of the whole profit. The same result also may be obtained, as follows:—

By proportion.

$$\left.\begin{matrix}275\\+475\\+500\end{matrix}\right\} = 1250 : 150 :: \left\{\begin{matrix}275\\475\\500\end{matrix}\right\} : x = Ans. \left\{\begin{matrix}\$33, \text{ A's profit.}\\57, \text{ B's profit.}\\60, \text{ C's profit.}\end{matrix}\right.$$

Proof, $150, whole profit.

403. RULE.—*The whole profit or loss, divided by the number denoting the whole stock, will give the profit or loss on each dollar of stock; and each partner's stock, multiplied by the number denoting the profit on* **$1**, *will give his share of the whole profit or loss.*

Or,

As the whole stock is to each partner's stock, so is the whole profit or loss to each partner's profit or loss.

EXAMPLES FOR PRACTICE.

1. With £200, two men gained £50; the first man contributed £125, the second, £75; what part of the gain is each entitled to?

Ans. The first, £31 5; the second, £18 15.

2. Four merchants associated and raised a capital of $45000, to which each man contributed equally. At the expiration of the partnership, the capital was found to be augmented by $26877. What shall be the part of each man, knowing that the 1st. ought to have 13 parts; the 2nd., 11; the 3rd., 8; and the 4th., 7?

Ans. 1st., $23959; 2nd., $20273; 3rd., $14744; 4th., $12901.

3. Three men associating together, gained £287 10; the 1st., put in 400 yd. of velvet at £1 per yard; the 2nd., 350 yd. of cloth at £2; the 3rd., 450 yd. of cassimere at 15s.; what part of the gain should each have? *Ans.* £80, £140, and £67 10.

4. Four persons having joined in partnership agree that the 1st. put in £1250; the 2nd., $\frac{1}{4}$ more than the first; the 3rd., as much as the two others together; and the 4th., his industry during the year, which was estimated at £2000; what share of the profits, £1525, shall each receive? *Ans.* £250, £312$\frac{1}{2}$, £562$\frac{1}{2}$, and £400.

5. Four associates made a profit of $1500. The first is to have 3 parts; the 2nd., 4; the 3rd., 5; and the 4th., 6. How much will each receive? *Ans.* $250, $333⅓, $416⅔, and $500.

6. The first of five men, associated in partnership, put in $800; the 2nd., $100 more than the first; the 3rd., 100 more than the second; and so on, with the others, always augmenting by $100. If the gain is $1800, what ought to be the part of each?

Ans. $288, $324, $360, $396, $432.

7. Three speculators have together a capital of $4928, which brings them a profit of $616; the 1st. received $150 for his share of the gain; the 2nd., $206; and the 3rd., $260. What was each one's investment? *Ans.* $1200, $1648, $2080.

8. Two speculators shipped 6000 tons of corn to Cuba. During the voyage 650 tons were thrown overboard on account of a storm which arose. If 250 tons were spoiled, how much did each man lose, knowing that 3500 tons belonged to the first? *Ans.* 525 and 375 tons.

9. Three farmers bought 148 sheep at $4.12½ per head, for the payment of which the 1st. furnished $218.85, the 2nd., $236.32½, and the 3rd. the remainder. They sold the sheep, after having nurtured them during 6 months, at a profit of $1.60 per head; how much did each receive of the profits? *Ans.* $84.88¾, $91.66½, $60.24¾.

10. Three lumber merchants bought 76500 saplings, on which they realized a profit of £296 8 9. The first man contributed £460 15 7½; the second, £527 6 10½; the third man's part is not known, but he received, however, £98 16 3 as his share of the profits. Tell us the contribution of the third merchant, the profits of the two others, and the price of the saplings per hundred? *Ans.* Third merchant's share £494 1 3. The profits of 1st., £92 3 1½; 2nd., £105 9 4½; £1 18 9 per hundred.

11. Two dealers in furs made a joint purchase of 268 assorted fox and beaver skins, at £112 10 per hundred; the first dealer advanced £48 10 more than the second, and, together they realize a profit of 18 % on the buying price. Required what is due to each, and at what price they sold the skins a piece? *Ans.* £149 5 4½ due to the 2nd.: £206 10, to the first. The skins cost £1 6 6⅗ a piece.

12. Three students in Astronomy join in raising $698.50 for the purchase of a telescope. The second furnished ⅔ of what the first gave, and the third furnished ¾ of what the two others had advanced; what was the contribution of each? *Ans.* $277.81¼, $166.68¾, $254.

13. Four farmers associated in furnishing a quantity of straw which they sold at $7 per hundred bundles; what did each receive, knowing that the 1st. furnished $\frac{3}{11}$ of it; that the 2nd. furnished a quantity not mentioned, and that the 3rd. furnished 600 bundles, which quantity equalled the delivery of the 1st. and 4th., who furnished 240 bundles? *Ans* $25.20, $8.40, $42.00, $16.80.

14. Two clockmakers joined in the purchase of 120 clock works at the average price of $7.37½; in the speculation, they lost $135. The loss of the 1st. surpassed that of the 2nd. by $33.50; what were the loss and investment of each? *Ans.* 1st. Inv. $552.30⅝, loss $84.25. 2nd. Inv. $332.69¼, loss $50.75.

15. Several persons agreed to conduct, during one year, a paper manufactory. The first put in $\frac{2}{3}$ of the stock; the second, \$4000 less than the first; the third, \$4000 less than the second, and so on until the last. If the investments had been in sums equal to the highest, the capital stock would be augmented by $\frac{1}{3}$. The merchandise sold produced a sum equal to the $\frac{5}{6}$ of what was put in, which was employed in buying rags. In admitting that the $\frac{13}{16}$ of the sum proceeding from sales serve to cover the expenses of fabrication and investment, it is required to ascertain how many persons there were, how much each one put in, and what part of the gain each is entitled to?

404. CASE II.—*To find each partner's share of the profit or loss, when the stock is employed for different periods of time.*

Ex. A and B entered into partnership; A furnished \$240 for 8 months, and B \$560 for 5 months. They lost \$118; what was each man's share of the loss?

OPERATION.

\$240 × 8 = \$1920. 560 × 5 = 2800. \$4720.

\$1920 × 0.025 = \$48, A's loss. 2800 × 0.025 = 70, B's loss. Proof, \$118, entire loss.

\$118.00 ÷ 4720 = \$0.025, loss on \$1.

ANALYSIS.—It is evident that \$240 for 8 mo. is the same as \$240 × 8 = \$1920 for 1 mo., since \$1920 would lose as much in 1 mo. as \$240 in 8 mo.; and \$560 for 5 mo. is the same as \$560 × 5 = \$2800 for 1 month. The question then is the same as if A had furnished \$1920, and B \$2800, for equal times. Then, if \$1620 + \$2800 = \$4720 lose \$118, \$1 will lose $\frac{1}{4720}$ of \$118 = \$0.025, and \$1920 × .025 = \$48, A's loss; \$2800 × .025 = \$70, B's loss. The same results may be obtained as follows :—

By proportion.

$$\left.\begin{matrix}\$240\times 8=1920\\ 560\times 5=2800\end{matrix}\right\} = 4720 : \left\{\begin{matrix}1920\\ 2800\end{matrix}\right\} :: 118 : x = Ans. \left\{\begin{matrix}\$48, \text{ A's loss.}\\ 70, \text{ B's loss.}\end{matrix}\right.$$

Proof, \$118.

405. RULE.—*Multiply each partner's stock by the time it was in trade, and divide the whole profit or loss by the sum of the several products; by the quotient, multiply the product of each partner's stock and time, and the result will be his share of the profit or loss.*

Or,

Multiply each partner's stock by the time it was in trade; then, as the sum of these products is to each product, so is the whole profit or loss to each partner's profit or loss.

EXAMPLES FOR PRACTICE.

1. Two persons contribute unequal sums towards a capital: the first puts in $2300 for 2 years; the second, $1500 for 18 months. What part of the gain, $1400, should each person receive?

Ans. $940.15, $459.85.

2. Three individuals raised a capital sum with which they gained £1137 10: the first contributed £200 for $2\frac{1}{2}$ years; the second, £125 for 25 months; and the third, £248 15 for 35 months. What part of the gain should each have?

Ans. 1st. £382 15 $1\frac{1}{5}$; 2nd. £199 7 $0\frac{3}{5}$; 3rd. £555 7 $10\frac{1}{4}$.

3. A porter associated with a pedler and raised a capital of $16000. After two years they divided the gain, and the pedler, who had contributed $9000, received $1800; what did his companion receive, knowing that the latter left his share in the business but during 20 months? *Ans.* 1166.66\frac{2}{3}$.

4. Four persons agree to form a partnership for 3 years. The first puts in at the beginning $350, and 5 months after $2400 more; the second puts in $8000 at first, and at the end of 20 months withdraws the half of his share, and 5 months after withdraws $2400 more; the third puts in $1500 in the beginning, and $5000 at the end of 2 years; the fourth puts in at first $600, and every six months augments his portion by a like amount; the gain being $80000, what part did each receive?

Ans. $14677.35+, $33336.15−, $19232.39+, $12754.11+.

5. Three merchants joined in business. The first put in £1001 12 for 10 months; the second, £1751 12 6 for $15\frac{1}{2}$ months; and the third, £2000 3 9 for 17 mo. and 20 days. Required each merchant's share of the profits which amount to £350 3?

Ans. £48 7 $5\frac{2}{5}$−; £131 2 $4\frac{4}{5}$+; £170 13 $1\frac{4}{5}$−.

6. Two clothiers associate together; one of them contributed a sum with which could be bought 90 yd. of Broadcloth at $6 per yard, the other put in a sum with which 60 yd. could be purchased at the same rate. In supposing the 1st. to have had $6 of the profits more than the 2nd., to how much did the profits amount? *Ans.* $30.

7. Four farmers rent a pasture for $975. The first put 5 beeves on it during 54 days; the second, 7 cows during 63 days; the third, 8 heifers during 75 days; and the fourth, 6 horses during 50 days. It was calculated that 1 beef consumed $1\frac{1}{2}$ times as much as a cow, or twice as much as a heifer, or $1\frac{1}{4}$ times as much as a horse; how much must each farmer pay?

Ans. $238.45+; $259.65−; $264.94+; $211.96−.

8. In the working of a mine during 6 years, three partners gain £21750. The first partner had put in £13437 10 in the beginning, but after $2\frac{1}{2}$ years, he withdrew £3275. The second put in his share, which was £41000, only $1\frac{1}{3}$ years after the commencement of the work. Finally, the third made his contribution of £53750, but 3 years after the installment of the first. What part of the profits should each receive? *Ans.* £3566 16 $0\frac{3}{4}$+; £9867 6 $4\frac{1}{4}$−; £8315 17 7+.

EXCHANGE.

406. Exchange is the process of remitting money from one place to another by **Drafts** and Bills of Exchange.

NOTE.—For a full treatment of this and of the following subjects, *see* the Commercial Arithmetic.

Form of a Draft.

$400. [STAMP.] *Quebec, P. Q., March 1, 1871.*

Thirty days after sight, pay to Henry Simms, or order, Four Hundred Dollars, and charge the same to my account.

To James Benton. *Louis O'Neil.*

No. 12, Richard Street, Toronto, P. O.

407. The **Drawer,** or **Maker,** is the person who signs the draft.

408. The **Drawee** is the person on whom the draft is made.

409. The **Payee** is the person to whom the draft is made payable.

410. An **Acceptance** is the promise of the Drawee, to pay the draft at maturity, and is usually acknowledged by writing the word "Accepted" with his signature, across the face of the draft.

411. An **Indorsement** of a draft, by the payee, is made in the same manner as the indorsement of a note.

412. A **Sight Draft** is an order to pay at sight.

413. A **Time Draft** is an order requiring payment at a specified time.

414. A Draft or Bill of Exchange is at a *Premium*, when the price paid is greater than its face; and at a *Discount*, when the price paid is less than its face.

415. Domestic, or **Inland Exchange,** is when both the drawer and drawee reside in the same country.

416. CASE I.—*Given the face of a draft, the rate per cent. of exchange, and the time, to find its cost.*

Ex. 1. What must I pay in Ottawa for a draft of $640 on Quebec, exchange being $1\frac{1}{2}\%$ premium?

OPERATION.

$640 × 1.015 = $649.60, *Ans.*

ANALYSIS.—The cost of exchange of $1 is $1 + $0.015 = $1.015, and of $640, 640 × $1.015 = $649.60.

Ex. 2. What must be paid in Montreal for a draft of $3500 on Halifax, at 33 days, exchange $2\frac{1}{2}\%$ premium.

OPERATION.

$1.000
.006 = disct. for 36da. at 6%.
$ .994 = cost at par of $1.
.022 = rate of exchange.
$1.016 = cost of $1 of the draft.
$3500 × 1.016 = $3556, *Ans.*

ANALYSIS.—The discount of $1 at 6% for 36 days is $0.006, which being subtracted from $1 leaves $0.994, the cost of $1 of the draft, if the exchange was at par. To this add the premium of $1, $0.022, and we have $1.016, the cost of $1 of the draft. Hence the cost of $3500, the draft, is $3500 × 1.016 = $3556.

417. RULE.—I. For sight drafts.—*Multiply the face of the draft by* 1 *plus the rate when exchange is at a premium, and by* 1 *minus the rate when exchange is at a discount.*

II. For drafts payable after sight.—*Find the cost of* $1 *at bank discount for the specified time, at the legal rate where the draft is purchased; then add the rate of exchange when at a premium, or subtract it when at a discount, and multiply the face of the draft by this result.*

EXAMPLES FOR PRACTICE.

1. A merchant in Toronto wishes to pay in Montreal $7930, and exchange is $\frac{3}{8}\%$ premium; what will be the cost of the draft?
Ans. $7989.47½.

2. A merchant in St. John, N. B., wishes to pay in Ottawa, $9e0, and exchange is $1\frac{3}{4}\%$ discount; required the cost of the draft?
Ans. $962.85.

3. What will be the cost, in Kingston, of a draft on Halifax for $800, payable 60 days after sight, exchange being at a premium of 2%?

4. A merchant in Kingston purchased a draft on Fredericton for $840, payable 30 days after sight, at 6%; what did it cost him, the rate of exchange being $1\frac{1}{2}\%$ discount? *Ans.* $822.78.

5. What will be the cost of a draft of $4250, for 60 days, at 6%, exchange being $1\frac{7}{8}\%$ premium? *Ans.* $4285.06¼.

6. A merchant in Quebec receives from his agent 1200 bushels red wheat, purchased in Toronto at 65 cts. per bushel; in payment for which he remits a draft on Toronto, at $\frac{5}{6}$ % discount. The transportation of his wheat cost $98. What must he sell it for per bushel to gain $225? *Ans.* $0.91\frac{3}{8}$.

418. CASE II.—*Given the cost of a draft, the rate per cent. of exchange, and the time, to find its face.*

Ex. A merchant in Three Rivers paid $6856.10 for a 60 days' draft on Toronto, exchange being $1\frac{7}{8}$ % premium, and interest 6 %; required the face of the draft.

OPERATION.

$1.0000
.0105 = the discount for 63 days.
$ .9895 = the cost of $1 at par.
.01875 = the rate of exchange.
$1.00825 = the cost of $1 of the draft.
$6856.10 ÷ $1.00825 = $6800, *Ans.*

ANALYSIS.—By 416, Case I., *Ex.* 2, we find the cost of $1 of the draft to be $1.00825. Hence, $6856.10 ÷ $1.00825 = $6800, is the face of the draft.

419. RULE.—*Divide the given cost by the cost of a draft for* $1, *at the given rate of exchange; the quotient will be the face of the required draft.*

EXAMPLES FOR PRACTICE.

1. What draft may be purchased for $16415.10, exchange being at $3\frac{1}{2}$ % premium? *Ans.* $15860.
2. Required the face of a draft for $158.40, exchange being at 1 % discount? *Ans.* $160.
3. An agent in Kingston is directed to make the remittance by draft, of $565.32, to his employer in Quebec, drawn at 60 days. What will be the face of the draft, exchange being at $1\frac{3}{4}$ % premium?
4. What will be the face of a draft for $962.85, exchange being at $\frac{3}{8}$ % discount?
5. A man in Halifax, has $4800 due him in Montreal; how much more will he realize by making a draft for this sum on Montreal and selling it at $\frac{1}{2}$ % discount, than by having a draft on Halifax remitted to him, purchased in Montreal for this sum, at $\frac{3}{4}$ % premium? *Ans.* $11.73+.

FOREIGN EXCHANGE.

420. A **Foreign Bill of Exchange** is a draft in which the drawer and drawee live in different countries.

Form of a Foreign Bill of Exchange.

£ 300. [STAMP.] *Montreal, P. Q., April 7, 1871.*

At sight of this, our first Bill of Exchange (second and third of same tenor and date unpaid), pay to the order of William C. Maguire, London, Three hundred pounds sterling, value received, and charge the same to my account.

To F. S. Moore & Co., *R. N. Wallace.*

36, Wellington Street, London.

NOTE.—In foreign exchange, to prevent loss or delay, two or three drafts of the same tenor and date are drawn up and sent by different conveyances, or at different times; on the payment of one, the other two are worthless. Each draft must have a stamp attached.

421. Foreign exchange is computed as inland exchange, except that the currency of one country must be reduced to that of another.

422. Rates of exchange between the Dominion of Canada and Great Britain are commonly reckoned, at a certain per cent. on the old par of exchange, instead of on the new par.

NOTE.—By an old act of Provincial Parliament, it was enacted that £1 sterling = $4.444 Canadian money. But by a recent act the value of the pound sterling was fixed at $4.866. Now, the *new par* is equal to the *old par* plus 9⅓ % of the old par, that is, $4.444 + 9⅓ % of $4.444, which is .422, equal to $4.866, the new par. Consequently, the rate of exchange between the Dominion of Canada and Great Britain, must reach the nominal premium, or 9⅓ % before it is at par, according to the new standard.

Ex. 1. A merchant in Quebec wishes to remit to London £560 3 6 sterling; exchange being at 11 % premium. How much must he pay for the bill of exchange?

OPERATION.

$\$\frac{4.0}{9} \times 1.11 = \$4.93\frac{1}{3}$;
£560 3 6 = £560.175;
£560.175 × $4.93\frac{1}{3}$ = $2763.53, *Ans.*

ANALYSIS.—Since the old par of £1 sterling = $4.444, or $\$\frac{4.0}{9}$, we multiply $\$\frac{4.0}{9}$ by 11 %, or $1.11, the given rate, decimally expressed, and we obtain $4.93⅓, the cost of £1 at that rate; multiplying the face of the bill, £560 3 6, decimally expressed by the cost of exchange of £1, we obtain $2763.53, the required cost of the bill.

Ex. 2. What will be the face of a bill of exchange on Liverpool, purchased in Montreal for $5537.40, exchange being at 10 % premium?

OPERATION.

$\$1\frac{10}{8} \times 1.10 = \$4.88\frac{8}{9}$;
$\$5537.40 \div 4.88\frac{8}{9} = £1132\ 13\ 0$.

ANALYSIS.—We find, as in the preceding example, the cost of £1, at the given rate of exchange; then we divide $5537.40, the given cost, by the cost of exchange for £1, and obtain £1132 13 0, the face.

Ex. 3. What is the cost in Toronto of a bill on Paris, for 1780 francs, exchange being at $2\frac{1}{2}$ % discount?

OPERATION.

Commercial value of the franc, = $0.186
Deduct $2\frac{1}{2}$ % discount, ... 0.00465

Value of 1 franc, ... $0.18135

$0.18135 × 1780 = $322.803, *Ans.*

423. From these illustrations we derive the following

RULE.—I. To find the cost of a bill, the face being given.—*Multiply the face by the cost of a unit of the currency in which the bill is expressed.*

II. To find the face of a bill, the cost being given.—*Divide the given cost by the cost of a unit of the currency in which the bill is to be expressed.*

REDUCTION OF THE STERLING MONEY TO THE OLD OR TO THE NEW CANADIAN CURRENCY, NEW PAR.

Ex. Reduce £560 3 4 sterling, to Old Canadian Currency.

OPERATION.

	£	s	d
	560	3	4
+ $\frac{1}{5}$ of £560 3 4 =	112	0	8
+ $\frac{1}{12}$ of 112 0 8 =	9	6	$8\frac{2}{3}$
	£681	10	$8\frac{2}{3}$, *Ans.*

And in Decimal Currency,
£681 10 $8\frac{2}{3}$ (233) = 2726.13\frac{8}{9}$.

ANALYSIS. — The pound sterling = $\$4.86\frac{2}{3}$, and the Old currency pound = $4; diff., $\$0.86\frac{2}{3}$. Then £1 ster. $= £1 + \frac{86\frac{2}{3}}{400} = £1\frac{13}{60}$ old currency. Now, $\frac{13}{60}$ of a number $= \frac{1}{5}$ plus $\frac{1}{12}$ of $\frac{1}{5}$ of that number. Hence the

424. RULE.—To reduce sterling money to Old Canadian Currency, new par,—*Add to the given sum its fifth plus one twelfth of the fifth.*

EXAMPLES FOR PRACTICE.

1. What will be the commercial value in Ottawa, of a bill of exchange on London for £390 10 ster., at 9 % prem. ? *Ans.* 1891.75\frac{5}{9}$.

2. What will cost, in Amsterdam, a bill on Montreal for $681.34, the course of exchange being at $0.38 per guilder? *Ans.* $717.20.

3. What must be paid in Kingston for a bill of exchange on Paris for 3000 francs, exchange being at 2 % above par?

4. What will be the cost in Montreal of a bill on Boston, for $2000, at $\frac{1}{2}$ % premium? *Ans.* $2010.

5. What will a bill of exchange on Hamburg, for 5000 marcs banco, cost in Quebec, at 1 % above par, the marc banco being equal to 35 cts.?

6. Bought in Toronto a bill of exchange on Glasgow for £675 2 6 sterling; what did it cost me at 8$\frac{1}{2}$ % premium? *Ans.* $3255.60 + .

7. What will cost in Halifax a bill of exchange on Rouen for 56245 francs, the course of exchange being 5 fr. 54 centimes per dollar? *Ans.* $10152.53 + .

8. What will be the face of a bill on Dublin, that may be bought in Montreal for $7125.50, exchange being at 9$\frac{1}{4}$ % premium?

9. Paid in Quebec £2170 15 7, old Canadian currency, for a bill on Lyons amounting to 49335 fr. 20 centimes; what was the rate of exchange below par? *Ans.* $0.053 + .

10. Received from L. Nelson & Co., London, a bill of exchange for £381 5 0, on J. Chalmers & Co., Quebec; what is its value in decimal currency of Canada, at 9 % premium? *Ans.* $1846.94 + .

EQUATION OF PAYMENTS.

425. Equation of Payments is the process of finding the mean or equitable time of payment of several sums, due at different times without interest.

426. The **Term of Credit** is the time to elapse before a debt becomes due.

427. The **Average Term of Credit** is the time to elapse before several debts, due at different times, may all be paid at once, without loss to debtor or creditor.

428. The **Equated Time** is the date at which the several debts may be cancelled by one payment.

429. CASE I.—*To find the average or equitable time of paying several debts due at different times.*

Ex. 1. On the first of March 1870, a man gave notes, as follows: the first for $250 payable in 30 days; the second for $200 payable in 60 days; the third for $300 payable in 90 days. What was the average term of credit, and what the equated time of payment?

OPERATION.

$250 × 30 = 7500
200 × 60 = 12000
300 × 90 = 27000

$750 46500

46500 ÷ 750 = 62da., average term of credit.

March 1 + 62da. = May 2, *Ans.*

ANALYSIS. — The interest of $250 for 30 days is the same as the interest of $1 for 7500 days; and of $200 for 60 days, the same as of $1 for 12000 days; and of $300 for 90 days, the same as of $1 for 27000 days. Hence, the interest of all the sums to the time of payment is the same as the interest of $1 for 7500da. + 12000da. + 27000da. = 46500 days. Now, if $1 require 46500 days to gain a certain sum, $250 + $200 + $300 = $750 will require $\frac{1}{750}$ of 46500 days; 46500da. ÷ 750 = 62 days, the average term of credit; and, March 1, the date at which the credits begin, + 62da. = May 2, the equated time of payment.

Ex. 2. Bought of D. I. Lyons several bills of goods, at different times, and on various terms of credit, as by the following statement. What is the equated time for the payment of the whole?

Jan. 1, a bill amounting to $300, on 4 months.
Feb. 7, " " " " 185, on 5 months.
March 15, " " " " 280, on 4 months.
April 20, " " " " 210, on 6 months.

OPERATION.

Due	May	1,	$300				
"	July	7,	185	×	67	=	12395
"	July	15,	280	×	75	=	21000
"	Oct.	20,	210	×	172	=	36120
			$975				69515 days.

69515 ÷ 975 = $71\frac{590}{975}$ days.

May 1 + 71 days = July 11, *Ans.*

ANALYSIS.—We first find the time when each of the bills will become due. Then, since it will shorten the operation and not change the result, we *take the first time when any bill becomes due*, instead of its *date*, or the point from which to compute the average time. Now, since May 1 is the period from which the average time is computed, no time will be reckoned on the first bill, but the time for the payment of the second bill extends 67 days beyond May 1, and we multiply its amount by 67. Proceeding in the same manner with the remaining bills, we find the average term of credit to be 71 days, and July 11, the equated time of payment.

430. RULE.—*Multiply each payment by its own time of credit, and divide the sum of the products by the sum of the payments.*

NOTE.—If the date of the average time of payment is required, as in Ex. 2, *find the time when each of the sums becomes due. Multiply each sum by the number of days intervening between the date of its becoming due and the earliest date on which any sum becomes due. Then proceed as in the rule, and the quotient will be the average time required, in days forward from the date of the earliest sum becoming due.*

EXAMPLES FOR PRACTICE.

1. A merchant purchased £4750 worth of cloth, and agreed to pay $\frac{1}{5}$ of the sum every month until the cancellation of his debt; what will be the amount of each payment? *Ans.* £950.

2. A man owes $15960 payable as follows: $\frac{1}{4}$ in cash, $\frac{2}{3}$ in 6 mo., and the remainder in 1 year; required the amount of each payment?

3. The sum of $1710 is to be paid in two installments, viz.: $\frac{1}{2}$ in 6 mo., and the other $\frac{1}{2}$ in 10 mo. At what time should it be paid so as to make but one payment? *Ans.* In 8 months.

4. Bought 25 casks of wine for $1125 which I agreed to pay as follows: $525 in 6 mo., and the balance in 9 mo. Wishing to make but one payment, how long should this payment be deferred?

Ans. 7mo. 18da.

5. On the 1st. of January, 1868, a merchant gave three notes: the first for $500 payable in 30 days; the second for $400 payable in 60 days; the third for $600 payable in 90 days. Required the equated time of payment? *Ans.* March 3, 1868.

6. A merchant bought, on the 15th. of May, 1868, $8000 worth of merchandise and agreed to pay $\frac{1}{3}$ of the price in 6 mo., $\frac{1}{5}$ in 8 mo., and the remainder in 10 mo. But wishing to cancel his debt by a single payment, how long should this payment be deferred?

Ans. 8mo. 24da.

431. CASE II.—*To find the time of paying the balance of a debt, when partial payments have been made before the debt is due.*

Ex. Bought $180 worth of goods, at 8 months' credit. At the end of 4 months, I paid $30, and 2 months later, $40; when, in equity, after the expiration of 8 months, shall I owe the balance?

OPERATION.

$30 \times 4 = 120$

$40 \times 2 = 80$

$70 \quad 200$

$\$180 - \$70 = \$110$;

$200 \div 110 =$ 1mo. 25da., *Ans.*

ANALYSIS.—The interest on the $30 for 4 months is equal to the interest of $1 for 120 months, and the int. of $40 for 2 months is equal to that of $1 for 80 months; and thus the int. on both partial payments, at the expiration of the 8 months, is equal to the int. of $1 for $120 + 80 = 200$ months. To equal this credit of int., the bal. of the debt, which we find to be $110, should remain unpaid, after the 8 months, $\frac{1}{110}$ of 200 months, or 1mo. 25da.

432. RULE.—*Multiply each payment by the time it was made before it becomes due, and divide the sum of the products by the balance remaining unpaid; and the quotient will be the required time.*

EXAMPLES FOR PRACTICE.

1. A vintner agreed to pay $1895.20 for 2369 gal. of cognac brandy in 12 mo.; but at the end of 10 mo. he paid for 633 gal. Required the equated time of the balance? *Ans.* 12 mo. 22 da.

2. Bought of C. Lyons, at 6 mo. £432 worth of goods; at the end of 1 mo. I paid him £75, and 4 mo. after £200 more. How long after the expiration of the 6 mo. should I pay the balance?

Ans. 3 mo. 20 da.

3. A grocer bought $2829.75 worth of coffee which he desires to pay in three different payments: the first is to the second as 4 is to 5, and the third is equal to half the second. The first payment should be made in 4 mo.; the second in 7 mo.; and the third in 1 year. But at the end of 6 mo. he paid $975, how long can he keep the balance? *Ans.* 7 mo. 18 da.

4. An undertaker built a house for £6035 payable in 15 mo.; but being in want of some money, the proprietor pays him £2847 10 eight months before the time. How long, in equity, can the proprietor keep the balance to compensate the advance he made the undertaker?

Ans. 22 mo. 4 da.

5. Andrew having sold $8400 worth of linen, at 12 mo. credit, received the $\frac{1}{3}$ of the price only 15 mo. after. When did he receive the $\frac{2}{3}$? *Ans.* In 10 mo. 15 da.

6. I owed $600 at 13 months; I paid $\frac{2}{3}$ of this sum before it was due, so that I can keep the remainder 2 years without injuring my creditor. Required the time when the $\frac{2}{3}$ were paid? *A.* 7mo. 15da.

7. A trader owes $3000 payable in 6 mo.; $4500 payable in 8 mo., and $9500 payable in 10 mo. At the end of 5 mo. he pays $12000. How long can he keep the balance? *Ans.* 17 mo. 24 da.

ALLIGATION.

433. Alligation treats of mixing or compounding articles or ingredients of different qualities or values. It is of two kinds—*Alligation Medial*, and *Alligation Alternate*.

ALLIGATION MEDIAL.

434. Alligation Medial is the process of finding the mean or average rate of a mixture composed of articles of different qualities or values, the quantity and rate of each being given.

435. *To find the average value of several articles mixed, the quantity and rate of each being given.*

Ex. A grocer mixed 2cwt. of sugar worth $9 per cwt. with 1cwt. worth $7 per cwt. and 2cwt. worth $10 per cwt.; what is 1cwt. of the mixture worth?

OPERATION.

$$\begin{array}{rcl} \$\ 9 \times 2 = & \$18 \\ 7 \times 1 = & 7 \\ 10 \times 2 = & 20 \\ \hline 5) & \$45 \\ & \$9, \textit{Ans.} \end{array}$$

ANALYSIS.—Since 2cwt. at $9 per cwt. is worth $18, 1cwt. at $7 per cwt. is worth $7, and 2cwt. at $10 per cwt. is worth $20; 2cwt. + 1cwt. + 2cwt. = 5cwt. is worth $18 + $7 + $20 = $45; and 1cwt. is worth as many dollars as 45 contains times 5, or $9.

436. RULE.—*Find the value of each of the articles, and divide the sum of their values by the number denoting the sum of the articles. The quotient will be the average value of the mixture.*

EXAMPLES FOR PRACTICE.

1. A farmer mixes together 10 bush. of oats at 40 cts. per bu., 15 bu. of corn at 50 cts. per bu., and 25 bu. of rye at 70 cts. per bu.; what is the value of a bushel of the mixture? *Ans.* 58 cts.

2. If I mix 20 pounds of tea at 70 cts. per pound with 15 pounds at 60 cts. per pound, and 80 pounds at 40 cts. per pound; what is the value of 1 lb. of this mixture? *Ans.* $47\frac{1}{23}$ cts.

3. A dealer in liquors would mix 14 gal. of water with 12 gal. of wine at $.75, 24 gal. at $.90, and 16 gal. at $1.10; how much is a gallon of the mixture worth? *Ans.* $\$0.73\frac{1}{3}$.

4. A man bought $3\frac{3}{4}$ dozen of eggs at 12 cts. a dozen, 4 dozen at $10\frac{1}{2}$ cts. a dozen, $4\frac{1}{2}$ dozen at 11 cts. a doz., and $5\frac{1}{4}$ doz. at 10 cts. a doz. He sells them so as to make 50 % on the cost; how much did he receive per dozen? *Ans.* $16\frac{1}{2}$ cts.

5. A goldsmith wishes to compound 3 lb. 6 oz. of gold 23 carats fine with 4 lb. 8 oz. 21 carats, 3 lb. 9 oz. 20 carats, and 2 lb. 2 oz. of alloy; what will be the fineness of the composition? *Ans.* 18 carats.

ALLIGATION ALTERNATE.

437. Alligation Alternate is the process of finding the proportional quantities to be taken of several articles or ingredients, whose prices or qualities are known to form a mixture of any given rate or quality.

438. *To find the proportional quantity to be used of each ingredient, when the mean price or quality of the mixture is given.*

Ex. 1. What relative quantities of timothy seed worth $2 a bushel, and clover seed worth $7 a bushel, must be used to form a mixture worth $5 a bushel?

OPERATION.

$$5 \left\{ \begin{array}{c|c|c} 2 & \frac{1}{3} & 2 \\ 7 & \frac{1}{2} & 3 \end{array} \right\} \textit{Ans.}$$

ANALYSIS.—Since on every ingredient used whose price or quality is *less* than the mean rate there will be a *gain*, and on every ingredient whose price or quality is *greater* than the mean rate there will be a *loss*, and since the gains and losses must be exactly equal, the relative quantities used of each should be such as represent the unit of *value*. By selling one bushel of timothy seed worth $2, for $5, there is a gain of $3; and to gain $1 would require $\frac{1}{3}$ of a bushel, which we place opposite the 2. By selling one bushel of clover seed worth $7, for $5, there is a loss of $2; and to lose $1 would require $\frac{1}{2}$ of a bushel, which we place opposite the 7.

In every case, to find the unit of value we must divide $1 by the gain or loss per bushel or pound, &c. Hence, if, every time we take $\frac{1}{3}$ of a bushel of timothy seed, we take $\frac{1}{2}$ of a bushel of clover seed, the gain and loss will be exactly equal, and we shall have $\frac{1}{3}$ and $\frac{1}{2}$ for the *proportional quantities*.

Ex. 2. What proportions of coffees worth respectively 3, 4, 7 and 10 shillings a pound, must be taken to form a mixture worth 6 shillings a pound?

OPERATION.

		1	2	3	4	5
6	3	$\frac{1}{3}$		4		4
	4		$\frac{1}{2}$		1	1
	7		1		2	2
	10	$\frac{1}{4}$		3		3

ANALYSIS.—To preserve the equality of gains and losses, we must always compare two prices or simples, one *greater* and one *less* than the mean rate, and treat each pair or couplet as a separate example. In the given example we form two couplets, and may compare either 3 and 10, 4 and 7, or 3 and 7, 4 and 10.

We find that $\frac{1}{3}$ of a lb. at 3s. must be taken to gain 1 shilling, and $\frac{1}{4}$ of a lb. at 10s. to lose 1 shilling; also $\frac{1}{2}$ of a lb. at 4s. to gain 1 shilling, and 1 lb. at 7s. to lose 1 shilling. These proportional numbers, obtained by comparing the two couplets, are placed in columns 1 and 2. If, now, we reduce the numbers in columns 1 and 2 to a common denominator, and use their numerators, we obtain the integral numbers in columns 3 and 4, which, being arranged in column 5, give the proportional quantities to be taken of each.

It will be seen that in comparing the simples of any pair or couplet, one of which is greater, and the other less than the mean rate, the proportional number finally obtained for either term is the difference between the mean rate and the other term. Thus, in comparing 3 and 10, the proportional number of the former is 4, which is the difference between 10 and the mean rate 6; and the proportional number of the latter is 3, which is the difference between 3 and the mean rate. The same is true of every other couplet. Hence, when the simples and the mean rate are integers, the intermediate steps taken to obtain the final proportional numbers as in columns 1, 2, 3, and 4, may be omitted, and the same results readily found by taking the difference between each simple and the mean rate, and placing it opposite the one with which it is compared.

From the foregoing examples and analyses we derive the following

439. RULE.—I. *Write the several prices or qualities in a column and the mean price or quality of the mixture at the left.*

II. *Form couplets by comparing any price or quality less, with one that is greater than the mean rate, placing the part which must be used to gain* 1 *of the mean rate opposite the less simple, and the part that must be used to lose* 1 *opposite the greater simple, and do the same for each simple in every couplet.*

III. *If the proportional numbers are fractional, they may be reduced to integers, and if two or more stand in the same horizontal line, they must be added; the final results will be the proportional quantities required.*

NOTES. 1. If the numbers in any couplet or column have a common factor, it may be rejected.

2. We may also multiply the numbers in any couplet or column by any multiplier we choose, without affecting the equality of the gains and losses, and thus obtain an indefinite number of results, any one of which being taken will give a correct final result.

EXAMPLES FOR PRACTICE.

1. A grocer has sugars worth 10 cents, 11 cents, and 14 cents per pound; in what proportions may he mix them to form a mixture worth 12 cts. per lb.? *Ans.* 1 lb. at 10 cts., and 2 lb. at 11 and 14 cts.

2. What proportions of water at no value, and wine worth $1.20 a gallon, must be used to form a mixture worth 90 cents a gallon? *Ans.* 1 gal. of water to 3 gal. of wine.

3. A farmer had sheep worth $2, $2½, $3, and $4 per head; what number could he sell of each, and realize an average price of $2¾ per head? *Ans.* 5 of the 1st. kind, and 1 each of the 2nd. and 3rd., and 3 of the 4th. kind.

4. What relative quantities of alcohol 80, 84, 87, 94, and 96 per cent. strong must be used to form a mixture 90 per cent. strong?

Ans. 6 of the first two kinds, four of the 3rd., 3 of the 4th. and 16 of the 5th.

440. *To find the proportional quantity to be used of each ingredient, when the quantity of one of the simples is limited.*

Ex. A miller has oats worth 30 cents, corn worth 45 cents, and barley worth 84 cents per bushel; he desires to form a mixture worth 60 cents per bushel, and which shall contain 40 bushels of corn; how many bushels of oats and barley must he take?

OPERATION.

$$60 \left\{ \begin{array}{c|c|c|c|c|c} 30 & \frac{1}{30} & & 4 & 8 & 20 \\ 45 & & \frac{1}{15} & & & 40 \\ 84 & \frac{1}{24} & & 5 & 5 & 50 \end{array} \right\} \text{Ans.}$$

ANALYSIS. By the same process as in Case I we find the proportional quantities of each to be 4 bushels of oats, 8 of corn, and 10 of barley. But we wish to use 40 bushels of corn, which is 5 times the proportional number 8, and to preserve the equality of gain and loss we must take 5 times the proportional quantity of each of the other simples, or $5 \times 4 = 20$ bushels of oats, and $5 \times 10 = 50$ bushels of barley. Hence the following

441. RULE.—*Find the proportional quantities as in* 438. *Divide the given quantity by the proportional quantity of the same ingredient, and multiply each of the other proportional quantities by the quotient thus obtained.*

EXAMPLES FOR PRACTICE.

1. A merchant has teas worth 40, 60, 75, and 90 cents per pound; how many pounds of each must he use with 20 pounds of that worth 75 cents, to form a mixture at 80 cents?

Ans. 20 lbs. each of the first three kinds, and 130 lbs. of the fourth.

2. A farmer bought 24 sheep at $2 a head; how many must he buy at $3 and $5 a head, that he may sell the whole at an average price of $4 a head, without loss? *Ans.* 24 at $3, and 72 at $5.

3. How much alcohol worth 60 cents a gallon, and how much water, must be mixed with 180 gallons of rum worth $1.30 a gallon, that the mixture may be worth 90 cents a gallon?

Ans. 60 gallons each of alcohol and water.

4. How many acres of land worth 35 dollars an acre must be added to a farm of 75 acres, worth $50 an acre, that the average value may be $40 an acre? *Ans.* 150 acres.

5. A merchant mixed 80 pounds of sugar worth $6\frac{1}{4}$ cents per pound with some worth $8\frac{1}{3}$ cents and 10 cents per pound, so that the mixture was worth $7\frac{1}{2}$ cents per pound; how much of each kind did he use?

442. *To find the proportional quantity to be used of each ingredient, when the quantity of the whole compound is limited.*

Ex. A grocer has sugars worth 6 cents, 7 cents, 12 cents, and 13 cents per pound. He wishes to make a mixture of 120 pounds worth 10 cents a pound; how many pounds of each kind must he use?

OPERATION.

10	6	$\frac{1}{4}$	3	2	3	30
	7	$\frac{1}{3}$			2	20
	12	$\frac{1}{2}$			3	30
	13	$\frac{1}{3}$	4		4	40
					12	120

ANALYSIS. By Case 1 we find the proportional quantities of each to be 3 lbs. at 6 cts., 2 lbs. at 7 cts., 3 lbs. at 12 cts., and 4 lbs. at 13 cts. By adding the proportional quantities, we find that the mixture would be but 12 lbs. while the required mixture is 120, or 10 times 12. If the whole mixture is to be 10 times as much as the *sum* of the proportional quantities, then the quantity of each simple used must be 10 times as much as its respective proportional, which would required 30 lbs. at 6 cts., 20 lbs. at 7 cts., 30 lbs. at 12 cts., and 40 lbs. at 13 cts. Hence we deduce the following

443. RULE.—*Find the proportional numbers as in* 438. *Divide the given quantity by the sum of the proportional quantities, and multiply each of the proportional quantities by the quotient thus obtained.*

EXAMPLES FOR PRACTICE.

1. A farmer sold 170 sheep at an average price of 14 shillings a head; for some he received 9s., for some 12s., for some 18s., and for others 20s.; how many of each did he sell?

Ans. 60 at 9s., 40 at 12s., 20 at 18s., and 50 at 20s.

2. A jeweler melted together gold 16, 18, 21, and 24 carats fine, so as to make a compound of 51 ounces 22 carats fine; how much of each sort did he take? *Ans.* 6 ounces each of the first three, and 33 ounces of the last.

3. A man bought 210 bushels of oats, corn, and wheat, and paid for the whole $178.50; for the oats he paid $\frac{1}{2}$, for the corn, $\frac{3}{4}$, and for the wheat $1\frac{1}{2}$ per bushel; how many bushels of each kind did he buy? *Ans.* 78 bu. each of oats and corn, and 54 bu. of wheat.

4. A, B, and C are under a joint contract to furnish 6000 bushels of corn, at 48 cts. a bushel; A's corn is worth 45 cts., B's 51 cts., and C's 54 cts.; how many bushels must each put into the mixture that the contract may be fulfilled?

5. One man and 3 boys received \$84 for 56 days' labor; the man received \$3 per day, and the boys \$$\frac{1}{2}$, \$$\frac{3}{4}$, and \$$1\frac{3}{4}$ respectively; how many days did each labor? *Ans.* The man 16 days, and the boys 24, 4, and 12 days respectively.

INVOLUTION.

444. Involution is the process of raising a number to a given power.

445. A Power is the product arising from multiplying a number by itself, or repeating it several times as a factor.

446. The **Index** or **Exponent** of a power is a small figure placed at the right and a little above the number, to show how many times it is used to produce the power:—

Thus,
- $3^1 =$ the first power of 3, or the root.
- $3^2 = 3 \times 3 = 9$, the second power, or square of 3.
- $3^3 = 3 \times 3 \times 3 = 27$, the third power, or cube of 3.
- $3^4 = 3 \times 3 \times 3 \times 3 = 81$, the fourth power of 3.
- $(\frac{2}{3})^5 = \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{32}{243}$, the fifth power of $\frac{2}{3}$.

Hence, from these several powers of 3, we derive the following

447. RULE.—*Multiply the given number by itself as many times, less* 1, *as there are units in the exponent of the required power.*

NOTE.—A mixed number may be either reduced to an improper fraction, or the fractional part reduced to a decimal, before involution.

EXAMPLES FOR PRACTICE.

1. Square 25.	*Ans.* 225.	7. $(1.06)^4 = ?$	*Ans.* 1.262476.
2. Square 79.	*Ans.* 6241.	8. $(\frac{2}{3})^3 = ?$	*Ans.* $\frac{8}{27}$.
3. Cube 47.	*Ans.* 103823.	9. $(\frac{1}{8})^3 = ?$	*Ans.* $\frac{1}{512}$.
4. Cube 39.	*Ans.* 59319.	10. $(2\frac{2}{3})^4 = ?$	*Ans.* $50\frac{46}{81}$.
5. $24^4 = ?$	*Ans.* 331776.	11. $(1\frac{3}{5})^6 = ?$	*Ans.* $16\frac{2144}{15625}$.
6. $(1.2)^5 = ?$	*Ans.* 2.48832.	12. $(2\frac{1}{4})^5 = ?$	*Ans.* $157\frac{283}{1024}$.

EVOLUTION.

448. Evolution is the process of extracting the root of a number considered as a power; it is the reverse of Involution.

449. The **Root** of a number is one of its equal factors.

450. The **First Root** of a number is the number itself.

451. The **Second Root,** or **Square Root,** of a number, is one of its two equal factors. Thus, 4 is the square root of $16 = 4 \times 4$.

452. The **Third Root,** or **Cube Root,** of a number, is one of its three equal factors. Thus, 4 is the cube root of $64 = 4 \times 4 \times 4$.

453. The **Radical Sign** is the character, $\surd$, which, placed before a number, indicates that its root is to be extracted.

454. The **Index** of the root is the figure placed above the radical sign, to denote what root is to be taken. When no index is written, the index, 2, is always understood.

455. The names of roots are derived from the corresponding powers, and are denoted by the indices of the radical sign. Thus, $\surd 36$ denotes the *square root* of 36; $\sqrt[3]{36}$ denotes the *cube root* of 36; $\sqrt[4]{36}$ denotes the *fourth root* of 36; etc.

456. A **Rational Root** is a root which can be exactly obtained.

457. A **Surd** is one which cannot be exactly obtained.

SQUARE ROOT.

The roots of the first ten integers and their squares are:

1,	2,	3,	4,	5,	6,	7,	8,	9,	10.
1,	4,	9,	16,	25,	36,	49,	64,	81,	100.

NOTES.—1. It will be observed that the second power or square of each of the numbers contains twice as many figures as the root, or twice as many wanting one. Hence, to ascertain the number of figures in the square root of a given number,—*Beginning at the right, point it off into as many periods as possible, of two figures each; and there will be as many figures in the root as there are periods.*

2. When the given number contains an *odd* number of figures, the period at the left can contain but *one* figure.

Ex. What is the square root of 4096?

OPERATION.

$$\begin{array}{r|l} \dot{4}\dot{0}\dot{9}\dot{6} & 64, \textit{Ans.} \\ 36 & \\ \hline 124 \mid 496 & \\ \quad 496 & \\ \hline \quad 0 & \end{array}$$

ANALYSIS.—Beginning at the right, we separate the number into periods of two figures each, by placing a point (.) over the right-hand figure of each period. Now, the greatest square of 40, the left-hand period, is 36, the root of which is 6. Placing the 6 on the right of the number, we subtract its square from the period 40, and to the right of the remainder bring down the next period. We then double the 6, the part of the root already found, and, placing it on the left of the dividend for a partial divisor, we perceive it is contained in the dividend, (omitting its right-hand figure), 4 times. Placing the 4 on the right of the root, also on the right of the partial divisor, we multiply the divisor thus completed by 4, and subtract the product from the dividend. The root or answer is 64.

458. RULE.—I. *Point off the given number into periods of two figures each, counting from units' place toward the left and right.*

II. *Find the greatest perfect square in the left-hand period and write its root on the right for the first figure in the root.*

III. *Subtract the square of the root figure from the left-hand period, and to the remainder annex the next period for a dividend.*

IV. *Double the part of the root already found for a trial divisor, and see how many times it is contained in the dividend, exclusive of the right-hand figure, and write the quotient as the next divisor of the root, and also at the right of the trial divisor.*

V. *Multiply the divisor thus formed by the figure of the root last found, and subtract the product from the dividend.*

VI. *To this remainder annex the next period for a new dividend, and divide the same by twice the root already found, and continue in this manner until all the periods are used.*

NOTES.—1. When any dividend, exclusive of its right-hand figure, does not contain the divisor, a cipher must be placed in the root, and also at the right of the divisor; then, after bringing down the next period, this last divisor must be used as the divisor of the new dividend.

2. When there is a remainder after all the periods have been brought down, periods of ciphers may be annexed, and the figures of the root thus obtained will be decimals.

3. If the given number is a decimal, or a whole number and a decimal, the root is extracted in the same manner as in whole numbers, except, in pointing off the decimals, either alone or in connection with the whole number, we place a point over every second figure toward the right, from the separatrix, filling the last period, if incomplete, with a cipher.

4. The square root of a common fraction may be obtained by extracting the square roots of the numerator and denominator separately, provided the terms are perfect squares; otherwise, the fraction may be reduced to a decimal.

5. Mixed numbers may be reduced to the decimal form before extracting the root; or, if the denominator of the fraction is a perfect square, to an improper fraction.

EXAMPLES FOR PRACTICE.

1. What is the square root of 133225? of 62.8?

OPERATION.

133225 (365, *Ans.*

3 × 3 = 9

3 × 2 = 6 6) 432

66 × 6 = 396

36 × 2 = 72 5) 3625

725 × 5 = 3625

0

OPERATION.

62.80(7.92+, *A.*

7 × 7 = 49

7 × 2 = 14.9) 13.80

14.9 × .9 = 13.41

7.9 × .2 = 15.82) .3900

15.82 × .02 = .3164

.0736

2. What is the square root of 169? of 576? of 1225? of 2401? of 3249? of 4096? of 5329? of 6724? of 9801? of 10816?

Ans. 13, 24, 35, 49, 57, 64, etc.

3. What is the square root of 61009? of 454276? of 505521? of 637821? of 648132? of 738417? of 809215? of 927748? of 977137? of 999999? *Ans.* 247, 674, 711, 798, 805, 859, 899, etc.

4. What is the square root of 234.09? of 5.4756? of 17.3056? of 256.6404? of 0.1024? of 0.120409? of 0.00008836? of 609151.76100?

Ans. 15.3, 2.34, 4.16, 16.02, 0.32, 0.347, 0.0094, 780.481.

5. What is the square root of $\frac{3}{4}$? of $6\frac{97}{144}$? of $\frac{49}{529}$? of $60\frac{1}{16}$? of $\frac{48}{147}$? of $\frac{3}{5}$? of $28\frac{57}{64}$? of $\frac{42}{57}$? of $\frac{1008}{1183}$? of $95\frac{1}{16}$?

Ans. 0.86602+, $2\frac{7}{12}$, $\frac{7}{23}$, $7\frac{3}{4}$, $\frac{4}{7}$, 0.7745+, $5\frac{3}{8}$, 0.858+, $1\frac{2}{13}$, $9\frac{3}{4}$.

APPLICATIONS OF THE SQUARE ROOT.

1. What is the length of one side of a square farm containing 90 acres? *Ans.* 120 rods.

2. A certain general has an army of 141376 men; how many must he place in rank and file to form them into a square? *Ans.* 376.

3. A company of persons spent \$75.69; each spending as many cents as there were persons in the company. How much did each expend? *Ans.* \$0.87.

4. Bought 200 yards of carpeting $1\frac{1}{8}$ yards wide; what is the length of one side of the square room which this carpet will cover? *A.* 45ft.

5. A man owns three pieces of land; the first is 125 rods long, and 53 wide; the second is $62\frac{1}{2}$ rods long, and 34 wide; and the third contains 37 acres: what will be the length of the side of a square field whose area will be equal to the three pieces? *Ans.* 121.11+ rods.

6. Purchased 2 house-lots; the first is 242 feet square, and the second contains 9 times the area of the first; how many feet square in the second? *Ans.* 726 feet.

7. Required the sides of a rectangular court-yard having an area of 432 rods, and whose breadth is only the $\frac{3}{4}$ of the length?

8. A certain field contains 48020 square rods; the length exceeds the breadth by 49 rods: what are the sides?

Ans. 245 rods long; 196 rods wide.

9. A school-master says that the number of his pupils multiplied by $\frac{1}{2}$ of itself is 2523; how many pupils has he? *Ans.* 87.

10. How much will it cost to roughcast the walls of a garden, having a surface of 8100 yards, at $87\frac{1}{2}$ cts. per yard, the walls being $2\frac{3}{10}$ yd. high? *Ans.* \$1449.

11. The greater of two numbers is 40, and the sum of their squares 1625; what is the smaller number? *Ans.* 5.

12. A clock-maker sold three watches whose prices are as 5 is to 6, and as 6 is to 9; the sum of the squares of the prices is \$3550. What is the price of each watch? *Ans.* \$25, \$30, \$45.

13. What is the price of a raking machine, knowing that the price added to its square gives \$186 for result? *Ans.* \$13.13$\frac{1}{2}$.

14. In dividing the square of the number of dollars that I have by $\frac{1}{4}$ of the number itself I obtain \$96 for result. How many barrels of codfish, at \$4 per barrel, can I buy with the money I possess?

Ans. 6 barrels.

CUBE ROOT.

The roots of the first ten integers and their cubes are:—

1,	2,	3,	4,	5,	6,	7,	8,	9,	10.
1,	8,	27,	64,	125,	216,	343,	512,	729,	1000.

NOTE.—It will be observed that the cube or third power of each of the numbers contains *three* times as many figures as the root, or *three* times as many wanting *one*, or *two* at most. Hence, to determine the number of figures in the cube root of a given number,—*Beginning at the right, point it off into as many periods as possible of three figures each, and there will be as many figures in the root as there are periods.*

Ex. What is the cube root of 157464?

OPERATION.

$$\begin{array}{rl|l} & & 157464\ (54 \\ & 5^3 = & 125 \\ \text{trial div.,} & 3 \times 50^2 = 7500 & 32464 \\ & 3 \times 50 \times 4 = 600 & \\ & 4^2 = 16 & \\ \text{True divisor,} & 8116 \times 4 = & 32464 \\ & & 0 \end{array}$$

PROOF.

$$54 \times 54 \times 54 = 157464.$$

ANALYSIS.—Beginning at the right, we separate the given number into periods, by placing a point over the units' figure, then over thousands. Since the number of periods is two, the root will consist of two figures, *tens* and *units*. Then 157464 = the cube of tens, plus three times the square of the tens into the units, plus three times the tens into the square of the units, plus the cube of the units. The cube of tens is thousands, and must therefore be found in the thousands of the number. The greatest number of tens whose cube does not exceed 157 thousands is 5, which we write as the tens figure of the root. We then subtract the 125 thousands, the cube of the 5 tens, from the 157 thousands, and there remain 32 thousands; and, annexing the next period, we have as the entire remainder, 32464, equal three times the square of the tens into the units, plus three times the tens into the square of the units, plus the cube of the units, or the product of three times the square of the tens, plus three times the tens into the units, plus the square of the units, multiplied by the units. By dividing this remainder by three times the square of the tens of the root, we obtain the units, or a number somewhat too large. Although it may be too large, it cannot be too small, since the remainder 32464 contains not only three times the square of the tens into the units, but three times the tens into the square of the units, plus the cube of the units. We therefore make three times the square of the tens of the root, = 75 hundreds, a trial divisor, with which we divide the 324 hundreds of the remainder, disregarding the 64 units, since they cannot form any part of the product of the square of the tens by the units. The quotient figure obtained, 5, must be the units figure of the root, or a number somewhat larger. But on undertaking to complete the divisor on the supposition that 5 is the true units figure of the root, we find a divisor too large for the remainder. We therefore take 4, a number one less, and to determine whether it expresses the real number of units in the root, we add to the 75 hundreds of the trial divisor three times the 3 tens of the root into the 4 units, plus the square of the 4 units; and multiplying the true divisor, 81116, thus formed, by the units, and subtracting the product, 32464, from the remainder, there is nothing left. Hence, 157464 is a perfect cube, and 54 its cube root.

459. RULE.—I. *Point off the given number into periods of three figures each, counting from units place toward the left and right.*

II. *Find the greatest cube that does not exceed the left-hand period, and write its root for the first figure in the required root; subtract the cube from the left-hand period, and to the remainder bring down the next period for a dividend.*

III. *At the left of the dividend write three times the square of the first figure of the root, and annex two ciphers, for a trial divisor; divide the dividend by the trial divisor, and write the quotient for a trial figure in the root.*

IV. *Add to the trial divisor three times the product of the tens figure of the root by the units figure with a cipher annexed, and the square of the last figure, for a true divisor.*

V. *Multiply the complete divisor by the trial figure; subtract the product from the dividend, and to the remainder bring down the next period for a new dividend.*

VI. *Multiply the square of the root figures already found, by 3, and to the product annex two ciphers for a new trial divisor; and proceed as before until all the periods are brought down.*

NOTE.—The observations made in Notes 1, 2, 3, 4, and 5, under the rule for the extraction of the square root (458), are equally applicable to the extraction of the cube root, except that two ciphers must be placed at the right of a true divisor when it is not contained in its corresponding dividend; and, in pointing off decimals, each period must contain three figures.

EXAMPLES FOR PRACTICE.

1. What is the cube root of 12326391?

OPERATION.

$$
\begin{array}{rrl}
 & & 1\,2\,3\,2\,6\,3\,9\,1 \mid 231 \\
2^3 = & & 8 \\
\text{Trial divisor, } 3 \times 20^2 = & 1200 & \\
3 \times 20 \times 3 = & 180 & 4326 \\
3^2 = & 9 & \\
\text{True divisor,} & 1389 \times 3 = & 4167 \\
\text{Trial divisor, } 3 \times 230^2 = & 158700 & 159391 \\
3 \times 230 \times 1 = & 690 & \\
1^2 = & 1 & \\
\text{True divisor,} & 159391 \times 1 = & 159391
\end{array}
$$

2. What is the cube root of 1331? of 3375? of 12167? of 32768? of 110592? *Ans.* 11, 15, 23, 32, etc.

3. What is the cube root of 185193? of 272144? of 456533? of 704969? of 970299? *Ans.* 57, 64, 77, 89, etc.

4. What is the cube root of 1367631? of 9938375? of 41781923? of 96071912? of 184220009? of 300763000? of 476379541? of 709732288? of 736314327? of 997002999?

Ans. 111, 215, 347, 458, 569, 670. 781, 892, 903, etc.

5. What is the cube root of 9.15? of 51662.1837824? of 11.03? of 0.518? of 0.12965? *Ans.* 2.091, 37.244, 2.22, 0.803, 0.2349.

6. What is the cube root of $34\frac{21}{64}$? of $\frac{2}{8}$? of $39\frac{38}{125}$? of $\frac{125}{343}$? of $81\frac{5}{11}$? of $166\frac{3}{8}$? of $11\frac{25}{64}$? of $\frac{216}{42875}$?

Ans. $3\frac{1}{4}$, .87+, $3\frac{2}{3}$, $\frac{5}{7}$, 4.334+, $5\frac{1}{2}$, $2\frac{1}{4}$, $\frac{6}{35}$.

APPLICATIONS IN CUBE ROOT.

1. A mason wishes to make a cubical cistern that shall contain 2744 cubic feet of water; what must be the length of one of its sides?

Ans. 14 feet.

2. A miller has a cubical box that will hold 400 bushels of wheat; what is the depth of the box?

3. What quantity of paper will be required to make a cubical box which shall contain $\frac{27}{64}$ of a solid foot? *Ans.* $\frac{3}{8}$ of a yd.

4. A carpenter has a plank 1 foot wide, $22\frac{37}{72}$ feet long, and $2\frac{1}{2}$ inches thick; and wishes to make a box whose width shall be twice its height, and whose length shall be twice its width. Required the contents of the box. *Ans.* 5719 cub. in.

5. How much must be paid for a certain number of pounds of linseed, bought at 35 cents per lb., knowing that the $\frac{2}{3}$ of the cube of the number equal 26509168?

6. A mattress-maker purchased 84 lb. of hair, for which he gave a sum such that the $\frac{1}{15}$ of the cube of the price, diminished by $\frac{7}{8}$ of the same cube equal \$0.6591. How much did the 84 lb. cost him?

Ans. \$163.80.

7. Required the value of the articles contained in 25 boxes, each containing as many articles, which cost as many cents as there are boxes? *Ans.* \$156.25.

8. What is that number, whose $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ multiplied together, give 9 for product? *Ans.* 6.

9. Bought \$164.64 worth of oranges packed up in a certain number of boxes, each containing three times as many oranges as there are boxes; and each orange costs twice as many cents as there are boxes. Required the number of boxes and oranges.

Ans. 14 boxes, 588 oranges.

10. In dividing the cube of a certain number by the $\frac{2}{3}$ of the square of the same number, we obtain $13\frac{1}{2}$ for quotient; what is this number? *Ans.* 9.

11. A reservoir, whose length is to its breadth as 13 is to 5, and depth as 13 is to 3, contains 99840 cubic feet of water; what are the dimensions of the reservoir?

Ans. length 104 ft., breadth 40 ft., depth 24 ft.

12. Some merchants formed a partnership in which each partner invested 1000 as many dollars as there were associates. Having made

a profit of $2560, they find that they have gained the half as much per cent. as there are associates. How many partners were there in the company? *Ans.* 8.

13. An inlayer bought a certain quantity of pearl-shells; by paying $1.35 per lb., and multiplying the square of the sum he laid out by the $\frac{5}{6}$ of itself, it gives a product of 59049. Required the number of lbs. he bought? *Ans.* $35\frac{5}{27}$ lb.

14. How much must a merchant pay, at 55 cents per lb., for a certain number of bales of wool, each bale containing 145 lb., the number of bales being such that in multiplying together its $\frac{3}{5}$, $\frac{5}{6}$, and $\frac{1}{4}$, the product will be 8640? *Ans.* $3828.

PROGRESSIONS.

ARITHMETICAL PROGRESSION.

460. An **Arithmetical Progression** is a series of numbers increasing or decreasing by a constant difference.

461. The **Terms** of a series are the numbers of which it is formed.

462. The **Extremes** are the first and last terms.

463. The **Means** are the intermediate terms.

464. The **Common Difference** is the number added or subtracted, in order to form each successive term.

465. An **Ascending Series** is produced by adding the common difference to each term successively; as, 1, 3, 5, 7, 9, 11, 13, 15, and 17.

466. A **Descending Series** is produced by subtracting the common difference from each term successively; as, 17, 15, 13, 11, 9, 7, 5, 3, and 1.

467. The sum of the extremes is equal to the sum of any two terms equally distant from them, or to double the middle term. Thus,

1	3	5	7	9
17	15	13	11	9
18	18	18	18	18

468. The following are the *five quantities* considered, *three* of which being given, the other *two* may be found:—

1. The first term,	denoted by	**a**.
2. The last term,	" "	**l**.
3. The common difference,	" "	**c**.
4. The number of terms,	" "	**n**.
5. The sum of all the terms,	" "	**s**.

NOTE.—Half the sum of any two numbers is called their *Arithmetical Mean*.

469. CASE I.—*Given the first term, the common difference, and the number of terms, to find the last term.*

Ex. The first term of an ascending series is 4, the common difference 3, and the number of terms 19; what is the last term?

$18 = 19 - 1$
3
54
4
58, the last term.

ANALYSIS.—The first term is 4, the second term = 4 + *once* the common difference, the third term = 4 + *twice* the common difference, etc. Therefore the last term = 4 + 18 *times* the common difference. Hence the Formula, $\mathbf{a + (n - 1)\,c = l}$, or the

470. RULE.—*To the first term add the product of the common difference and the number of terms less one.*

NOTE.—If the series is descending, subtract the product from the first term.

EXAMPLES FOR PRACTICE.

1. The first term of an ascending series is 8, the common difference 5, and the number of terms 40; what is the last term? *Ans.* 203.

2. I bought 16 acres of land, giving \$1 for the first acre, \$9 for the second, \$17 for the third, and so on; what did the last acre cost at this rate? *Ans.* \$121.

3. The first term of a descending series is 75, and the common difference 5; what is the 13th. term? *Ans.* 15.

4. A board, $2\frac{1}{2}$ inches wide at the narrow end, and 10 feet long, increases in width $1\frac{1}{2}$ inches for every foot in length. What is the width of the wide end? *Ans.* $17\frac{1}{2}$ in.

5. If the first term of an ascending series be $\frac{2}{3}$, the common difference $\frac{3}{8}$, and the number of terms 20, what is the last term? *Ans.* $7\frac{1}{24}$.

471. CASE II.—*Given the extremes and number of terms, to find the common difference.*

ANALYSIS.—Since $\mathbf{a + (n - 1)\,c = l}$, $\mathbf{c = \frac{l - a}{n - 1}}$. Hence, the

472. RULE.—*Divide the difference of the extremes by the number of terms less one.*

EXAMPLES FOR PRACTICE.

1. The first term is 3, the last term is 15, and the number of terms is 7; what is the common difference? *Ans.* 2.

2. The extremes are 2 and 17, and the number of terms is 6; what is the common difference? *Ans.* 3.

3. A man has 10 sons; the youngest is 8, and the eldest 44 years old; their ages increase in arithmetical progression. Required the difference of their ages? *Ans.* 4 years.

4. If the extremes are 0 and $2\frac{1}{2}$, and the number of terms is 18, what is the common difference? *Ans.* $\frac{5}{34}$.

473. CASE III.—*Given the extremes, and the common difference, to find the number of terms.*

ANALYSIS. — Since, $\mathbf{a + (n - 1)\,c = l}$, $\mathbf{n = \frac{l - a}{c} + 1}$. Hence, the

474. RULE.—*Divide the difference of the extremes by the common difference, and increase the quotient by* **1**.

EXAMPLES.

1. The first term is 8, the last term 203, and the common difference 5; what is the number of terms? *Ans.* 40.

2. A man going a journey travelled the first day 7 miles, the last day 51 miles, and each day increased his journey by 4 miles; how many days did he travel? *Ans.* 12.

3. The extremes are $2\frac{1}{2}$ and 40, and the common difference is $7\frac{1}{2}$; what is the number of terms? *Ans.* 6.

4. In what time can a debt be discharged, supposing the first week's payment to be \$1, and the payment of every succeeding week to increase by \$2, till the last payment shall be \$103? *Ans.* 52 weeks.

475. CASE IV.—*Given the extremes, and the number of terms, to find the sum of all the terms.*

ANALYSIS.—Since, the sum of the extremes of an arithmetical progression is equal to the sum of any two terms equally distant from them, it follows that the terms must average half the sum of the extremes. Hence, $\mathbf{s = \frac{1}{2}\,(a + l)\,n}$.

476. RULE.—*Multiply half of the sum of the extremes by the number of terms.*

EXAMPLES.

1. The extremes of an arithmetical series are 3 and 19, and the number of terms 9; what is the sum of the series? *Ans.* 99.

2. A man bought 16 acres of land, giving \$1 for the first acre, and \$121 for the last acre; the prices of the successive acres form an arithmetical progression. How much did the 16 acres cost? *Ans.* \$976.

3. A gentleman wishes to discharge a debt in 11 annual payments such that the last payment shall be \$220, and each payment greater than the preceding by \$17; what is the amount of the debt, and the first payment? *Ans.* 1st. payment, \$50.

4. A merchant bought 20 pieces of cloth, giving for the first, \$2, and for the last \$40; the prices of the pieces form an arithmetical series; how much did the cloth cost? *Ans.* \$420.

5. If 100 oranges are placed in a line, exactly 2 yards from each other, and the first 2 yards from a basket; what distance must a boy travel, starting from the basket, to gather them up singly, and return with each to the basket?

GEOMETRICAL PROGRESSION.

477. A **Geometrical Progression** is a series of numbers increasing or decreasing by a constant ratio.

478. The **Ratio** is the constant multiplier or divisor.

479. An **Ascending Series** is produced by any ratio greater than 1, as 2 4, 8, 16. 32, 64, etc.

480. A **Descending Series** is produced when the ratio is less than 1; as 1, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, $\frac{1}{32}$, $\frac{1}{64}$, etc.

481. The following are the *five quantities* considered, *three* of which being given, the other *two* may be found:—

1. The first term,	denoted by	**a**.
2. The last term,	" "	**l**.
3. The ratio,	" "	**r**.
4. The number of terms,	" "	**n**.
5. The sum of all the terms,	" "	**s**.

NOTE.—The *Geometrical Mean* between two numbers is the square root of their product.

482. CASE I.—*Given the first term, the ratio, and the number of terms, to find the last term.*

Ex. The first term of a geometrical series is 4, and the ratio is 3; what is the 9th. term?

ANALYSIS.—The first term $= 4$, and from the nature of the series,
The second term $= 4 \times 3^1$
The third term $= 4 \times 3^2$ OPERATION.
The fourth term $= 4 \times 3^3$ $4 \times 3^8 = 26244$, *Ans.*
and so on. Hence, the last term, $\mathbf{l} = \mathbf{a} \times \mathbf{r}^{n-1}$.

483. RULE.—*Multiply the first term by that power of the ratio denoted by the number of terms, less one.*

NOTE.—If the series is decreasing, consider the first term as the last, and the last as the first, the ratio will then be greater than 1.

EXAMPLES.

1. The first term of a geometrical series is 2, and the ratio 3; what is the 10th. term? *Ans.* 39366.

2. The first term of a series is 1458, the number of terms 7, and the ratio $\frac{1}{3}$; what is the last term? *Ans.* 2.

3. A woman bought 9 eggs, agreeing to pay 1 mill for the first egg, 2 mills for the second, and so on; what did the last egg cost her? *Ans.* $0.256.

4. If the first term of a series is 30, the ratio 1.06, and the number of terms 6; what is the last term? *Ans.* 40.146767328.

5. A person traveling goes 2 miles the first, 4 miles the second, 8 miles the third day, and so on, increasing in geometrical progression for 10 days. How far did he travel the last day? *Ans.* 1024 miles

6. Bought a lot of land containing 15 acres, agreeing to pay for the whole what the last acre would come to, reckoning 5 cts. for the first acre, 15 cts. for the second, and so on, in a threefold ratio. What did the lot cost me? *Ans.* $239148.45.

484. CASE II.—*Given the extremes and ratio, to find the sum of all the terms.*

Ex. The first term is 2, the last term is 128, and the ratio 4; required the sum of all the terms.

OPERATION.

$$\begin{aligned} 8 + 32 + 128 + 512 &= 4 \times \text{sum of the series.} \\ 2 + 8 + 32 + 128 &= 1 \times \text{sum of the series.} \\ \hline 512 - 2 &= 3 \times \text{sum of the series.} \end{aligned}$$

Hence $\frac{512 - 2}{3} = 170$, the sum of the series.

ANALYSIS.—Since $512 = \mathbf{lr}$, $2 = \mathbf{a}$, and $3 = \mathbf{r} - 1$, $\mathbf{s} = \dfrac{\mathbf{lr} - \mathbf{a}}{\mathbf{r} - 1}$. Hence, the

485. RULE.—*Multiply the last term by the ratio, subtract the first term from the product, and divide the remainder by the ratio less one.*

NOTES.—1. If the ratio is less than 1, the product of the last term, multiplied by the ratio, must be subtracted from the first term; and, to obtain the divisor, the ratio must be subtracted from the unity, or 1.

2. When a descending series is continued to infinity, it becomes what is called an INFINITE SERIES, whose last term must be regarded as 0, and its ratio as a fraction.

To find the sum of an Infinite Series,—*Divide the first term by a unit diminished by the fraction denoting the ratio.*

EXAMPLES.

1. The first term of a series is 4, the last term is 62500, and the ratio 5; what is the sum of all the terms? *Ans.* 78124.

2. If the first term of a series is 12, the ratio 3, and the number of terms 8; what is the sum of the series? *Ans.* 39360.

3. The first term of a decreasing series is 102, the last term 4, and the ratio $\frac{1}{3}$; what is the sum of the series? *Ans.* 151.

4. If the first term of a series is 5, the ratio $\frac{2}{3}$, and the number of terms 6; required the sum of the series. *Ans.* $13\frac{168}{243}$.

5. The first term of a decreasing series is 106, the last term 10, and the ratio $\frac{1}{5}$; required the sum of the terms. *Ans.* 130.

6. In what time will a certain debt be discharged by monthly payments in geometrical progression, if the first and last payments are $1 and $2048, and the ratio 2? *Ans.* In 12 months.

7. A young man agreed to serve in a store for 6 months. For the first month he was to receive $3, and each succeeding month's wages were to be increased by $\frac{2}{3}$ of his wages for the month next preceding; what sum did he receive for the 6 months? *Ans.* $91.95 + .

8. A gentleman wishing to purchase a piece of ground, measuring 10 square rods, thought $1 per sq. rod too high a price; he, nevertheless, agreed to give 1 cent for the first sq. rod, 4 for the second, 16 for the third, and so on, in a fourfold ratio; how much did that ground cost him? *Ans.* $3495.25.

MEASUREMENT OF LUMBER.

486. Boards are usually measured by the square foot. The board is considered to be 1 inch in thickness.

487. Planks, Beams, Joists, etc., are usually measured by board measure.

Round timber is sometimes measured by the ton, and sometimes by board measure.

488. To find the contents of a board.

RULE.—*Multiply the length of the board, taken in feet, by its width in feet, and the product will be the contents in sq. feet.* Or, *Multiply its length in feet by its width in inches, and the product divided by* 12 *will give the contents in square feet.*

NOTE.—If the board is tapering, take half the sum of the width of its ends for the width.

Ex. 1. What are the contents of a board 36 feet long, and $1\frac{1}{2}$ feet wide? *Ans.* 54 sq. feet.

2. What are the contents of a board 24 feet long, and 15 inches wide? *Ans.* 30 sq. feet.

3. What are the contents of a tapering board, 20 feet long, whose ends are, the one 24 inches, and the other 13 inches wide?

489. To find the contents of planks, beams, joists, etc.

RULE.—*Multiply the width, taken in inches, by the thickness, in inches, and this product by the length, in feet; and the last product divided by* 12 *will give the contents in feet, board measure.*

NOTE.—If the plank, beam, etc. is tapering in width, take half the sum of the width of the ends for the width; and if the taper be both of the width and the thickness, the common rule of obtaining the contents in cubic feet is, to *multiply half the sum of the areas of the two ends by the length, and divide the product by* 144.

Ex. 1. What are the contents of a plank 40 feet long, 2 feet wide, and 3 inches thick? *Ans.* 240 sq. ft.

2. How many feet are there in 9 joists, which are 15 feet long, 5 inches wide, and 3 inches thick? *Ans.* $168\frac{3}{4}$ feet.

3. How many feet in 3 beams 24 feet long, 10 inches thick, whose width tapers from 18 to 16 inches? *Ans.* 858 feet.

490. To find the contents of round timber.

RULE.—*Multiply the length, taken in feet, by the square of one fourth of the mean girth, taken in inches; and, this product, divided by* 144, *will give the contents in cubic feet.*

NOTES.—1. The girth of tapering timber is usually taken about one third the distance from the larger to the smaller end.

2. This rule is that in common use, though very far from giving the actual number of cubic feet in round lumber measured by it. 40 cubic feet, as given by the rule, are in fact equal to $50\frac{92}{100}$ true cubic feet. The following rule gives results more nearly accurate, requiring to be diminished by only one foot in 190, to give exact contents. *Multiply the square of one fifth of the mean girth, taken in inches, by twice the length, in feet; and divide by* 144.

Ex. 1. How many cubic feet in a stick of timber which is 50 feet long, and whose girth is 60 inches? *Ans.* $78\frac{1}{8}$ cub. ft.

2. What are the contents of a stick of timber whose length is 30 feet, and girth 40 inches? *Ans.* $20\frac{5}{9}$ feet.

3. How many cubic feet in a log 90 feet long, and whose circumference is 120 inches? *Ans.* $562\frac{1}{2}$ cub. feet.

NOTE.—For the culling and measuring of Timber, Masts, Spars, Deals, Staves, &c., according to the Consolidated Statutes of Canada, Cap. 46, *See page* 361.

MISCELLANEOUS EXAMPLES.

1. A man sleeps $7\frac{1}{2}$ hours each day. What per cent. of his time does he sleep? *Ans.* $31\frac{1}{4}\%$.

2. What number is that to which, if you add $\frac{2}{5}$ of $\frac{3}{11}$ of itself, the sum will be 61? *Ans.* 55.

3. A gentleman bought 95 yards of cloth, $\frac{3}{4}$ of a yard wide, for \$100, and gave the same and \$25 for cloth of the same quality, 1 yard wide. How many yards did he buy? *Ans.* $89\frac{1}{16}$ yd.

4. A father devised $\frac{7}{18}$ of his estate to one of his sons, and $\frac{7}{15}$ of the residue to the other, and the remainder to his wife. The difference of his sons' legacies was found to be £257 3 4. What money did he leave for his widow? *Ans.* £635 0 $10\frac{8}{9}$.

5. How many bricks 8 inches long, 4 inches wide, and 2 inches thick, will it take to build a wall 40 feet long, 20 feet high, and 2 feet thick? *Ans.* 43200 bricks.

6. If a man can paint 4 square yards in one hour, and is 31 h. 6 min. 40 sec. in painting two sides of a wall 7 feet high; how long is the wall? *Ans.* 80 ft.

7. By selling wheat at 12 s. 6 d. per bushel, I gain £30 on the quantity purchased; but if I sell it for 13 s. 6 d. a bushel, I shall gain £42 on the same quantity. How many bushels were bought? *Ans.* 240.

8. A grocer bought a hogshead of wine for $28.35; how much water must be added to reduce the first to 35 cts. per gal.? *Ans.* 18 gal.

9. A father, dying, left his son a legacy, $\frac{1}{4}$ of which he spent in 8 months; $\frac{3}{5}$ of the remainder lasted him 12 months longer, after which he had only $410 left. What amount did his father bequeath him? *Ans.* 956.66\frac{2}{3}$.

10. A man had $\frac{1}{3}$ of a yard of broadcloth, for which he paid at the rate of 8\frac{1}{2}$ per yard; he gave the broadcloth and 50 cents for $1\frac{1}{4}$ yards of cassimere. What did the cassimere cost him per yd.? *A.* 2.66\frac{2}{3}$.

11. How many dollars, Canada currency, are equal to $160 United States currency?

12. A grocer wishes to mix together brandy at 80 cts. a gal., wine at 70 cts., cider at 10 cts., and water, in such proportions that the mixture may be worth 50 cts. a gal.; what quantity of each must be used? *Ans.* 3 gal. of water, 2 of cider, 4 of wine, and 5 of brandy.

13. If the longitude of Boston is 70° 4' west, what will be the time in that place when it is 3 h. 35 min. A. M. in London?

Ans. 10 h. 54 min. 44 sec. P. M. of the previous day.

14. A merchant sold goods to a certain amount, on a commission of 4 %; and, having remitted the net proceeds to the owner, received $\frac{1}{2}$ % for prompt payment, which amounted to $15.60. What was his commission? *Ans.* $260.

15. I purchased railroad stock to the amount of $2356.80, and found that the sum invested was 40 % of what I had left; what sum had I at first? *Ans.* $8248.80.

16. If $13\frac{1}{2}$ bushels of wheat make 3 barrels of flour, how many bushels of wheat will be required to make 40 barrels of flour? *Ans.* 180.

17. The capital of an insurance company is $250000; its receipts for one year are $58760; its losses and expenses are $40010. What rate of dividend can it declare? *Ans.* $7\frac{1}{2}$ %.

18. By selling a lot of books for $438, a bookseller loses 10 %; how much should the books have been sold for, to gain $12\frac{1}{2}$ %?

19. What is the difference between the interest and the discount of $540 at 6 %, for 6 years 10 months?

20. I own 25 shares of $50 each in the Gosford Railroad Co., which has declared a semi-annual dividend of $3\frac{1}{2}$ %. How much do I receive? *Ans.* $43.75.

21. If 12 boarders eat $25 worth of bread in 2 mo., when flour is $9.50 per bbl.; in how many months will 15 boarders eat $60 worth of bread, when flour is $12 per bbl.? *Ans.* $3\frac{1}{25}$ mo.

22. B hired a house for one year for $300; at the end of 4 months he takes in C as a partner, and at the end of 8 months he takes in D. At the end of the year, what rent must each pay?

Ans. B 183\frac{1}{3}$; C 83\frac{1}{3}$; D 33\frac{1}{3}$.

23. A person mixed 12 cwt. of sugar at $10, with 3 cwt. at 8\frac{2}{3}$, and 8 cwt. at 7\frac{1}{2}$; how much was 1 cwt. of the mixture worth?

Ans. 8\frac{23}{23}$.

24. A shipment of wheat was insured at $2\frac{3}{5}$ %, to cover $\frac{3}{4}$ of its value; the premium paid was \$44.07; the wheat being worth 80 cts. per bushel, how many bushels were shipped? *Ans.* 2825 bush.

25. A stack of hay will keep 24 cows or 18 horses one week. How many days will it keep 5 cows and 5 horses? *Ans.* $14\frac{2}{3}$ da.

26. C, of Montreal, remits to D, of Quebec, a bill of exchange on Liverpool, the avails of which he wishes to be invested in goods on his account. D, having disposed of the bill at $7\frac{1}{2}$ % advance, received \$9675; and, having reserved for himself $\frac{1}{4}$ % on the sale of the bill, and 2 % for commission, he invests the remainder. What is the amount invested, and for how much was the bill drawn?

Ans. Investment, \$9461.58$\frac{3}{4}$; the bill was £2025.

27. What per cent. is gained by buying oil at 80 cents a gallon, and selling it at 12 cents a pint? *Ans.* 20 %.

28. A merchant pays \$10050 for a stock of goods; he sells them at an advance of $33\frac{1}{3}$ %; the expenses connected with the business are \$1750. How much does he gain? *Ans.* \$1600.

29. What o'clock is it when the time from noon is $\frac{9}{11}$ of the time to midnight? *Ans.* 5 o'cl. 24 min. P. M.

30. A merchant receives on commission three kinds of flour; from C he receives 20 bbl., from D 25 bbl., and from E 40 bbl. He finds that C's flour is 10 % better than D's, and that D's is 20 % better than E's. He sells the whole at \$6 per bbl. What in justice should each man receive? *Ans.* C receives \$139$\frac{61}{141}$; D, \$158$\frac{72}{141}$; E, \$211$\frac{132}{141}$.

31. For what sum must a note be drawn at 4 mo., that the proceeds of it, when discounted at bank, at 7 %, shall be \$875.50?

32. If $2\frac{1}{2}$ yards of merino $1\frac{2}{5}$ yards wide cost \$3.37$\frac{2}{5}$, what will be the cost of $36\frac{1}{2}$ yards $1\frac{1}{2}$ yards wide? *Ans.* \$52.779.

33. What must be the face of a note at 60 days, the proceeds of which, when discounted at Bank, at 6 %, are \$100? *Ans.* \$101.06 +

34. A merchant sold a piece of cloth for \$24, and thereby lost 25 %; what would he have gained had he sold it for \$34? *Ans.* $6\frac{1}{4}$ %.

35. A bankrupt compromises with his creditors for $37\frac{1}{2}$ %; how much will he pay on a claim of \$3656? *Ans.* \$1371.

36. A man, dying, left \$3565 to be placed at interest for his son, who was 16 yr. 5 mo. 15 da. old; how much will he receive when he is 21 years old, allowing 7 % interest? *Ans.* \$4698.37 +.

37. A garrison, consisting of 360 men, was provisioned for 6 months; but at the end of 5 months they dismissed so many of the men that the remaining provision lasted 5 months longer; how many men were sent away? *Ans.* 288.

38. What sum must I invest in the New Brunswick 6 % stock, selling at $2\frac{1}{2}$ % premium, to secure an annual income of \$840? *Ans.* \$14350.

39. A grocer divided a barrel of flour into two parts, so that the smaller contained $\frac{2}{3}$ as much as the other; how many pounds were there in each? *Ans.* $78\frac{2}{5}$ lb., $117\frac{3}{5}$ lb.

40. A sportsman spends $\frac{1}{3}$ of his time in smoking, $\frac{1}{4}$ in gunning, 2 ho. per day in loafing, and 6 ho. in eating, drinking, and sleeping; how much remains for useful purposes? *Ans.* 2 ho.

41. Exchanged 250 shares of 6 % stock, at 70 %, for stock bearing 8 %, at 120 %; what is the difference in my income? *Ans.* \$333.33$\frac{1}{3}$.

42. Purchased 100 barrels herrings, at $5 per bbl. and immediately sold them on a credit of six months. The note which I received for pay, I got discounted at the Union Bank; and, on examining my money, I found that I had gained 20 % on my purchase. What did I receive per bbl. for the herrings? *Ans.* $6.18 + .

43. How many bricks are required to build the front of a house 50 ft. 8 in. in length, 15 ft. 8 in. in height, and 1 ft. 6 in. in thickness, the dimensions of a brick being 8, 4 and 2 inches? *Ans.* 32148 bricks.

44. A woman buys apples at the rate of 5 for 2 cts., and sells them at the rate of 4 for 3 cts.; how many must she buy and sell to make a profit of $4.20? *Ans.* 1200.

45. Sent $12300 to my agent in Toronto, with which to purchase flour at $10 per bbl., after deducting his commission of 2½ %. How many barrels of flour did I receive? *Ans.* 1200.

46. Borrowed of A $150 for six months; afterwards I lent him $100; how long shall he keep it to compensate him for the use of the sum he lent me? *Ans.* 9 mo.

47. A broker charges me 1⅛ % for purchasing some uncurrent bank bills at 25 % discount; of these bills, three of $10 each, and one of $50 became worthless; I dispose of the remainder at par, and thus make $520. What was the amount of bills purchased? *Ans.* $2500.

48. A grocer mixed 5 lbs. of sugar, at 8¾ cts. per lb., with 80 lbs., at 7¾ cts. per lb., and 60 lbs. at such a price that the mixture was worth 9¾ cts. per lb. Required the price per lb. of the last kind of sugar. *Ans.* 12½ cts.

49. A gentleman's garden is 23¼ rods long, and 13½ rods wide, and is surrounded by a good fence 7½ ft. high. Now, if he shall make a walk around his garden within the fence 7$\frac{5}{12}$ ft. wide; how much will remain for cultivation? *Ans.* 1 A. 3 R. 7 p. 85$\frac{129}{176}$ ft.

50. A certain principal, at compound interest for 5 years, at 6 %, will amount to $669.113; in what time will the same principal amount to the same sum, at 6 % simple interest? *Ans.* 5 yr. 7 mo. 19 + da.

51. I invested ⅜ of my money in R. R. stock, which depreciated 6¼ %; the remainder I invested in real estate, which advanced 15 %, and thereby I gained $1500. How much did I gain in both investments? *Ans.* $250.

52. What % in advance of the cost must a merchant mark his goods, so that, after allowing 5 % of his sales for bad debts, an average credit of 6 months, and 7 % of the cost of the goods for his expenses, he may make a clear gain of 12½ % on the first cost of the goods, money being worth 6 %? *Ans.* 29.56 + %.

53. What is the greatest possible number of hills of rye that can be planted on a square acre, the hills to occupy only a mathematical point, and no two hills to be nearer than 3½ feet? *Ans.* 4165.

54. I wish to line the carpet of a room, 6½ yd. long and 5½ yd. wide, with duck, ⅞ yd. wide. How many yards of lining must I purchase, if it will shrink 4 % in length and 5 % in width? *Ans.* 43$\frac{4}{13}$ yd.

55. A man bequeathed ¼ of his estate to his son, and ⅕ of the remainder to his daughter, and the residue to his wife; the difference between his son and daughter's portion was $100; what did he give his wife? *Ans.* $600.

56. Eight workmen, laboring 7 hours a day for 15 days, were able to execute $\frac{1}{3}$ of a job; in how many days can they complete the residue, by working 9 hours a day, if 4 workmen be added to their number? *Ans.* $15\frac{5}{8}$ days.

57. Exchanged 60 Ontario bonds of \$1000 each, at $8\frac{1}{2}\%$ premium, for Nova Scotia bonds of \$200 each, at 5 % premium. How many of the latter did I receive? *Ans.* 310.

58. I lent a friend \$700, which he kept 20 months. Some years after I borrowed of him \$300; how long should I keep it to balance the favor? *Ans.* $46\frac{2}{3}$ months.

59. Bought merchandise as follows: July 3, \$35.26; July 4, \$48.65, on 30 da.; Aug. 17, \$6.48; Sept. 12, \$50. What is due on the account Oct. 12, interest at 9 %? *Ans.* \$142.60.

60. Lent a certain sum of money to A, and at the end of 3 yr. 7 mo. 20 da., I received for interest and principal \$1000; what sum did I lend? *Ans.* \$820.79 +.

61. If 12 oz. of wool make $2\frac{7}{8}$ yd. of cloth, $1\frac{1}{4}$ wide, how many pounds of wool are required to make 115 yd. of cloth 1 yd. wide? *Ans.* 24.

62. Bought goods for \$1500, $\frac{1}{3}$ payable in 3 months, $\frac{1}{3}$ in 6 months, and the remainder in 9 months. How much ready cash ought I to pay for the goods, money being worth 6 %? *Ans.* \$1456.52—.

63. Purchased a quantity of oats, April 1; May 1 its value had increased 25 %; June 1 its value was 30 % more than May 1; July 1 I sold it for 15 % less than its value June 1, receiving in payment a 6 months' note, which I got discounted at a bank, at 7 %, receiving \$12950 on it. How much was my profit on the oats? *Ans.* \$3238.52.

64. If 24.4 cubic inches of lead weigh 16 lb., required the number of feet of lead pipe that can be made from 80 lb. of lead, the caliber of the pipe to be 1 inch, and the thickness of it $\frac{1}{4}$ of an inch.

Ans. 10.35 + feet.

65. One-third of a quantity of goods was sold to gain a certain %, one-fourth to gain $1\frac{1}{2}$ times as much %, and the remainder to gain $2\frac{1}{2}$ times as much %. What is the gain % on each part, the gain upon the whole being 21 %? *Ans.* 1st., 12 %; 2nd., 18 %; 3rd., 30 %.

66. A merchant in Kingston has 5000 francs due him on account in Paris. He can draw on Paris for this amount, and negotiate the bill at $19\frac{2}{3}$ cts. per franc; or he can advise his correspondent in Paris to remit a draft on Canada, purchased with the sum due him, exchange on Canada being at the rate of 5 fr. 20 centimes per dollar. What sum will the merchant receive by each method?

Ans. By draft on Paris, \$970; by remittance from Paris, \$961.53.

67. A miller is required to grind 160 bushels of provender, worth \$1 a bushel, from oats worth \$.40, corn worth \$.80, barley worth \$.90, and rye worth \$1.10, and wheat worth \$1.30 per bushel. How many bushels of each kind may he take?

Ans. 20, 20, 20, 60, and 40, respectively.

68. How much coffee at $\$.37\frac{1}{2}$ a lb., must be given for 12 gal. 3 qt. of sirup, at \$.75 a gallon? *Ans.* $25\frac{1}{2}$ lb.

69. A servant draws off a gallon on each day, for 20 days, from a cask containing 10 gallons of wine, each time supplying the deficiency by the addition of a gallon of water; and then, to escape detection, he

again draws off 20 gal., supplying the deficiency each time by a gallon of wine. How much water still remains in the cask?

Ans. 1.0679577 gal., or more than a gallon and half a pint.

70. A merchant has $216 due him, to be paid in 7 months; but the debtor agrees to pay one-half ready money, and $\frac{3}{4}$ of the remainder in 6 months. What time should he be allowed for paying the balance? *Ans.* 3 yr. 2 mo.

71. A house that cost £3931 5 0, rents for £369 10 9; the insurance is $\frac{4}{5}$ %, and the repairs $\frac{2}{5}$ % each year. What rate of interest does it pay? *Ans.* 8 %.

72. I owe a man the following notes: one of $500, due April 1; one of $750, due July 15; and one of $1750, due Sept. 10. The holder wishes to exchange them for two notes of $1500 each, and wants one to fall due May 10; when should the other be made payable? *Ans.* Oct. 20.

73. A trader bought merchandise as follows: April 8, $150.22; May 28, $55.64, on 30 da.; June 2, $82.60, on 2 mo., and July 14, $90. What was due on the account Sept. 26, money being worth 7 %? *Ans.* $386.67.

74. By working 9 hours a day, for $15\frac{5}{8}$ days, 12 men were able to execute $\frac{3}{8}$ of a job, how many men may be withdrawn, and the residue be finished in 15 days more, if the laborers are employed only 7 hours a day? *Ans.* 4 men.

75. At a certain time between 2 and 3 o'clock, the minute-hand was between 3 and 4. Within an hour after, the hour-hand and minute-hand had exactly changed places with each other. What was the precise time when the hands were in the first position?

Ans. 2 hr. 15 min. $56\frac{22}{143}$ sec.

76. D and E traded together; D put in £100 for 512 days, and received $\frac{1}{3}$ of the gain; the number of dollars which E put in was equal to the number of days it was employed in trade. What was E's capital? *Ans.* £320.

77. If stock bought at 8 % discount will pay 7 % on the investment, at what rate should it be bought to pay 10 %? *Ans.* 35.6 % disct.

78. An importer sold cloth to a wholesale dealer at 10 % advance; the wholesale dealer sold it to a clothier at $12\frac{1}{2}$ % advance; the clothier sold it at a farther advance of 25 %, and received $1452. How much did it cost the importer? *Ans.* 938.66\frac{2}{3}$.

79. What is the difference between the interest and discount of $730, for 5 yr. 9 mo., at 8 %? *Ans.* $105.80.

80. A merchant sold $\frac{1}{4}$ of his goods at an advance of 25 %: $\frac{3}{10}$ of them at a loss of 8 %; $\frac{1}{20}$ of them at a profit of 30 %, and $\frac{1}{5}$ of them at a discount of 20 %. For what % of the cost must the remainder be sold in order to lose 5 % on the whole? *Ans.* $68\frac{1}{4}$ %.

81. I received an 8 % dividend on Montreal city railroad stock, and invested the money in the same stock at 80 %. My stock having increased to $13750, what was the amount of my dividend? *A.* $1000.

82. A tailor bought 40 yards of broadcloth, $2\frac{1}{4}$ yd. wide; but on sponging it, it shrunk in length upon every 4 yd. half a quarter.. and in width, one nail and a half upon every $1\frac{1}{2}$ yd. To line this cloth, he bought flannel 5 quarters wide, which, being wet, shrunk the whole

width on every 20 yards in length, and in width it shrunk half a nail. Required the number of yards of flannel used in lining the cloth. *Ans.* $71\frac{7}{13}$ yards.

83. Stock purchased at 5 % premium pays 6 % on the investment, what % will it pay if purchased at 15 % discount? *Ans.* $7\frac{7}{17}$ %.

84. A merchant failing in business can pay 76 cts. on a dollar. He offers, to pay his whole indebtedness without interest in 5 years if his creditors will allow him to go on with his business; his offer being accepted, how much will his creditors lose in the 5 years, money being worth 7 %? *Ans.* $.026 on a dollar.

85. Purchased a quantity of wine for $675.32\frac{1}{2}$, at 85 cents per gallon; but a part having leaked out, the remainder was sold at 40 % advance, and the original cost was realized. What quantity leaked out? *Ans.* 227 gal.

86. A owes B $600 due in 4 months, and $840 due in 6 months; B owes A $1600 due in 7 months. If A should make present payment of his debts, when ought B in justice to pay A? *Ans.* In 2mo. $10\frac{4}{7}$da.

87. How many pounds of sugar at 8, 13, and 14 cts. per pound, may be mixed with 3 lb. at $9\frac{1}{4}$ cts., 2 lb. at $8\frac{1}{2}$ cts., and 4 lb. at 14 cts. a lb., so as to gain 16 % by selling the mixture at $14\frac{1}{2}$ cts. per lb.? *Ans.* 1 lb. at 8; $8\frac{1}{2}$ lb. at 13; 8 lb. at 14.

88. What is the difference between the true and bank discount of $3000, payable in 120 days, at $8\frac{1}{2}$ %? *Ans.* $4.467 —.

89. A general, forming his army into a square, had 284 men remaining; but increasing each side by one man, he wanted 25 men to complete the square. How many men had he? *Ans.* 24000.

90. C bought a house of D, and gave him his bond for $6000, dated April 1, 1866, payable in 5 equal annual installments of $1200, the first to be paid April 1, 1867; C took up his bond April 1, 1869, semi-annual discount at the rate of 7 % per annum on the payments due after April 1, 1869, being deducted. What sum cancelled the bond? *Ans.* $3365.94 +.

91. I have a plank $42\frac{1}{4}$ feet in length, 24 inches wide, and 3 inches thick; required the side of a cubical box that can be made from it? *Ans.* 48 inches.

92. If B owes $500 due in 6 months, $400 due in 4 months, and $300 due in 7 months, and pays $\frac{2}{3}$ of the whole in 3 months, when ought the remainder to be paid? *Ans.* In $10\frac{3}{4}$ mo.

93. A wholesale merchant sent a quantity of goods into the country to be sold at auction, on a commission of $4\frac{1}{2}$ %. What amount of goods must be sold, that his agent may buy produce with the avails to the amount of $1910, after retaining a commission of 2 %? *Ans.* $2040.

94. If the annual rent of 23 A. 1 R. 27 per. of land be $187.35, how much will be the rent of 71 A. 20 per.? *Ans.* $569.

95. A Halifax merchant shipped 1000 barrels of salmon to his agent in New Orleans, directing him to sell it, and invest the proceeds in cotton; his agent sold the salmon at $14 per bbl., paid $274 charges, and bought cotton at $.65 per lb., charging 3 % commission for selling the salmon and 5 % for buying the cotton. How many pounds of cotton did he buy? *Ans.* 19495.9 + lb.

96. A man owes a debt to be paid in 4 equal installments at 4, 9, 12, and 20 months, respectively; discount being allowed at 5 %. he finds that $750 ready money will pay the debt; how much did he owe? *Ans.* $784.74 +.

97. D's money was to E's as 2 to 3; when D had spent $40, and E had spent 40 % more than D, D's money, minus $20, was to E's money, plus $2, as 4 to 9. How much had each at first?

Ans. D, $108; E, $162.

98. What is the cost of a 90 days' bill on Montreal, to the amount of $1000, at $\frac{3}{8}$ % premium, and int. off at 6 %? *Ans.* $990.75.

99. Three men engaged in the lumber trade; A furnished $4000, and B $6000; they gained $1680, of which C's share was $840. Required C's stock and A's and B's gain.

Ans. C's stock, $10000; A's gain, $336; B's $504.

100. A man having lost $\frac{2}{3}$ of his money, found he had remaining only $672; how much had he at first? *Ans.* $1792.

101. A speculator invested a certain amount in railroad stocks. By selling these stocks at a deteriorated price he lost $\frac{2}{5}$ of his investment; by investing the remainder he cleared $240, and afterward lost $\frac{2}{3}$ of the money he had remaining, which left him possessed of $480; how much did he invest? *Ans.* $3600.

102. Bought a certain number of horses for $2600; had I bought 8 more at $10 less each, all would have cost $3360; how many horses did I buy? *Ans.* 20.

103. Louis can do a piece of work in 8 days, and John in 12 days; in how many days can both do it? *Ans.* $4\frac{4}{5}$ days.

104. A grocer bought 11 bushels of chestnuts at $3 a bushel, and retailed them at 3 cents a half pint. What per cent. profit was his gain? *Ans.* 28 %.

105. The head of a fish is 12 inches long, the tail is as long as the head + $\frac{1}{2}$ of the body, and the body is as long as the head and tail together; what is the length of the fish? *Ans.* 96 inches.

106. A consignor sends 500 barrels of flour to a commission merchant, with instructions to sell it and remit the net proceeds by draft. The consignee pays $120.40 for freight and expenses, sells the flour at $8.40 per bbl., charges $2\frac{1}{2}$ % commission, and pays $\frac{3}{8}$ % premium for draft; how much does the consignor receive? *Ans.* $3959.75.

107. How many horses could be kept on 25 acres of land, if for every 3 horses there is of the 25 acres, 1 acre of plowed land, and for every 2 horses, 1 acre of pasture? *Ans.* 30 horses.

108. Purchased 240 bushels of oats at the rate of 18 bushels for $22.50, and sold it at the rate of $22\frac{1}{2}$ bu. for $33\frac{3}{4}$; how much did I gain on the whole? *Ans.* $60.

109. I paid £93 15 0, at the rate of $2\frac{1}{2}$ %, for insurance on a shoe factory; for what amount was the policy given?

110. Exchanged 75 railroad bonds of $500 each, at 36 % below par, for bank stock at 5 % premium, how many shares of $100 each did I receive? *Ans.* $228\frac{4}{7}$.

111. Invested £858 in Government bonds at 106 %, paying $1\frac{1}{4}$ % brokerage, and afterward sold the stock at 12 % premium, brokerage $1\frac{1}{2}$ %. What was my gain? *Ans.* £26.

112. The longitude of Paris is 2° 20' 22'' E., and of Constantinople, 8° 59' E. When it is 1 A. M. at the latter place, what time is it at the former? *Ans.* 33 min. $25\frac{7}{15}$ sec. past midnight.

113. Having placed a bill of $775 in the hands of a collector, who succeeded in obtaining 75 % of it, and charged 8 % commission, how much did I receive?

114. Suppose that the earnings of the Grand Trunk R. R. for December 1870 were $472240, which was an increase of $11\frac{1}{9}$ % over the earnings for the same month in 1869. How much was the increase? *Ans.* $47224.

115. In a cask containing brandy and water, $\frac{2}{3}$ of the whole + 3 gal. is brandy, and $\frac{1}{4}$ of the whole + 2 gal. is water; required the number of gal. of each. *Ans.* 43 gal. brandy, 17 gal. water.

116. Hamel, Perry, Lane, and Garneau are partners; Hamel takes $\frac{1}{3}$ of the gains or losses; Perry $\frac{1}{4}$, Lane $\frac{1}{5}$, and Garneau the remainder. At the close of the year, the resources of the firm are: Cash $10312.50, Merchandise $13447.50, Bonds and Mortgages $11470, Bank Stock $4500; Hamel has drawn from the business $900, Perry $525, and Lane $285; the liabilities are: Notes outstanding $5460; Balance in favor of Ross & Co., $1120; Balance in favor of J. L. Murphy, $3967.50; Hamel invested $9547.50, Perry $7905, Lane $6270, and Garneau $3480. What is each partner's *interest* in the business at the close of the year? *Ans.* Hamel, $9877.50; Perry, $8302.50; Lane, $6723; Garneau, $4279.50.

117. What is the difference in cost between a draft on Toronto of $17302.80, at $1\frac{1}{4}$ % premium, and one on St. John, N. B., for the same amount, at $\frac{1}{2}$ % discount? *Ans.* $302.80.

118. A mechanic received $3.75 a day for his labor, and paid $1.25 a day for his board; at the expiration of 100 days he had saved $200; how many days did he work? *Ans.* $86\frac{2}{3}$ days.

119. For two successive years, a merchant annually contributed $100 for charitable purposes, and added yearly to that part of his capital not thus expended, a sum equal to its half; at the end of the second year his capital was doubled. Required his capital. *Ans.* $1500.

120. A merchant in Halifax purchased 350 bales of cotton, each containing 450 pounds, at $.80 a lb., and shipped them to Liverpool at a cost of 16 % for freight and duties. How much in Canada currency did he gain by selling them at 2s. 10d. sterling per lb., rate of exchange 171 %? *Ans.* $23415.

121. A piece of merino cost $.80 per yard; at what price shall it be marked, that the merchant may sell it at 10 % less than the marked price, and still make 20 % profit? *Ans.* 1.06\frac{2}{3}$.

122. A merchant in Quebec gave $2000 for a bill of exchange of £400 to remit to London; what was the rate in favor of England?

123. What yearly debt can be discharged by monthly payments, the first being $2, the second $6, and the third $18, and so on, in geometrical progression? *Ans.* $531440.

124. A farmer sold one hog, weighing 250 lb., at 4 cts. per lb.; a second, weighing 300 lb., at $4\frac{1}{2}$ cts.; and a third, weighing 369 lb., at 5 cts.; what was the average price per lb. for the whole? *A.* $4\frac{51}{119}$ cts.

125. John's age is 4 times Mary's, but in 12 years John's age will be only $2\frac{1}{2}$ times Mary's; required the age of each.

Ans. Mary's age is 12 yr.; John's is 48.

126. A company of 50 men drank wine at 2s. 6d. per bottle, to the amount of £10. How many men at the same rate will £18 worth of wine supply, when wine is worth 2s. 3d. per bottle? *Ans.* 100 men.

127. The sales of a clothing house amount to $100000 a year; $\frac{1}{4}$ of the sales are made at a profit of 25 %, $\frac{6}{10}$ at a profit of 20 %, and the remainder at a loss of 4 %. Required the cost of the goods. *Ans.* $88750.

128. A merchant in Toronto purchased a draft on Quebec for $2660, drawn at 60 days, paying $2570.89. What was the course of exchange? *Ans.* $2\frac{1}{2}$ % disct.

129. A man gave $\frac{1}{3}$ of his estate to his wife, $\frac{1}{5}$ of the remainder to his oldest son, $\frac{1}{4}$ of the residue to his oldest daughter, and $\frac{1}{2}$ of what then remained, which was $1500, was to be equally distributed among his other children, who received $150 each; required the number of his children, and the value of his estate.

Ans. 12 children; $10000.

130. A merchant by selling a lot of goods for $438, loses 10 %; how much should the goods have been sold for, to gain $12\frac{1}{2}$ %?

131. An agent received $65 for collecting a debt of $1300. What was the rate of his commission?

132. A merchant marked a piece of goods 25 % above the cost, but its season passing, he determined to sell it 20 % below the marked price, supposing he should make 5 %. Did he make or lose?

133. A man can sell his farm for $4000 cash, or for $5000, payable in 2 years. If he accept the last offer, and received instead its present worth, at 10 % discount; how much more would he receive than by the former? *Ans.* 166.66\frac{2}{3}$.

134. A laborer worked 3 months, 25 days each month, 10 hours each day, for $.08 an hour, and received in payment 2 loads of grain, each containing 15 bags of $2\frac{1}{2}$ bushels each. What was the price of the grain per bushel? *Ans.* $0.80.

135. Sold goods to the amount $348.25, taking in payment, April 6, a promissory note for sixty days, which I indorsed and had discounted at the bank, April 20, at 7 %; how much cash did I receive? *Ans.* $344.93 +.

136. Suppose bank stock is purchased at 28 % premium, and the bank declares a dividend of 9 % per share, what % is that on the cost price of the stock? *Ans.* $7\frac{1}{32}$ %.

137. A person, wishing to buy wheat with the proceeds of cotton, sends to his agent 32 bales, each weighing 380 lb. The agent sells the cotton at 26 cts. per lb., for which he charges $2\frac{1}{4}$ % commission; pays for freight and charges, $34.60; and expends the remainder, less his commissions for selling and buying, in wheat at 85 cts. per bushel, for which he charges $1\frac{1}{2}$ % commission; how much wheat is obtained through this factor? *Ans.* 3542 + bu.

138. A pole 63 feet long, in falling, was broken into two pieces; the shorter piece being $\frac{2}{5}$ of the longer; what is the length of each piece? *Ans.* 18 and 45 ft.

139. A farmer had a dairy of 48 cows, each furnishing 18 qt. of milk a day, from which he made 40 tubs of butter of 60 lb. each in 30 days. He made a contract to deliver 100 tubs of 96 pounds each in 80 days. How many cows must he add to his dairy provided each additional cow furnish 4 gallons of milk daily? *Ans.* 27.

140. In what time will \$3045.20 gain \$190.32$\frac{1}{2}$ if the gain of \$2494.75 for 1 yr. 13 da., is \$258.48, and what is the rate per annum? *Ans.* 7 mo. 15 da.; rate 10 %.

141. Andrews, Baker, and Childs entered into partnership. Andrews put in £3000, Baker £2000, and Childs £1750. At the end of the first year Andrews drew out £500, Baker £250, and Childs put in £750. At the close of the second year, Andrews and Baker each drew out £250, and Childs put in £500 more. At the end of the third year they dissolved partnership, and found that their joint property was £7125. What was each partner's share? *Ans.* Andrews', £2393 10 4$\frac{4}{9}$; Baker's, £1597 4 5$\frac{1}{3}$; Childs', £3134 5 2$\frac{2}{9}$.

142. If I buy 50 shares Grand Trunk railroad stock at 141 %, and 50 shares Canada Central railroad stock at 139 %, the former paying a semi-annual dividend of 4$\frac{1}{10}$ %, the latter of 5 %; what rate of interest shall I realize on my investment? *Ans.* 6$\frac{1}{2}$ %.

143. What is the cost of a bill on London for £800 17 6 sterling, when the rate of exchange is 9$\frac{7}{8}$ % premium?

144. J. Sheridan bought of L. H. Miles & Co., the following bills of goods; Nov. 1, 1870, a bill of \$750, on 6 mo. credit; Dec. 15, 1870, a bill of \$300, on 5 mo.; Jan. 1, 1871, a bill of \$425, on 4 mo.; Feb. 5, 1871, a bill of \$275, on 2 mo. What sum would settle the account, May 29, 1871, interest at 7 %? *Ans.* \$1760.10.

145. When exchange on England is at 10 % premium, and freight at 1s. 3d. sterling per Winchester bushel, how much can be paid in Montreal for wheat per bushel, in answering an order from London limited to £3 10 per Imperial quarter?

146. The duty on an invoice of 300 dozen London porter, at 30 %, was \$190.512; breakage, 2 %. Required the invoiced price per dozen. *Ans.* \$2.16.

147. Three merchants have an interest in a steam vessel; A puts in \$960 for 6 months; B, a sum unknown, for 12 months; C, \$640 for a time not known when the accounts were settled; A received \$1200 for his share, stock and profit; B, \$2400 for his, and C, \$1040 for his. What was B's stock, and C's time?

Ans. B's stock, \$1600; C's time, 15 mo.

148. Merrill, Wells and Roche were partners in the grain business; Merrill had invested $\frac{1}{3}$, Wells $\frac{1}{3}$, and Roche $\frac{7}{15}$ of the capital. They were to share equally the gains or losses. The business not being successful, the partnership was dissolved at the close of the year, when the resources of the firm were found to be: Cash, \$1785; barley on hand, \$2500; corn, \$1752; rye, \$350; oats, \$1650; wheat, \$5000. The liabilities were: Notes outstanding, \$1562; they owed S. Myler, \$1200, and P. Riley, \$1875. The net losses were \$750. What was the net capital of the firm at commencing, and what was each partner's net capital?

ACCOUNTS OF STORAGE.

491. Storage is the price charged for the safe keeping of goods in a store or warehouse.

There is no uniform method of computing storage. The Chambers of Commerce of the different cities, adopt such rules and rates for storage as they deem equitable. The charges for storage are usually, however, a certain rate per month for each box, bale, cask, etc.

NOTES.—1. When goods are withdrawn before the close of the month, no deduction is made, but storage is charged for the full month. After the first month, for a part of a month less than one half, charge is made for a month. In some cities, all fractional parts of a month are considered full months.

2. If, however, goods are received and sold on account, as in the commission business, or are received and delivered at the place of the consignor, an account is kept, showing the date and number of boxes, etc., received, and the date and number sold or delivered. In computing the storage on such an account it is customary to average the time, and charge a certain rate per month of 30 days. If there is a fractional part of a barrel, etc., in the average, it is treated as in the case of parts of months above.

E.x. What will be the cost for the storage of flour at 6 cents per barrel, which was received and delivered as follows: Received May 1, 1871, 1000 barrels; May 26, 2000 barrels. Delivered May 16, 500 barrels; June 1, 1000 barrels; June 12, 1100 bbl.; July 2, 400 bbl.

OPERATION.

1871.	bbl.		da.		prod.
May 1, Rec.	1000	×	15	=	15000
" 16, Deliv.	500				
Bal.	500	×	10	=	5000
" 26, Rec.	2000				
Bal.	2500	×	5	=	12500
June 1, Deliv.	1000				
Bal.	1500	×	11	=	16500
" 12, Deliv.	1100				
Bal.	400	×	20	=	8000
July 2, Deliv.	400		30)		57000
Chargeable for 1 month,					1900

1900 bbl. × .06 = $114, cost of storage.

ANALYSIS. — The storage of 1000 bbl. for 15 da., + 500 bbl. for 10 da., + 2500 bbl. for 5 da., + 1500 bbl. for 11 da., + 400 bbl. for 20 da., is the same as the storage of 57000 bbl. for 1 da., or 1900 bbl. for a month of 30 days. And the storage of 1900 bbl. at 6 cts. each = $114, *Ans.*

492. RULE.—*Commencing with the first date and ending with the last, multiply the number of barrels, or other articles in store, from each date to the one* NEXT *following it, by the number of days between these dates. Divide the sum of the several products by* 30, *and the quotient will be the number of articles stored for one month, and this number multiplied by the rate of storage for each article will give the amount of storage charged.*

EXAMPLES FOR PRACTICE.

1. What will be the cost of storing salt at 2 cts. per barrel, received and delivered as follows: June 6, 1871, 120 bbl.; June 16, 140 bbl.; June 26, 600 bbl.; July 5, 300 bbl.; July 15, 180 bbl.; July 20, 160 bbl. All delivered Aug. 1. *Ans.* $21.44.

2. What will be the storage of flour at 5 cents per bbl. per month, received and delivered as follows?

Received July 1, 1871, 400 bbl.; July 15, 350 bbl.; July 26, 450 bbl. Delivered, July 12, 200 bbl.; July 20, 400 bbl.; Aug. 1, 200 bbl.; and Aug. 8, 400 bbl. *Ans.* $25.10.

3. Received, and delivered, on account of James O'Neil, sundry bales of cotton, as follows: Received, May 1, 1871, 1848 bales; May 16, 96 bales; June 1, 240 bales. Delivered, June 12, 800 bales; July 1, 480 bales; Aug. 3, 320 bales; Aug. 10, 250 bales. Required the number of bales remaining in store on September 1, and the cost of storage up to that date, at the rate of 5 cents a bale per month.

Ans. In store, 334; cost, $240.75.

4. Received, July 3, 1871, 256 casks of wine, on storage, and on July 15, 381 more were added; July 18, delivered 261, and July 26, 312 casks; July 30, received 321 casks, and Aug. 8, 163 casks; delivered, Aug. 16, 208 casks, Aug. 18, 103 casks, and Aug. 19, 115 casks; received, Sept. 1, 320 casks, Sept. 2, 106 casks, Sept. 7, 342 casks; delivered, Sept. 12, 250 casks, Sept. 18, 321 casks, Sept. 21, 133 casks, and the balance, Sept. 27. What was the cost for the storage of the above, the charge being 6 cents per cask monthly?

GENERAL AVERAGE.

493. General Average is the process of computing the loss to be sustained by the owners of the ship, cargo, and freight, respectively,—when, owing to common peril at sea, any portion of the property has been damaged or destroyed for the common safety.

494. Jetson is the portion of the cargo or of the equipment of the vessel thrown overboard.

495. The **Contributory Interests** are the three kinds of property which are taxed to cover the loss. These are, 1st. the vessel, at its value before the loss; 2nd. the cargo, including the part sacrificed; 3rd. the freight, less $\frac{1}{3}$ as an allowance for seamen's wages.

496. The loss which is subject to general average includes, 1st. Jetson; 2nd. Repairs to the vessel; 3rd. Expense of detention to which the vessel is subject in port.

NOTES.—1. The goods, whether saved, injured, or destroyed, are estimated at their value at the port of destination, except when the adjustment of the general average is made at the port of lading.

2. Only ⅔ of the cost of repairs to the vessel is allowed, as it is assumed that the new work and material are worth ⅓ more than the old.

Ex. The ship "Armeda", on a voyage from Calcutta to Quebec with a cargo of silk, teas, etc., valued at $59215, sprung a leak in a gale, and the captain was compelled to throw overboard a portion of her cargo valued at $6375 to save the vessel and the remainder of the cargo. The vessel was valued at $75000; the freight was $9600; and of the value of the cargo $17560 belonged to P. N. Garneau, $11600 to J. Benson & Co., $8500 to Ross & Timms, and the remaining $21555 to Murphy & Field. Of the cargo thrown overboard $2156 belonged to P. N. Garneau, $1560 to J. Benson & Co., $895 to Ross and Timms, and $1770 to Murphy and Field. The necessary repairs of the vessel were made at Good Hope, costing $750; and the expenses of the detention at that port were $155.75. How should the loss be distributed among the owners of the several contributory interests?

OPERATION.

LOSSES.		CONTRIBUTORY INTERESTS.	
Jetson	$6375.00	Vessel	$ 75000.00
⅔ cost of repairs	500.00	Cargo	59215.00
Cost of detention	155.75	⅔ freight	6400.00
Total	$7030.75	Total	$140615.00

$7030.75 ÷ $140615 = .05, rate per cent. of loss.

$75000 × .05 = $3750.00, amount payable by vessel.
59215 × .05 = 2960.75, " " " cargo.
6400 × .05 = 320.00, " " " freight.

Total contribution $7030.75, to be distributed as follows:

$75000 × .05 = $3750.00, amount payable by vessel.
6400 × .05 = 320.00, " " " freight.
17560 × .05 = 878.00, " " " P. N. Garneau.
11600 × .05 = 580.00, " " " J. Benson & Co.
8500 × .05 = 425.00, " " " Ross & Timms.
21555 × .05 = 1077.75, " " " Murphy & Field.

From the amount payable by the vessel must be deducted $500 + $155.75 = $655.75, the cost of repairs, less ⅓, and the expenses of detention. Hence, the net amount that the vessel must contribute to the general loss is $3750 — $655.75 = $3094.25. So each of the other owners of contributory interests must have his loss deducted from the amount of his payment. Hence,

$3750 — $655.75 = $3094.25, balance payable by vessel.
320 320.00, " " " freight.
2150 — 878.00 = 1272.00, " " to P. N. Garneau.
1560 — 580.00 = 980.00, " " " J. Benson & Co.
895 — 425.00 = 470.00, " " " Ross & Timms.
1770 — 1077.75 = 692.25, " " " Murphy & Field.

From the analysis of this example we deduce the

497. RULE.—I. *Divide the entire loss by the sum of the contributory interests; the quotient will be the loss per cent.*

II. *Multiply each contributory interest by the loss per cent.; the product will be the amount of its contribution to the general loss.*

III. *The difference between the loss of each contributory interest and the amount of its contribution will be the balance to be paid by it if its contribution exceeds its loss, and the amount to be received by it if its loss exceeds its contribution.*

EXAMPLES FOR PRACTICE.

1. The ship Nestor, in her passage from Antwerp to Quebec, was crippled in a storm, in consequence of which the captain had $4300 worth of the cargo thrown overboard, and put into port for the necessary repairs, which cost $1260. The charges for board of seamen, pilotage, and dockage, amounted to $170.40. The contributory interests were: vessel, $37800; gross amount of freight $4992; cargo shipped by S. Keller & Co., $2574; by Shiller & Morse, $1752; by Krauss & Herr, $1152; by Lebrun & Co., $804; and by Ross & Daller, $1200. In adjusting the general average in Quebec, the deduction made from the gross amount of freight on account of seamen's wages was one third. Required the several shares of the general loss.

2. A vessel valued at $35000, having been disabled in a storm, entered port, and was refitted at an expense of $337.50 for repairs, and $150 for board of seamen, pilotage, and dockage. Of the cargo, valued at $6250, $3000 belonged to A, $2312.50 to B, and $937.50 to C; and the amount sacrificed for the ship's safety was $1750 of A's property, and $212.50 of B's; the gross charges for freight were $1875. Required the balance, payable or receivable, by each of the parties, the loss being apportioned by general average.

Ans. { $1618.75 payable by ship owners; $1585, receivable by A;
$51.56¼ " " C; $85.31¼ " " B.

AVERAGING OF ACCOUNTS.

498. Averaging of Accounts (also called "Equation of Accounts," and "Compound Equation of Payments") is the process of finding the equated time for the payment of the balance of an account that contains both **debits** and **credits.**

The debit and credit sides of an account being respectively equivalent to the sum of their several items, due at the *equated time*, the *first step* in equating accounts is to find the time when each side of the account becomes due.

This may be found by equating each side of the account, *without any reference to the other*, commencing either at the *first* or the *last* date of each, or by using the *first* or *last* date of the account as a common *starting-point* for both sides.

The solution of the following example will sufficiently illustrate these two methods of equating the debit and credit sides of an account.

In the following solution we have commenced at the first date and discounted.

Dr. R. Seeley & Co. in account with L. N. Thompson. *Cr.*

1871			Time of credit.	1871		
April 3	To Mdse.	$220	3 months.	July 1	By Cash	$200 00
May 1	"	125	5 "	Oct. 3	"	150 00
" 15	"	200	6 "	Dec. 20	"	300 00
June 24	"	140	8 "			
July 1	"	190	9 "			

FIRST METHOD.

Debits.

Due,				
1871				
July 3,	$220	× 00	=	
Oct. 1,	125	× 90	=	11250
Nov. 15,	200	× 135	=	27000
1872				
Feb. 24,	140	× 236	=	33040
April 1,	190	× 272	=	51680
	$875		)	122970
				141 da.

Debits are due 141 days from July 3, which is Nov. 21.

Credits.

Due,				
1871				
July 1,	$200	× 00	=	
Oct. 3,	150	× 94	=	14100
Dec. 20,	300	× 172	=	51600
	$650		)	65700
				101 da.

Credits are due 101 days from July 1, which is Oct. 10.

The above account thus equated will stand as follows:

Dr.

Due, Nov. 21, 1871, $875.

Cr.

Due, Oct. 10, 1871, $650.

Or thus:

Debits.

Due,				
1871				
July 3,	$220	× 2	=	440
Oct. 1,	125	× 92	=	11500
Nov. 15,	200	× 137	=	27400
1872				
Feb. 24,	140	× 238	=	33320
April 1,	190	× 274	=	52060
	$875		)	124720
				143 da.

Debits due 143 days from July 1, which is Nov. 21.

Credits.

Due,				
1871				
July 1,	$200	× 00	=	
Oct. 3,	150	× 94	=	14100
Dec. 20,	300	× 172	=	51600
	$650		)	65700
				101 da.

Credits due 101 days from July 1, which is Oct. 10.

The account thus equated stands as before:

Dr.	*Cr.*
Due, Nov. 21, $875.	Due, Oct. 10, $650.

NOTE.—In the above operation, we start from the earliest date upon which any item of either side of the account *becomes due.*

The next step is to find when the balance of the account, as thus equated, becomes due.

Debits, $875
Credits, 650

Balance, $225

$(650 \times 42) \div 225 = 121$ days.

Difference in time 42 days.

Or thus, by Discount:

$6.50

3.25, dis. for 30 days.
1.30, " " 12 "

$4.55 ÷ .0375 (dis. of $225 for 1 da.) = 121 days.

$4.55, " " 42 days.

The balance is due 121 days from Nov. 21, 1871, which is March 22, 1872.

EXPLANATION.—Assume the account settled Nov. 21, the *latest* date. The credit side of the account has been due from Oct. 10 to Nov. 21, or 42 days. Nov. 21, the credit side, is equal to $650, and the interest of the same 42 days. That the debit side of the account may be increased by an equal amount of interest, it is evident that the balance of the account must remain unpaid 121 days, or the 121 days must be counted *forward* from Nov. 21. *Or thus:*

The above account may be stated as follows: Oct. 10, 1871, L. N. Thompson paid R. Seeley & Co. $650; Nov. 21, 1871, R. Seeley & Co. paid L. N. Thompson $875. Now, since R. S. & Co. had the use of $650 for 42 days L. N. T. is entitled to the use of $225 (the balance) until its interest equals the interest of $650 for 42 days, which is 121 days. 121 days from Nov. 21, 1871, is March 21, 1872.

PROOF.

Dr.		*Cr.*	
Due, Nov. 21,	$875.00	Due, Oct. 10	$650.00
Int. to March 21, 1872	17.65	Int. to March 21, 1872,	17.65
		Balance,	225.00
	$892.65		$892.65

Suppose the debit and credit side of the above account, when equated, to stand as follows:

Dr.	*Cr.*
Due, Nov. 21, 1871, $650.	Due Oct. 10, 1871, $875.

What is the equated time for the payment of the balance?

Credits,	$875	$(875 \times 42) \div 225 = 163$ days.
Debits,	650	Balance due 163 days *previous* to
		Nov. 21, 1871, which is June 11, 1871.
Balance,	$225	

Difference in time, 42 days.

EXPLANATION.—Suppose the account settled Nov. 21. The credit side is equal to $875, and its interest from Oct. 10, to Nov. 21, or 42 days. That the debit side of the account may be increased by an equal amount of interest, the balance of the account must be regarded as due 163 days *previous* to Nov. 21, or June 11. *Or thus:*

Oct. 10, 1871, L. N. Thompson paid R. Seeley & Co. $875; Nov. 21, 1871, R. S. & Co. paid L. N. T $650. Since R. S. & Co. had the use of $875 for 42 days, L. N. T. is entitled to the interest of $225 (the balance) for 163 days. Hence, the balance must be regarded as due 163 days *previous* to Nov. 21. The simple question is: How long must $225 be on interest to equal the interest of $875 for 42 days.

NOTE.—If R. Seeley & Co. should wish to give their note for the balance, it is evident the note must be dated June 11, 1871.

499. RULE.—*First find the equated time for each side of the account without any reference to the other. Then multiply the side of the account which falls due* FIRST *by the number of days between the dates of equated time, and divide the product by the balance of the account. The quotient will be the number of days to be counted* FORWARD *from the* LATEST DATE *when the* SMALLER *side of the account falls due* FIRST; *and* BACKWARD *when the* LARGER *side falls due* FIRST.

NOTE.—Some authors give the following RULE:—*Multiply the* smaller *side of the account by the number of days between the dates of equated time, and divide the product by the balance of the account. The quotient will be the time for consideration. From the equated date of the* larger *side, count* FORWARD *when that side becomes due* last, *but* BACKWARD *when it becomes due* first.

ANOTHER METHOD.

500. The equated time for the payment of the balance of an account may be found *directly* without first averaging the debit and credit items, by the following method:

Due,				Due,			
1871				1871			
July 3,	$220 ×	2 =	440	July 1,	$200 ×	0 =	
Oct. 1,	125 ×	92 =	11500	Oct. 3,	150 ×	94 =	14100
Nov. 15,	200 ×	137 =	27400	Dec. 20,	300 ×	172 =	51600
1872							
Feb. 24,	140 ×	238 =	33320		$650		65700
April 1,	190 ×	274 =	52060				
	$875		124720		59020 ÷ 225 = 262.		
	650		65700				
	$225		59020	262 days from July 1, 1871, is March 21, 1872.			

EXPLANATION.—We assume July 1, 1871 (the earliest date upon which any item becomes due), as the time upon which *all* the items of the account become due. The interest of the debit items, from this assumed date of maturity to the time they respectively become due, equals the interest of $1 for 124720 days; the interest of the credit items equals the interest of $1 for 65700 days. Hence, the balance of interest in favor of the debit side equals the interest of $1 for 59020 days, or $225 for $\frac{1}{225}$ of 59020 days = 262 days. Since the balance of items is also in favor of the debit side, it is evident it can remain unpaid 262 da. without interest, or will become due 262 days from July 1, 1871, which is March 21, 1872. If the balance of interest had been on the credit side, it would have been due 262 days *previous* to July 1, 1871.

501. RULE.—I. *Assume the earliest date upon which any item of the account becomes due to be the time of maturity for all the items.*

II. *Multiply each item by the number of days intervening between this assumed date and the date upon which it becomes due, and find the sum of these products on each side of the account. Then divide the* DIFFERENCE *between the sums of the debit and credit products by the balance of the account; the quotient will be the time for consideration.*

III. *When the difference of products and the balance of the account fall on the* SAME *side, count* FORWARD; *when on opposite sides, count* BACKWARD.

NOTES.—1. The *latest* date may be used as a starting-point.

2. In finding the equated time, when the cents, if any, are less than 50, reject them; when more, add $1. The work will be sufficiently accurate.

EXAMPLES FOR PRACTICE.

1. J. Murphy has with C. Duval an account, which, when each side is equated, stands as follows:

Dr.	*Cr.*
Due, Sept. 5, $1542.	Due, Sept. 24, $1296.

What is the equated time of payment for the bal.? *Ans.* May 28.

2. L. N. Carroll has with Simms & Norris an account, the debit and credit sides of which, when equated, are as follows:

Dr.	*Cr.*
Due, Feb. 8, $650.	Due, Feb. 12, $2180.

What must be the date of a note for the balance? *Ans.* Feb. 14.

3. What is the equated time for the payment of the balance of an account, which, when the two sides are equated, stands as follows:

Dr.	*Cr.*
Due, June 12, $540.	Due, Aug 1, $960.

Ans. Oct. 4.

4. At what time will the balance of the following account commence drawing interest?

Dr.	*Cr.*
Due, Oct. 20, $2520.	Due, Nov. 25, $1800.

Ans. July 22.

. What is the balance of the following account, and when is it due?

Dr. JOHN WOODLEY. *Cr.*

1871				1871			
April 10	To Mdse., on 2 mo.,	$ 450	00	April 25	By Cash	$615	00
May 20	" Cash	300	00	June 10	" "	180	00
June 15	" Mdse.,	1200	00	July 20	" Mdse.	540	00

Ans. Balance, $615; due June 20, 1871.

6. At what time did the balance of the following account become due, allowing that each item drew interest from its date?

Dr. C. RYAN IN ACCT. WITH N. MILLER & Co. *Cr.*

1871				1871			
Feb. 22	To Mdse.	$ 44	70	Feb. 4	By Mdse.	$38	50
" 24	" "	38	00	May 16	" "	20	00
March 20	" "	12	50	June 14	" "	76	60
" 23	" "	105	00				
May 4	" "	94	30				
June 21	" "	15	00				

Ans. 25 days back of April 1st. = March 7.

7. Required the balance of the following account, and when it is due.

Dr. A. E. ROY IN ACCT. WITH T. LYONS & Co. *Cr.*

1871			1871		
March 14	To Mdse. on 6 mo.	$560	June 10	By Mdse. on 2 mo.	$600
April 20	" " " "	650	Aug. 5	" Cash	400
May 10	" Cash	540	Sept. 20	" Mdse. on 1 mo.	300
June 15	" "	350	Nov. 20	" Cash	300

Due 208 days prior to Nov. 20, 1871, or on April 26, 1871.

8. What is the equated time for the payment of the balance of the following account, the merchandise items having a credit of 4 mo.?

Dr. S. THOMAS & SON IN ACCT. WITH R. HILL. *Cr.*

1871				1872			
May 11	To Mdse.	$680	56	Jan. 11	By Cash.	$400	00
June 16	" "	272	60	" 29	" "	352	00
July 13	" "	144	20	Feb. 11	" "	80	00
Aug. 23	" "	400	00	" 25	" "	784	00
" 30	" "	272	32				
Sept. 9	" "	64	00				

Ans. 808 days back of Feb. 7, 1872, or on Nov. 21, 1869.

9. What is the balance of the following acct., and when is it due?

Dr. L. MURPHY IN ACCOUNT WITH A. KELLY. *Cr.*

1871				1871			
May 1	To Mdse.	$218	00	May 25	By draft, at 60 da.	$200	00
June 12	" "	274	00	June 6	" Cash	325	00
Sept. 16	" Sundries	156	00	Aug. 20	" draft, at 30 da.	100	00
Nov. 14	" Mdse.	268	00	Oct. 3	" Cash	42	00

Ans. Bal., $249, due Sept. 22, 1871.

10. Suppose the following account was settled May 6, 1871, by draft on time, how many days' credit should be given?

Dr. P. ROBINSON IN ACCT. WITH O'NEIL & Co. *Cr.*

1871				1871			
Feb. 1	To Mdse.	$ 73	44	Feb. 10	By Cash	$197	44
March 1	" "	96	50	" 21	" "	51	68
April 17	" "	144	72	April 23	" Sundries	30	34
May 1	" "	196	96	May 6	" Mdse.	17	92

Ans. 19 days.

11. When shall a draft for the settlement of the following account be made payable?

Dr. S. T. MITCHELL IN ACCT. WITH R. S. LEE. *Cr.*

1871				1871			
June 1	To Mdse. on 2 mo.	$108	72	Sept. 1	By Cash	$100	
July 14	" " on 40 da.	56	90	Oct. 15	" draft, at 30 da.	60	
Aug 16	" "	191	50	Nov. 10	" "	250	
Nov. 25	" Sundries	52	44	" 20	" Cash	300	

Ans. Feb. 10, 1872.

12. When shall a note be made payable, to balance the following account?

Dr. C. R. MARTEL IN ACCT. WITH N. SCOTT & CO. *Cr.*

1871					1871				
April 12	To Mdse. on 3 mo.	$265	42	July 14	By Cash	$218	00
" 20	" " " "	118	24	" 25	" "	160	00
" 30	" " " "	369	78	Aug. 3	" "	280	00
May 5	" " " "	136	72	" 17	" "	185	00
" 16	" " " "	96	39	Sept. 16	" "	420	00
" 29	" " " "	268	21	" 24	" "	280	00

Ans. Nov. 16, 1871.

13. A merchant owes $1368, payable July 14, and $936, payable Sept. 16. If he pays $1678, July 2, what will be the equated time for the payment of the balance? *Ans.* Nov. 19.

14. S. Jordan owes G. Peters, 1871, May 1, for merchandise, $1000; May 15, for coal, $800; June 14, for horses, $600; July 24, for timber, $200. Peters owes Jordan, 1871, March 7, for flour, $800; April 2, for sundries, $400; May 6, for oats, $600; June 13, for two carriages, $240. Allowing all the items to be on 6 months' credit, when will the balance of the acct. become due? *Ans.* April 27, 1872.

15. A farmer owes $750, due June 10, and $1500, due Nov. 18, and wishes to discharge the obligation by two equal payments, made at an interval of 40 days; when must the two payments be made?

16. A merchant holds 3 notes, the first for $600, due March 10, the second for $960, due June 10, and the third for $720, due Aug. 10. He wishes to exchange them for two others, one of which shall be for $1200, payable April 10; what shall be the face and when the maturity of the other? *Ans.* Face, $1080; maturity Aug. 6.

CASH BALANCE.

502. When an account current is settled *by cash*, it is not necessary to find the equated time. The *true* or *cash* balance of an account at a particular date may be found directly as follows:

Ex. 1. What will be the true balance of the following account June 1, 1871, the time of settlement, allowing that each item draws interest from its date, at 6 %?

Dr. C. JOHNSON IN ACCT. WITH ANDREWS & SON. *Cr.*

1871				1871			
March 5	To Mdse.	$160	00	March 7	By Cash	$270	00
" 25	"	440	00	" 28	"	200	00
April 11	"	100	00	May 2	"	440	00
" 19	"	110	00	" 20	"	720	00
May 1	"	330	00				
" 4	"	370	00				
" 21	"	220	00				

OPERATION.

Debits.					*Credits.*				
Due,			*Days.*		*Due,*			*Days.*	
March 5,	$ 160	×	88	= 14080	March 7,	$ 270	×	86	= 23220
" 25,	440	×	68	= 29920	" 28,	200	×	65	= 13000
April 11,	100	×	51	= 5100	May 2,	440	×	30	= 13200
" 19,	110	×	43	= 4730	" 20,	720	×	12	= 8640
May 1,	330	×	31	= 10230					
" 4,	370	×	28	= 10360		$1630			6) 58060
" 21,	220	×	11	= 2420					$9.677
	$1730			6) 76840					
				$12.807					

Sum of debit items,	$1730	Interest of debit items,	$12.807
" " credit items,	1630	" " credit "	9.677
Balance of items,	$100	Balance of interest,	$3.130

True balance June 1, $100 + $3.13 = $103.13.

EXPLANATION. Since each item of the debit side of the account was on interest from its date to the time of settlement, the total interest of the several debit items equals the interest of $1 for 76840 days, which, at 6 %, gives $12.807. (The int. of $1 for 6 days is 1 mill; hence, the interest of $1 for 76840 days is found by dividing 76840 by 6, and pointing off three decimal places.) The total interest of the several credit items equals the interest of $1 for 58060 days, which is $9.677. Now, instead of increasing each side of the account by its interest, and then finding the balance, this same result may be obtained by finding separately the balance of items and the balance of interests. If the two balances fall on the same side of the account, it is evident the *true balance* will be their *sum*; if, on different sides, their *difference*.

METHOD BY INTEREST.

Due,			*Days.*	*Int.*	*Due,*			*Days,*	*Int.*
March 5,	$ 160	for	88	= $2.347	March 7,	$ 270	for	86	= $3.870
" 25,	440	"	68	= 4.987	" 28,	200	"	65	= 2.167
April 11,	100	"	51	= .850	May 2,	440	"	30	= 2.200
" 19,	110	"	43	= .788	" 20,	720	"	12	= 1.440
May 1,	330	"	31	= 1.705					
" 4,	370	"	28	= 1.727		$1630			$9.677
" 21,	220	"	11	= .403					
	$1730			$12.807					

Balance of items = $1730 — $1630 = $100.
" " interest = $12.807 — $9.677 = $3.13.
True balance, $100 + $3.13 = $103.13.

NOTE.—The "Method by interest" will generally be found most convenient either for finding the equated time for the payment of the balance of accounts, or in finding the cash balance.

The above account, when balanced by interest, may be presented as follows:

Dr. C. JOHNSON IN ACCT. WITH ANDREWS & SON. *Cr.*

1871		Amount.	Da.	Int.	1871		Amount.	Da.	Int.
Mar. 5	To Mdse.	$ 160.00	88	$ 2.347	Mar. 7	By Cash.	$ 270.00	86	$ 3.870
" 25	"	440.00	68	4.987	" 28	"	200.00	65	2.167
April 11	"	100.60	51	.850	May 2	"	440.00	30	2.200
" 19	"	110.00	43	.788	" 20	"	720.00	12	1.440
May 1	"	330.00	31	1.705	June 1	bal. acc't.	103.13		
" 4	"	370.00	28	1.727					$ 9.677
" 21	"	220.00	11	.403					
June 1	By int.	3.13							
				$12.807					
		$1733.13					$1733.13		

Errors excepted.

Quebec, June 1, 1871.

ANDREWS & SON.

ANOTHER METHOD BY INTEREST.

Dr. GIBB & HUGHES IN ACCT. CURRENT WITH J. HUNTER. *Cr.*

1871				1871		
Mar. 1	To bal. of all acct.	$100	15	Mar. 15	By Mdse on 6 mo.	$160 00
" 15	" Cash, paid draft	40	00	Apr. 1	" Cash	100 00
" 20	" Mdse. on 6 mo.	180	85	June 1	" draft	120 00
				Au. 10	" Mdse. on 2 mo.	80.00
				Sept. 1	" cash paid draft	50.00

Required the cash value of the above account, Sept. 5, 1871, int. at 6 %.

OPERATION.

Dr. *Cr.*

Due.	Da.	Items. Int.	Cash value.	Due.	Da.	Items. Int.	Cash value.
Mar. 1	188	100.15 + 3.138	103.288	April 1	157	100.00 + 2.617	102.617
" 15	174	40.00 + 1.160	41.160	June 1	96	120.00 + 1.920	121.920
Sep. 20	15	180.85 — .452	180.398	Sept. 1	4	50.00 + .033	50.033
				" 15	10	160.00 — .267	159.733
				Oct. 10	35	80.00 — .467	79.533
			324.846				513.836

$513.836 — $324.846 = $188.99, *Ans.*

503. RULE.—*Multiply each item of the account by the number of days intervening between the date on which it becomes due and the time of settlement. Divide the sums of the debit and credit products respectively by 6: the quotient will be the interest of the two sides of the account, at 6 %, expressed in mills. Find the balance of items and also the balance of interests.*

When the two balances fall on the SAME *side of the account, the cash balance will be their* SUM; *when on opposite sides, their* DIFFERENCE. Or,

Find the interest of each item from the date on which it becomes due, to the time of settlement. The difference between the sums of interests on the debit and credit sides of the account will be the BALANCE OF INTEREST.

When the balance of interest falls on the same side as the balance of items, the cash balance will be their SUM; *when on opposite sides, their* DIFFERENCE. Or,

Find the number of days intervening between each maturity and the day of settlement.

Compute the interest on each item for the corresponding interval of time; add the interest to the item if the maturity is before the day of settlement, and subtract it from the item if the maturity is after the day of settlement; the results will be the cash values of the several items.

Add each column of cash values, and the difference of the amounts will be the cash balance required.

EXAMPLES FOR PRACTICE.

1. The following account was settled Nov. 16, 1871. What was the cash balance, interest being computed on each item from date at 6 %? *Ans.* $215.54.

Dr. JOHN FRASER IN ACCT. WITH L. R. BARRY. *Cr.*

1871				1871			
Feb. 1	To Merchandise	$ 72	00	Feb. 16	By Cash	$100	00
April 4	" "	187	00	March 24	" "	150	00
May 22	" "	250	00	April 16	" "	300	00
July 19	" Cash	50	00	" 20	" Mdse.	90	00
" 22	" Merchandise	300	00	June 27	" "	350	00
Oct. 10	" "	125	00	Sept. 3	" Cash	200	00
Nov. 16	" *bal. new acc't.*			*Nov.* 16	" *bal. of int.*		

Errors excepted. Quebec, Nov. 16, 1871. L. R. BARRY.

2. D. A. Hamel is in account current with Lynch & Co., as follows: Debtor, March 17, 1871, to merchandise, on 3 months, $721.50; April 9, to mdse., on 3 months, $481.12; May 16, to mdse, on 3 mo., $1750.48; July 14, to mdse, on 2 months, $470.50. Creditor, March 21, 1871, by cash on acct., $500; May 10, by acceptance at 30 days,

$600; July 2. by sundries, $750; July 17, by cash on acct., $890. Required the cash value, if settled on July 21, 1871, interest at 6 %.

Ans. $749.49

3. N. L. Roy is in account and interest with Stevens & Co., as follows: Debtor, January 11, 1871, to mdse., on 6 months, $187.32; Feb. 13, to cash, paid draft, $120; March 30, to mdse., on 4 mo., $380.28; April 9, to mdse., on 4 mo., $194.40; May 25, to cash, paid draft, $120; Aug. 30. to mdse., on 6 mo., $255.60. Creditor, Feb. 12, by cash, $144; March 30, by mdse., on 4 months, $504.20; May 11, by mdse., on 6 mo., $360; July 12, by mdse., on 4 mo., $60; Sept. 20, by mdse., on 4 mo., $119.80. Required the true balance, if settled on Dec. 10, 1871, int. at 6 %. *Ans.* $73.73 + .

4. Required the cash balance due on the following account, on July 1, 1871, interest being computed on each item from date at 7 %. James Fisher in account and interest with Henry S. Lane: Debtor, Jan 7, 1871, to bal. of acct., $120; Jan. 15, to mdse., $96.75; Jan. 24, to bills payable, $130.50; Feb. 27, to mdse., $200.80; March 7, to mdse., $80; May 10, to mdse., $300; June 9, to mdse., $340.75. Creditor, April _, 1871, by cash, $140; April 30, by cash, $50; May 20, by order on L. R., $140; May 31, by cash, $450; June 11, by mdse., $500. *Ans.* $23.52.

ACCOUNT OF SALES.

504. An **Account of Sales** is a statement of the quantity and price of goods sold, the charges incurred in the sales, and the net proceeds, which a commission merchant or consignee makes to his employer or consignor.

The *net proceeds* is the sum to which the employer is entitled after all charges are deducted. The net proceeds are due as cash at the equated time of the different sales.

Ex. 1. Account of Sales of grain for C. Morgan & Co.

Date.	Purchaser.	Description.	Bush.	Price	$
1871					
Jan. 30	L. N. Maguire	Wheat, white	200	$.95	190.00
Feb. 3	Keller & Lee	Wheat, Ont.	800	.88	704.00
" 16	T. A. Thibodeau	Corn	1600	.55	880.00
" 28	D. L. Morris	Oats	1200	.37½	450.00
March 20	Vanner & Simms	Wheat, white	600	1.00	600.00
April 9	S. E. Lyman	" , red	1160	.85	986.00
" 28	A. Hamilton	Corn	1040	.58	603.20
May 7	R. F. Burton	"	360	.60	216.00
" 30	P. Sullivan	Wheat, med.	764	.90	687.60
					$5316.80

Charges.

	Commission on $5316.80, at 2½ %,	$132.92	
May 30,	Freight on 764 bushels of wheat,	38.20	
	Drayage and sacks,	40.80	
	Advertising in "Mercury",	6.25	
			218.17
	Net proceeds to credit of C. Morgan & Co.,		$5098.63

Errors excepted.

Quebec, June 1, 1871.

JOHN LAIRD & O'NEIL.

Ex. 2. ACCOUNT SALES of 1300 barrels of flour, sold for E. A. O'Dowl, Montreal, P. Q.

1871							
March	23	500 barrels Flour, at $6.00			cash		
"	26	400	"	" 6.00	60 days		
"	27	200	"	" 6.00	60 days	$500.00	
					Cash	700.00	
"	28	200	"	"	3 months	1250	
						7850	00
		CHARGES.					
		Storage on 1300 bbls. 1 mo., at 3 cts.....					
		Commission on $7850, at 2½ %,.....					
		Net proceeds, due.....				7614	75

E. E.

Quebec, March 30, 1871.

L. RUSSELL & Co.

Per Louis Bilodeau.

Date.		To whom sold.	Time.	Mark.	No.	Description of goods.	Yds.	Price.	Amount.		Total.	
1871												
March	4	P. Jamison	6 mo.	E	210	1 Case Satinets	600	$ .70				
"	"	L. Meehan	"	A	208	1 " "	720	.65				
"	"	Lyons & Todd	"	E	211	1 " Cassimeres	500	1.00				
"	"	"	"	"	212	1 " R.bb'd "	500	1.25				
"	6	C. Brown & Co.	"	B	209	1 " Satinets	550	.70				
"	10	E. Lee & Co. (note)	4 mo.	"	201, 202	2 " "	1200	.65				
"	12	Jobin & Roche	"	E	204, 205	2 " Cassimeres	800	1.50				
"	"	"	"	"	203	1 " "	400	1.25				
"	15	R. Chambers	6 mo.	A	206	1 " Satinets	425	.72				
"	"	"	"	"	207	1 " "	450	.50				
						Charges.						
						Ins. $22.14, Freight $22.04, Drayage $1.00			45	18		
						Int. on $45.18 from March 3rd., at 6 %.				21		
						Commission & Guarantee, at 5 %.....						
						Net proceeds, due.....					$5093	16

E. E. Quebec, March 31, 1871. L. O. Ross & Co.

EXAMPLES FOR PRACTICE.

1. Sales of 544 barrels flour, for account of P. S. Renaud & Co., Montreal, by L. J. McGreevy & Son, Quebec, viz.: July 2, 1871, to Jos. White, 400 bbl. Ohio extra, at $8.30, 19 bbl. fine, at $5; July 5, to Sweeney & Co., 125 bbl. Canada extra, at $7.50. Charges as follows: June 15, freight per Steamboat "Quebec", 544 bbl. at 16 cts., ss $10 for damage on the same; storage, 3 cts. per bbl.; insurance, 10.88; commission on $4352.50, at 2½ %. Required the net proceeds, and the date when they shall be accredited to the owner.

Ans. Net proceeds, $4139.45; due, July 10, 1871.

2. L. R. Doyle & Co. of Toronto, received into their store an Invoice of Fruit per Grand Trunk, from the United States, on acct. of T. A. Kane, New Orleans, and sold it as follows: Aug. 3, 1871, 100 boxes raisins, at $3, cash, 52 boxes lemons, at $3, cash; Aug. 4, 25 boxes oranges, at $3, and at 60 da., 200 jars olives, at $0.50, and at 60 da., 100 boxes oranges, at $3, cash; Aug. 9, 25 boxes oranges, at $3, and at 90 da., 25 boxes lemons, at $3, and at 90 days; Aug. 10, 150 boxes oranges, at $5, cash, 110 boxes, lemons, at $3.80, cash, 220 jars olives, at $0.50, cash. Sold at auction the three last items amounting to $1278; auctioneer's commission on $1278, at 3½ %, deducted; Aug. 10, 4000 lb. plums, at $0.50, and at 4 months. The charges were: duties and permit, $340; freight and primage, $108; cartage and labor, $12; refunded for damages, $55; storage and advertising, $52.54; commission on $4314.27, at 5 %. Required the net proceeds and when due? *Ans.* Net proceeds, $3531.22; due, Oct. 24, 1871.

TABLE

OF FOREIGN MONEYS OR CURRENCIES, WITH THE PAR VALUE OF THE UNIT, AS FIXED BY COMMERCIAL USAGE.

Cities and Countries.	Denominations and Metal.	Value.
Argentine Rep.	100 centesimos = 1 real; 8 reals = 1 dollar (silver) =	$1.016
Austria.	60 kreutzers = 1 florin " =	0.485
	120 " = 1 rix-dollar " =	0.971
	60 batzen = 1 ducat (gold) =	2.278
Azores.	1000 reas = 1 milrea (silver) =	0.830
Baden.	60 kreutzers = 1 florin " =	0.397
Batavia.	48 stivers = 1 rix-dollar " =	0.782
Bavaria.	60 kreutzers = 1 florin (silver) =	0.395
	crown, " =	1.072
	ducat, (gold =	2.274
Belgium.	100 centimes = 1 franc (silver) =	0.186

Cities and Countries.	Denominations and Metal.	Value.
Bolivia.	8 reals = 1 dollar (silver) =	1.011
	doubloon (gold) =	15.580
Brazil.	1000 reas = 1 milrea (silver) =	0.830
	Exchange on London, 30d sterling per milrea in bank notes.	
	Exch. on Paris, fr. 3.15 to fr. 3.20 per 1000 reis.	
Bremen.	5 schwares = 1 grote;	
Brunswick.	72 grotes = 1 thaler (silver) =	0.788
	30 groschen = 1 thaler " =	0.692
Chili.	100 cents = 1 dollar " =	1.011
	doubloon (gold) =	15.660
China.	10 cash = 1 candarine; 10 can. = 1 mace; 10 mace = 1 tael. (silver) =	1.480
	The exchange on London is 4s. 8d., more or less, for one Spanish dollar.	
Columbia.	8 reals = 1 dollar (silver) =	1.022
	doubloon (gold) =	15.617
Darmstadt.	60 kreutzers = 1 guilder (silver) =	0.397
Denmark.	12 pfennings = 1 skilling; 16 skillings = 1 mark; 1½ mark = 1 rigsbank daler; 6 marks = 1 rix-dollar (silver) =	1.051
	Frederick d'or (gold) =	3.932
	Exchange on London is 9¼ rigsbank daler for £1 sterling.	
	Exchange on Paris (rarely) from fr. 2.60 to fr. 270 per rigsbank daler.	
Egypt.	100 aspers = 1 piaster; 20 piasters = 1 real (silver) =	0.968
	Exchange on London, 80 piasters, more or less, for £1 sterling.	
	Exchange on Paris, 315 a 320 aspers per 100 fr.	
England.	12 pence = 1 shilling; 20 shillings = £1 sterling, £1 or 1 sovereign = (gold)	4.866
	Exchange on London is $4.86⅔ in Canada.	
	In New York it is usually 7 to 10 %, i. e., a pound sterling in London is worth $4.44⅘ and 7 to 10 % additional, in New York.	
France.	100 centimes = 1 franc (silver) =	0.186
	Exchange on London, fr. 25.50 for £1 sterling.	
	Exchange on New York and Canada, fr. 5.25 to 5.30 for $1.	
Frankfort.	60 kreutzers = 1 guilder or florin (silver) =	0.397
	Exchange on London, 120 florins, more or less, for £10 sterling.	
	Exchange on Paris, fr. 2.10 a 2.15 per florin.	
Genoa and Piedmont. (Italy.)	100 centesimi = 20 soldi = 1 lira (silver) =	0.186
	Exchange on London, 25.50 lira, more or less, for £1 sterling.	
	Exchange on Paris, 21 lira per fr. 20.	

Cities and Countries.	Denominations and Metal.	Value.
Greece.	100 lepta = 1 dragma; 1 dragma (silver) =	0.163
Hamburg and Lubeck.	12 pfennings = 1 skilling; 16 skillings = 1 mark banco (silver) =	0.350
	1 ducat (gold) =	2.257
	Exchange on London, 14 marks banco, more or less, for £1 sterling.	
	Exchange on Paris, fr. 1.50 to fr. 1.70 per mark banco.	
Hanover.	30 groshen = 1 florin (silver) =	0.547
	30 groschen = 1 thaler " =	0.694
Hindostan.	12 pice = 1 anna; 16 annas = 1 rupee (sil.) =	0.445
	16 rupees = 1 mohur (gold) =	7.109
	Exchange on London, at Bombay, 2s., more or less, for 1 Company's rupee.	
Holland.	100 cts. = 20 stivers = 1 guilder or florin (silver) =	0.400
	Exchange on London, 11 g. 80 cts., more or less, for £1 sterling.	
	Exchange on Paris, 2 fr. 10 cts., more or less, per guilder.	
Italy, (Florence, Leghorn, Lombardy, Venice.)	100 centesimi = 1 lira (silver) =	0.162
	Exchange on London, 30 lira, more or less, for £1 sterl., in Venice and Milan; 30 lira, more or less, per £1, in Florence and Leghorn.	
	Exchange on Paris, fr. 85, more or less, 100 lira, in Venice and Milan; 80 to 85 centimes per lira, in Florence and Leghorn.	
Japan.	10 mace = 100 candarines (silver) =	0.750
Madeira.	1000 reas = 1 milrea " =	1.000
Madras.	42 fanams = 1 pagoda (gold) =	1.840
Mecklemburg.	1 florin (silver) =	0.541
Mexico.	8 reals = 1 dollar " =	1.005
	1 doubloon (gold.) =	15.534
Monte Video.	100 centesimos = 1 rial; 8 rials = 1 dollar or 4 pesos duro =	1.000
	Exchange on London = 52d. sterling for 1 peso duro.	
Naples.	10 grani = 1 carlino; 12 carlini = 1 scudo (silver) =	0.950
	10 carlini = 1 ducat; 3 ducat = 1 ounce (gold) =	2.485
	Exchange on London, 575 grani per £1 sterling.	
	Exchange on Paris, 22 a 25 grani per 1 fr.	
Norway.	16 skillings = 1 mark; 6 marks = 1 rix-dollar (silver) =	1.051
Persia.	100 maravodis = 1 tomaum (gold) =	2.233
Peru.	8 reals = 1 dollar (silver) =	1.005
Portugal.	400 reas = 1 cruzado; 1000 reas = 1 milrea (silver) =	1.120
	1 crown (gold) =	5.813

Cities and Countries.	Denominations and Metal.	Value.
	Exchange on Paris, fr. 6.20 a fr. 6.30 per milrea.	
	Exchange on London, 1 milrea for 59 pence.	
Prussia.	12 pfennings = 1 groshen; 30 groshen = 1 florin (silver) =	0.227
	30 groschen = 1 thaler " =	0.692
	Exchange on London, 6 thalers 25 gr., more or less, for £1 sterling.	
	Exch. on Paris, fr. 3.75, more or less, per thaler.	
Rome.	10 bajocchi = 1 paolo; 10 paoli = 1 scudo; 1 scudo (silver) =	1.000
Russia.	100 copecks = 1 rouble " =	0.754
	Exchange on London, from 39d. to 42d. for 1 rouble silver.	
	Exchange on Paris, from fr. 4.10 to 4.20 per rouble silver.	
St. Domingo.	100 centimes = 1 dollar =	0.333
Sardinia.	100 centesimi = 1 lira (silver) =	0.186
	Same exchange on London and Paris, as for Genoa, (*see* above).	
Saxony.	30 groschen = 1 thaler (silver) =	0.694
	Exchange on London, 6 thaler 25 groschen, more or less, per £1.	
	Exchange on Paris, fr. 3.50 a fr. 3.75 per thaler.	
Sicily.	20 grani = 1 taro; 12 tari = 1 scudi (sil.) =	0.985
	30 tari = 1 ounce (gold) =	2.400
Smyrna and the Levant.	Like Constantinople.	
	In the Levant are likewise used to a great extent, Spanish dollars and Dutch, Hungarian, and Venitian ducats. Likewise German Conventions thaler = $0.96 to $1, being subject to variation.	
	Exch. on London, 105 piasters, more or less, for £1.	
	Exchange on Paris, fr. 4.75 to fr. 5 per piaster.	
Spain.	4 reals vellon = 1 pistareen (silver) =	0.197
	10 reals plate = 1 dollar " =	1.003
	1 doubloon (gold) =	15.570
	1 pistole " =	3.904
	Exchange on London, 40d. sterling, more or less, per peso duro or Spanish dollar = 48d. to 52d. Eng. per dollar.	
	Exchange on Paris, fr. 5.10 a fr. 5.30 per peso duro or dollar.	
Sweden.	48 skillings = 6 marks = 1 rix-dollar specie (silver) =	1.059
	12 marks = 1 ducat (gold) =	2.267
	Exch. on London, 12 rix-dollars banco for £1 ster.	
	Exch. on Paris, fr. 2.10 to fr. 2.15 for 1 rix-dollar.	
Switzerland.	100 centesimi = 1 livre (silver) =	0.273
	Exchange of Basle on London, fr. 17.5, more or less, for £1.	
	Exchange on Paris, fr. 1.50 per fr. 1, or 50 per cent. premium, more or less in favor of Basle.	

Cities and Countries.	Denominations and Metal.	Value.
Tripoli.	120 paras = 1 utchlik (silver) =	0.149
Tunis.	16 carobas = 1 piaster " =	0.124
Turkey.	100 aspers = 1 piaster " =	0.026
	20 piasters = 1 yormilik (gold) =	0.877
	Exchange on London, 104 piasters, more or less, for £1.	
	Exch. on Paris, from 400 to 410 piasters for 100 fr.	
Tuscany.	12 soldi = 1 florin (silver) =	0.262
	1 crown or corona " =	1.050
	1 ruspone (gold) =	6.925
	1 sequin " =	2.301
	NOTA.—For Exchange on London and Paris, (*see* Italy.)	
United States.	10 mills = 1 cent; 10 cts. = 1 dime; 10 dimes = 1 dollar (gold) =	1.000
Wurtemburg.	60 kreutzers = 1 guilder (silver) =	0.395
	1 crown " =	1.070
	1 ducat (gold) =	2.236
	Exchange on London and Paris, the same as for Frankfort.	

EXAMPLES FOR PRACTICE ON EXCHANGE (*see* 406).

1. A draft on Toronto cost £187 10s. in Liverpool, exchange being at 8 % premium for sterling; required the face of the draft?

2. What is the cost of a draft on St. Perersbourg for 6915 roubles 50 copecks, exchange being at 74 cts. a rouble? *Ans.* $5117.47.

3. Received of J. Walter & Son, Glasgow, a bill on Messrs. S. Ross & Co., of Montreal, for £1143 15s. What was its value in Canada currency, the premium being 9 % in favor of sterling currency?

Ans. $5540.833+.

4. What is the value in francs of a bill for $975.60, allowing a premium of 3 %, and 5⅛ fr. to the dollar? *Ans.* 5359 fr. 29 cen.

5. A merchant in Halifax has 8250 guilders 5 stivers due him in Amsterdam, and requests the remittance by draft; what sum will he receive, exchange on Canada being in Amsterdam at 2¼ guilders

Ans. $3666.77+.

6. A broker paid in Ottawa $8030 for £1650 draft on Dublin; at what per cent. of premium did he purchase it? *Ans.* 9½%.

7. What is the value in Canada currency, of 2000 florins in the Netherlands, at 2½ % premium? *Ans.* $820.

8. Twenty days after the date of a draft drawn at Genoa, Dec. 3, 1871, at ninety days, for 1820 liras 15 soldi, C. Jenkins to whose order it was drawn, requests payment, and proposes for prepayment a discount of 3 %. What is the value of the same in Canada currency, allowing that the corona bears a premium of 5 %? *Ans.* $1948.91.

9. A trader in Liverpool wishes to invest £3000 in merchandise in Venice; if he remits to his correspondent at Venice a bill purchased for this sum, at the rate of 8d. sterling per lira, what sum in the currency of Venice will the agent receive? *Ans.* 90000 liras.

10. L. O'Brien of Montreal, has consigned a cargo of oats, valued at £6500, to Payne & Moss, London. J. Wilson & Co., being about to import an invoice of dry goods, have purchased of Brook & Son, a bill of exchange, at 8½ % premium, for the value of the said cargo. What should they pay for the bill? *Ans.* $31344.44+.

11. To Messrs. Newman & Walter, Hamburg.

GENTLEMEN: Please pay to the order of P. Krauss, $1301.50, and place the same to the account of

Yours, &c.

Toronto, Sept. 14, 1871. MANNING & LEWIS.

Required the value of the above in mark bancos, allowing a premium of 2 %. *Ans.* 3792. m. b. 15 skill.

12. To Messrs. Woodley & Lyman, Quebec.

GENTLEMEN: Ninety days after date, please pay to the order of T. Sharples & Co., 2130 scudi, and charge the same to the acct. of

Your ob't servants,

Naples, Jan. 3, 1871. ALMERAS & ORSINI.

What is the value of the above draft, if paid 30 days after date, at a discount of 2 % being allowed for prepayment, and the scudi commanding a premium of 5 %? *Ans.* 2083\frac{1}{3}$.

ARBITRATION OF EXCHANGE.

505. Arbitration of Exchange is the process of computing exchange between two places by means of bills of exchange drawn on one or more intermediate places.

NOTES.—1. Exchange thus made between two places is called *Indirect* or *Circular* Exchange. When there is but one intermediate place on which a bill is drawn, the computation is called *Simple Arbitration*; but when there are two or more intervening places on which bills are drawn, the computation is called *Compound Arbitration*.

2. The main object of arbitration of exchange is to ascertain the cheapest route for making drafts or remittances.

Ex. 1. When exchange at Montreal on London is at 9½ % premium, and that at London on Amsterdam is at the rate of 12.5 guilders to the £, what must a person in Montreal, remitting through London, pay for a bill of exchange on Amsterdam for 1200 guilders?

OPERATION.

$$\frac{1200}{12.5} \times \$\frac{40}{9} \times 1.095 = \$467.20.$$

Or thus:

$\$ x$	1200 guilders
12.5 g'ders	1 £
9 £.	$\$40 \times 1.095$

$\$ x = \$467.20.$

ANALYSIS.—Since it takes £1 in London to buy a bill of exchange for 12.5 guilders on Amsterdam, it will take $\frac{£1200}{12.5}$ to buy a bill for 1200 guilders; but a bill on London for £1 costs in Montreal $\$\frac{40}{9} \times 1.095$ (422).

In the second form we draw a vertical line, and place equivalents with

symbols of their units directly opposite each other, beginning with that of the required quantity, which, for convenience is denoted by x, and so arrange the terms that the second on the left shall be of the same denomination as the first on the right; the third on the left, the same as the second on the right; and so on should there be a greater number of terms.

Ex. 2. A merchant in Toronto wishes to remit 8000 francs to Paris by circular exchange through London. If exchange at Toronto on Paris is at the rate of 5 francs 30 centimes to the dollar, that at London on Paris at the rate of 26 francs 20 centimes to the £, and that at Toronto on London at 9 % premium, how much less than by direct exchange, will it cost him, knowing that he pays his agent in London ½ % commission?

OPERATION.

$\frac{8000}{5.3}$ = \$1509.43+, the cost of direct exchange.

\$ x	8000 fr.
26.2 fr.	1 £
9 £	\$40 × 1.09
	1.005

\$ x = \$1486.61+.

\$1509.43 — \$1486.61 = \$22.82.

= difference in favor of indirect exchange.

ANALYSIS.—The cost of the direct exchange would be as many dollars as 8000 contains 5.3 = 1509.43.

To find the cost of the circular exchange we proceed as in *Ex.* 1, except that in this case the factor 1 + ½ % = 1.005, must be included among the factors on the right of the vertical line to cover the commission paid to the agent in London.

506. RULE.—I. *Draw a vertical line, and place the equivalent sums with the characters denoting their respective units directly opposite each other on the left and right of this line, representing the required sum by* x, *and writing it first and on the left, and arranging the other terms so that the second on the left shall be of the same denomination as the first on the right, the third on the left the same as that of the second on the right, and so on.*

II. *When a commission is allowed for remitting, put* 1 *plus the rate on the right* (Ex. 2) *if the cost, and on the left if the proceeds, of the exchange is required. When a commission is allowed for drawing, put* 1 *minus the rate on the left if the cost, and on the right if the proceeds, of the exchange is required.*

III. *Divide the product of the terms on the right by the product of the terms on the left, and the quotient will be the answer.*

NOTE.—Commission for remitting is a percentage on the price the agent who remits pays for a bill of exchange; and commission on drawing is a percentage on the value of the bill at the place where the agent besides.

EXAMPLES FOR PRACTICE.

1. When exchange at Quebec on Liverpool is at 9 % premium, and at Liverpool on Brussels 25 francs per £ sterling; what will be the arbitrated price in Quebec for a bill of exchange on Brussels for 2000 francs? *Ans.* \$387.54+.

2. If at Ottawa exchange on London is at $9\frac{3}{8}$ % premium, and at London on Paris 26 francs 86 centimes per £1; what is the arbitrated course of exchange between Ottawa and Paris, through London?
Ans. $0.181.

3. Exchange between Paris and Amsterdam being at the rate of 2 francs 20 centimes to the guilder, that between London and Paris at the rate of 25 francs 80 centimes to the £, and that from Halifax on London at $9\frac{1}{2}$ % premium, what will be the cost of a remittance for 900 guilders from Halifax to Amsterdam by bills of exchange through London and Paris? *Ans.* $373.48+.

4. A merchant in Kingston owes a debt of 4880 thalers in Bremen, to pay which he purchases a bill on London, at a premium of 9 %, and remits the same to his agent in England, on whom his creditor is requested to draw. If the exchange between London and Bremen be at the rate of 34d. sterling per thaler, and the charges for brokerage $\frac{1}{2}$ %, how much must have been the cost of the bill in Kingston?
Ans. $3365.87+.

5. When exchange in Quebec on Toronto is at $\frac{1}{4}$ % premium, and on Halifax at 2 % discount; if the exchange between Halifax and Toronto is at par, how much better is the circuitous route of exchange between Quebec and Toronto than the direct? *Ans.* $2\frac{1}{4}$ %.

6. When exchange between Toronto and Liverpool is at $8\frac{1}{2}$ % premium, and between Liverpool and Paris 25 francs 25 centimes per £ sterling, what sum in Toronto is equal to 6500 francs in Paris?
Ans. $1241.36+.

7. A merchant in Montreal wishes to remit to Dublin $2580. If he remit through Paris, what will his remittance amount to in sterling money, allowing $3 to be equal to 16 francs; and 24 francs, to 1 pound sterling? *Ans.* £573 6 8.

8. A man in Kingston wishes to draw on Hamilton for a railroad stock dividend of $1125, and exchange direct on Hamilton is $1\frac{1}{4}$ % discount; how much will he save by drawing on his agent in Toronto at $1\frac{1}{2}$ % premium, allowing his agent to draw on Hamilton at 1 % discount, brokerage at $\frac{1}{4}$ %? *Ans.* $11.03+.

9. A banker in Quebec remits $4500 to Edimburg, by arbitration, as follows: first to Lyons at 5 francs 30 centimes per $1; thence to Hamburg at 184.50 francs per 100 marks; thence to Amsterdam at 35 stivers per 2 marks; thence to Edimburg at 220 stivers per £ sterling. How much sterling money will he have in bank at Edimburg, and what will be his gain over direct exch. at $10\frac{1}{2}$ % premium?
Ans. { Proceeds in Edimburg, £916 5 $9\frac{1}{4}$.
Gain by arbitration, 111 19 $7\frac{1}{4}$.

10. A merchant in Paris desiring to remit to Montreal 14320 francs, remits through London. Required the value of the same in Canada currency, allowing 25.80 francs to the £ ster., and £1 = 4.86\frac{2}{3}$.

11. A merchant in Hamburg, wishing to cancel a claim of $2940 in Ottawa, and for that purpose buys a bill of exchange on Paris, at the rate of 2 francs per mark banco, which he forwards to Ottawa, brokerage $\frac{1}{2}$ %. Allowing $35 to 100 mark bancos, what did the bill cost him? *Ans.* 16884 francs.

12. A of Barcelona owes B of Liverpool, £1900. B of Liverpool

draws on C of Amsterdam, C of Amsterdam on D of Bordeaux, and D of Bordeaux on A of Barcelona. Allowing £1 exchanges for 12 florins; 19 florins for 40 francs; and 100 francs for 19 Spanish dollars, how many dollars will pay the bill? *Ans.* $9120.

13. I wish to remit from Glasgow to Quebec £1275 15s. What will be its value in Canada currency, remitting through Paris at the following rates: £1 equals 25 francs 80 centimes; and 5 francs 30 centimes, equal $1? *Ans.* $6210.25+.

14. A merchant in Halifax wishes to remit to London $6250, so as to receive the largest possible returns for the same. If he remits directly to London, the sterling currency will command a premium of 9 %; if through Paris, it must be at the rate of 5 francs 20 centimes to the dollar, and 25 francs 80 centimes to the pound; but if through Hamburg, at the rate of 35 cents per mark banco, and 55 mark bancos per £4. Which is the most desirable course?

Ans. The course through Hamburg is preferable by £8 11 $3\frac{1}{4}$, to the direct course, and by £39 0 $2\frac{3}{4}$, to that through France.

ARBITRATION OF MERCHANDISE.

507. Arbitration in Merchandise consists in comparing the weights and measures of different countries; also, in finding from the value of any particular weight or measure of one country, the value of the corresponding weight or measure of another country.

By the operations herein involved, the merchant is enabled to determine in what way he can most advantageously export or import any species of merchandise. The operation obviously consists, not only in the comparison of the weights and measures of different countries, but also in the exchange of currencies.

TABLE

OF THE PRINCIPAL WEIGHTS AND MEASURES OF THE MOST IMPORTANT COMMERCIAL COUNTRIES IN THE WORLD REDUCED TO THEIR ENGLISH EQUIVALENTS.

AUSTRIA.

(*Chief commercial cities*, VIENNA *and* TRIESTE.)

100 commercial lb. = 123.6 Avdp.
1 staro = 2.34 Winch. bu.
1 polonick = 0.861 " "
1 eimer = 15 wine gal.
1 barile = 173 " "
1 ell, woollen meas. = 26.6 in.
1 ell, silk = 25.2 "

BADEN AND BAVARIA.

(*Principal commercial city*, AUGSBURG.)

1 pound = 560 gram. French = 1.25 lb. Avdp.
1 Augsburg mark = 3643 gr. Troy.

1 foot = 11.5 in.
1 ell = 33.75 in.
1 klafter = 6feet = 5.75 feet.
1 scheffel for corn = 6.125 bu.
1 eimer of wine = 14.062 gal.
1 maas = 1.875 pint.

BELGIUM.

(*Principal commercial city,* ANTWERP.)

Weights and Measures the same as in France.

BRAZIL.

(*Principal commercial city,* RIO DE JANEIRO.)

Weights and Measures the same as in Portugal.

BREMEN.

(*One of the four Free Cities of Germany.*)

1 pound = 1.09 lb.
1 centner = 116 lb.
1 viertel of wine = 1.95 gal.
1 anker = 5 viertels = 9.65 gal.
1 oxhoft = 6 ankers = 58 gal.
1 scheffel of grain = 2 bu.
1 last = 40 sheffels = 80.70 bu.
1 stone flax = 20 lbs

CHINA.

(*Principal commercial city,* CANTON.)

1 catty = 1.33 lb.
1 pecul = 133.33 lb.
1 covid = 14.62 in.

CUBA.

(*Principal commercial city,* HAVANA.)

1 quintal = 101.75 lb.
1 arroba of wine = 4.1 gal.
1 fanega of grain = 3 bu.
1 vara = 33.34 in.

DENMARK AND NORWAY.

(*Principal commercial cities,* COPENHAGEN *and* CHRISTIANA.)

1 pound = 1.10 lb.
1 centner = 100 lb. = 110.28 lb.
1 viertel of wine = 2.04 gal.
1 anker of wine = 10 gal.
1 ahm = 4 ankers = 40 gal.
1 fuder of wine = 237.16 gal.
1 toende or bbl. of grain = 3.95 bu.
1 last = 12 toende = 47.50 bu.
1 danish ell = 24.66 in.

EGYPT.

(*Principal commercial city,* ALEXANDRIA.)

1 rotolo forforo = 15 oz.
1 rotolo zauro = 33.33 oz.
1 rotolo zadino = 21.3125 oz.
1 rotolo mina = 26.714 oz.
1 quintal coffee in Cairo = 103.6 lb.
1 oka = 3.239 lb. Tr.
1 dragma = 1.9375 dwt.
1 pik of corn = 26.8 in.
1 rebebe of corn = 36 gal.
1 kisloz = 39 gal.

ENGLAND.

(*Principal commercial cities,* LONDON *and* LIVERPOOL.)

The English Weights and Measures are the same as in Canada.

FRANCE.

(*Principal commercial cities,* PARIS, LYONS, *and* MARSEILLES.)

Weights and Measures, *see* p. 126.

FRANKFORT on the Main,

AND THE SOUTHERN PARTS OF GERMANY.

1 lb. heavy = 17.625 oz. Avdp.
1 lb. light = 15.05 oz. Tr.
1 mark = 525 oz. Tr.
1 cwt. of 100 heavy, or 108 light lb. = 111 lb. Avdp.
1 carat of jewels = 1.321 dwt. Tr.
1 foot = 11.25 in.
1 ell = 21.555 in.
1 Frankf. Brabant ell = 27.666 in.
1 malter of corn = 3.156 bush.
1 simmer " = 6.312 gal.
1 maas of wine = 3.156 pints.
1 ohm = 31.312 gal.
1 fuder = 6 ohms = 187.873 gal.

HAMBURG AND LUBECK.

(*Commercial cities of* GERMANY.)

1 pound = 1.068 lb.

100 commercial lb. = 106.838 lb.
1 foot = 11.289 in.
1 ahm of wine = 38.25 gal.
1 fuder = 6 ahms = 229.5 gal.
1 last of grain = 89.64 bu.
1 stock = 1½ last = 134.4 bu.
1 Brabant ell = 27.58 in.

HINDOSTAN.

(Principal commercial cities, Bombay, Bengal, Calcutta, *and* Madras.)

1 maund = 74.625 lb. Avdp.
1 seer = 29.875 oz. Avdp.
1 sicca = 178.666 gr. Tr.
1 cubit, or 1 covid = 18 in.
1 guz = 36 in.
1 coss = 4000 cubits = 1.125 mi.
1 pallie of corn = 9.5 lb. Avdp.
1 candy = 500 lb. Avdp.
1 garee of corn = 135 bu.
1 candy of corn = 24.5 bu.

HOLLAND.

(Principal commercial cities, Amsterdam, Haarlem, The Hague, Rotterdam, Leyden, etc.)

1 foot = 11.142 in.
1 ell = 27.083 in.
1 last for corn = 85.25 bu.
1 aam of wine = 41 gal.
1 vat = 100 kan = 1 hectol. Fr. = 26.42 gal.
1 muddle = 100 hop = 1 hectol. = 2.84 bu.
1 pound = 1.08 lb.
1 Fr. kilogramme = 2.20 lb.
1 last, marine = 4410 lb.

LOMBARDY (Italy.)

(Principal commercial cities, Venice & Milan.)

1 libra = 1 kilogramme = 2 lb. 3¼ oz. Avdp.
The Measures are equal to the French.

NAPLES (Italy.)

(Principal commercial city, Naples.)

1 rottolo = 1.96 lb.
1 cantaro grosso = 100 rottolo = 196.50 lb.
1 cantaro piccolo = 106 lb.
1 salma of oil = 42.75 gal.
1 carro of wine = 264 gal.
1 carro of grain = 52.20 bu.
1 canna = 83 in.

PORTUGAL.

(Principal commercial city, Lisbon.)

1 libra or arratel = 1.01 lb.
1 arroba = 22 arratels = 22.26 lb.
1 quintal = 4 arrobas = 89.05 lb.
100 libras or arratels = 101.19 lb.
1 almude of wine = 4.37 gal.
1 tonelado = 227.25 gal.
1 canada = 13.06 pints.
1 moyo of corn = 23.03 bu.
1 vara = 43.20 in.

PRUSSIA.

(Principal commercial city, Berlin.)

1 pound = 1.03 lb.
100 pounds Dantzic = 103.3 lb.
1 quintal = 110 lb. = 113.42 lb.
1 eimer of wine = 18.14 gal.
1 ahm = 39.66 gal.
1 scheffel of grain = 1.52 bu.
1 last of grain = 91 bu.
1 Berlin ell = 25.5 in.
1 Prussian ell = 26.28 in.

RUSSIA.

(Principal commercial cities, St. Petersburg *and* Warsaw.)

1 pound (funt) = 0.90 lb.
1 pood = 40 pounds = 36 lb.
100 pounds = 90.26 lb.
1 wedro of wine = 3.25 gal.
1 sorokovy = 40 wedros = 130 gal.
1 chetwert of corn = 5.95 bu.
1 arsheen = 28 in.
1 sashen = 7 feet.

SARDINIA (Italy.)

(Principal commercial cities, Genoa *and* Turin.)

1 peso grosso (Genoa) = 12.166 oz. Avdp.
1 libra (Turin) = 13 oz. Avdp.
1 palmo (Genoa) = 9.75 in.
1 mina of corn " = 3.50 bu.
1 barile of wine " = 16.33 gal.

1 barile of oil " = 14.25 gal.
1 piede liprando (Turin) = 20.5 in.
1 piede manelle " = 12.75 in.
1 raso (ell) " = 23.5 in.
1 sacco for wine " = 25.5 gal.
1 brenta " = 10.80 gal.

SAXONY.

(Principal commercial cities, DRESDEN *and* LEIPSIC.)

1 pound = 17.625 oz. Avdp.
1 foot = 11.875 in.
1 ell = 24 in.
1 schaffel of corn = 22 gal.
1 wispel " = 66 bu.
1 kanne = 1 litre = 1.75 pint.

SMYRNA AND THE LEVANT.

1 oka = 3.25 lb. Tr.
1 cantaro = 127.5 lb. Tr.
1 rotolo = 16.33 oz. Tr.
1 drachm = 49.6 gr. Tr.
1 pik = 27 in.
1 killow of corn = 11.625 gal.

SPAIN.

(Principal commercial city, MADRID.)

1 pound = 1.01 lb.
1 arroba = 25 pounds = 25.38 lb.
1 quintal = 4 arrobas = 101.52 lb.
1 cantaro or arroba of oil = 3.75 gal.
1 cantaro or arroba of wine = 4.25 gal.
1 moyo of wine = 16 arrobas = 68 gal.
1 botta = 38 arrobas of wine = 38½ arrobas of oil = 127.5 gal.
1 fanega of corn = 1.57 bu.
1 cahiz = 12 fanegas = 18.91 bu.
1 vara or yard = 33.37 in.

SWEDEN.

(Chief commer. city, STOCKHOLM.)

1 pound = 0.93 lb.
1 pound of iron = 0.75 lb.
1 anker of wine = 10.35 gal.
1 eimer of wine = 20.75 gal.
1 ahm = 2 eimers = 41.50 gal.
1 pipe = 3 ahms = 124.25 gal.
1 tun or barrel of corn = 4.16 bu.
1 ell = 23.36 in.

SWITZERLAND.

(Principal commercial cities, GENEVA, BERN, & BASLE.)

1 cwt. = 100 lb. = 50 kilogr. = 110.25 lb. Avdp.
1 lb. = ½ kilog. = 17.625 oz. Avdp.
1 foot = 0.3 meter = 11.85 in.
1 ell = 2 feet.
1 stab or staff = 4 feet.
1 malter of corn = 4.125 bu.
1 immir of " = 3.5 pints.
1 ohm of wine = 33 gal.
1 maas " = 3.5 pints.

TURKEY.

(Principal commercial city, CONSTANTINOPLE.)

1 pound, chequi, = 11.33 oz. Avd.
1 oka = 14 oz. Avdp.
1 pik, commercial = 27 in.
1 killow of corn = 7.5 gal.
1 fortin = 4 killows = 30 gal.
1 almud for liquids = 1.37 gal.

TUSCANY.

(Principal commercial cities, FLORENCE *and* LEGHORN.)

1 pound = 12 oz. Avdp.
1 quintal = 74⅞ lb. Avdp.
1 braccio = 23 in.
1 mile = 1 mile 48 yd.
100 sacchi of corn = 201 bu.
160 quartuzzi of wine = 10 1/10 gal.
1 barile of oil = 7⅞ gal.

UNITED STATES.

(Principal commercial cities, NEW YORK, BOSTON, CHICAGO, NEW ORLEANS, etc.)

The Weights and Measures are the same as in England.

NOTE.—The weights and measures of Mexico, Central America, and of the Republics of South America are the same generally as those of Spain; of Brazil, the same as those of Portugal; of the British North American Provinces, of the United States, and of Hayti, the same, in general, as in England; but the weights of Hayti are about 8% heavier, and its measures the same as in France.

The metric system has of late been adopted by Spain and Portugal, to the exclusion of other weights and measures. In 1864, it was legalized in Great Britain; and its use, either as a whole or in some of its parts, has been authorized in Greece, Holland, Italy, Norway, Sweden, Mexico, Guatemala, Venezuala, Ecuador, Columbia, Brazil, Chili, San Salvador, Argentine Republic, and the United States.

The following examples will embrace operations analogous to what we have already had, in addition to the exchange of weights and measures.

Ex. 1. A Montreal merchant imports from Holland 2550 ells of linen, which he finds costs him 2 florins per yard. In payment of the same, he remits through London. The amount of the remittance is required, allowing $9\frac{1}{2}\%$ premium in favor of sterling currency, that £1 exchanges for 14 florins of Amsterdam, the agent at London charging $\frac{1}{4}\%$ commission.

OPERATION.

$ x	2550 ells
3 ft.	2.3 × 4 fl.
14 fl.	£1
£9	$40 × 1.095
	1.0025

$ x = $2725.176+, *Ans.*

ANALYSIS.—Since 1 ell equals 2.3 ft., 2550 ells × 2.3 ÷ 3 = the same in yards; the yards multiplied by 4, equal the whole cost in florins, which, divided by 14, are reduced to sterling currency; and this in turn is exchanged to dollars by multiplying by 40 and dividing by 9, and this value is increased $9\frac{1}{2}\%$ by multiplying by 1.095, and finally the brokerage is added by multiplying by 1.0025.

Ex. 2. A merchant of Toronto sends lard to Hamburg at $10 per cwt., and orders remittance through Liverpool, expense of remittance to be paid by N. Ashley of Hamburg. Allowing $7 exchange for 20 mark bancos, and $13\frac{5}{7}$ mark bancos exchange for £1; also, that the sterling £ bears a premium of 9 % in Toronto, and that 105 lb. Hamburg equals 112 lb. Toronto. What is the cost of 1 lb. Hamburg, charges for commission being 2 %, insurance 1 %?

OPERATION.

105	10
40	9
109	100
100	103
7	96
x	16

x = 4 skillings 5+ pfennings per lb., *Ans.*

ANALYSIS. $10 ÷ 105 = price of 1 lb. Hamburg in Canada currency, which × $\frac{9}{40}$ or £1, and $\frac{100}{109}$ makes the required deduction in favor of Sterling currency; then the remaining value × $\frac{103}{100}$, is increased by the percentage of expense, and that value so increased × $13\frac{5}{7}$ or $\frac{96}{7}$ is exchanged to mark bancos; if £1 = $13\frac{5}{7}$ marks, £7 = 96 marks; and the marks × 16 skill. = 4 skillings 5+ pfennings per lb., *Ans.*

EXAMPLES FOR PRACTICE.

1. A merchant in Quebec ships 2000 lb. of butter to Bremen, and sells the same at 12 grotes per Bremen lb. The total receipts are remitted to Paris, and the merchant of Quebec draws on his agent there. For how many francs at 5.25 to $1 must he draw, allowing his agent in Paris charges 2 % commission? *Ans.* 1290.44 + fr.

2. L. Enright of Halifax imports from Lisbon 18 quintals of raisins, for which he pays 50 rees per arrated. He sells the same in the

Halifax market at 7 cts. per lb. What is his total gain, allowing he pays 2 % commission for remittance, etc.? *Ans.* $22.16.

3. R. N. Burke of Montreal imports from Belgium 2500 kilogr. of flax, purchased at 60 centimes per kilogr. The expenses of importation, including commission, etc., amount to 10 % on the first cost. How many dollars must R. N. B. remit through Liverpool, to pay for the same, allowing a premium of 8 % in favor of Sterling currency, and 25 francs to £1? Also, allowing 7 % of the whole for waste, and that R. N. B. sold the flax for 8 cts. per net pound; what per cent. did he gain on the total cost?

Ans. $316.80 to be remitted, and 29 % gained.

4. Maple sugar is bought in Quebec at 8 cts. a pound, and exported to Naples, and sold at 3.5 carlini per rottolo, and remittance is ordered through Paris, the exchange between Naples and Paris being at the rate of 24 grani to 1 franc, and between Paris and Quebec at the rate of 5.25 francs to $1. Allowing $1\frac{3}{4}$ % tare in Naples, and that freight, insurance, etc., amount to 5 %, what is the profit or loss of the transaction? *Ans.* 65% gained.

5. Wishing to import broadcloth, I find that the quality I desire will cost me at Liverpool 12s. 6d. sterling, and at Paris 16 francs 12 centimes. Allowing a premium of 9 % in favor of Sterling currency, and 5.25 francs to the dollar, at which market can I purchase most advantageously, the brokerage, etc., being the same in each place?

Ans. $4\frac{1}{2}$ cts. per yd. in favor of Liverpool.

6. Having a quantity of wheat, which I wish to export to the best market, I make inquiries and find London and Amsterdam the two most favorable places, and proceed to investigate their comparative merits. I find that in Amsterdam it commands $127\frac{1}{2}$ florins per last, and remittance can be effected through Paris at the rate of 37 florins to 80 francs, 5.20 francs being equal to $1; and at London the price is 3s. 4d. sterling per bushel, Sterling currency bearing a premium of 9 % in the Canada market; also that expense of remittance, etc., through Paris, amount to $1\frac{1}{2}$ %, while from London it amounts to only $\frac{5}{8}$ %. To which market shall I send my wheat?

Ans. 18 cts. per bushel in favor of London.

7. A merchant in Kingston imported 372 stone of flax from Bremen at 4 grotes per lb. Which is the better course of remittance: through Hamburg and Liverpool, at the rate of 4 thalers to 9 mark bancos, and 97 mark bancos to £7, the pound sterling bearing a premium of 9 %; or, through Paris and Naples, at the rate of 76 thalers to 63 scudi of Naples, and 1 scudo to 5 francs, and 5.25 francs to $1; the expenses of transmission through the former course being $2\frac{1}{2}$ %, and through the latter $3\frac{1}{2}$ %? Also, how much is gained by so remitting? *Ans.* Hamburg and Liverpool is $3.54 less than Paris and Naples.

8. Imported flax from Amsterdam; cost 8 centesimi per pound of Netherlands. To pay for the same, remittance was made through London and Bremen, with a premium of 8 % in favor of Sterling currency, £21 exchanging for 128 thalers, and 2 florins of Netherlands to 1 thaler of Bremen, expense of remittance being 5 %. What shall I gain per lb., and also at what rate %, if I sell the same at 4 cts. per pound of Canada? *Ans.* Gained 1 cent per lb., or 33 %.

SUPPLEMENT TO PROGRESSIONS.

ARITHMETICAL PROGRESSION (460).

508. CASE V.—*Given the common difference, the number of terms, and the sum of all the terms, to find the first term.*

Ex. The number of terms is 34, the common difference 6, and the sum of the terms, of a series of numbers in arithmetical progression is 3536; what is the first term?

OPERATION. $\mathbf{a} = \frac{\mathbf{s}}{\mathbf{n}} - [(\mathbf{n} - 1) \times \frac{1}{2}\,\mathbf{c}] = \frac{3536}{34} - [(34 - 1) \times 3] = 5$, the first term, *Ans.*

509. RULE.—*Divide the sum of the terms by the number of terms; subtract from the quotient, if the series be ascending, otherwise add to it, half the product of the common difference into the number of terms less one.*

EXAMPLES FOR PRACTICE.

1. If the number of terms be 22, the common difference 5, and the sum of the terms 1221; what is the first term? *Ans.* 3.

2. A farmer is to receive $300 in 12 payments, each succeeding payment to exceed the former by $4; what will his first payment be? *Ans.* 3.

510. CASE VI.—*Given the first term, the common difference, and the number of terms, to find the sum of all the terms.*

Ex. If the first term of a series of numbers in arithmetical progression be 5, the number of terms 34, and the common difference 6; what is the last term?

OPERATION. $\mathbf{l} = \mathbf{a} + [(\mathbf{n} - 1) \times \mathbf{c}] = 5 + [(34 - 1) \times 6] = 203$, the last term, *Ans.*

511. RULE.—*Add to twice the first term, if the series be ascending; otherwise subtract from it the product of the common difference into the number of terms, less one; multiply the sum or difference by half the number of terms.*

EXAMPLES FOR PRACTICE.

1. If the first term be 3, the number of terms 22, and the common difference 5, what is the last term? *Ans.* 108.

2. A man purchased 100 yards of cloth; the first yard cost him 40 cts., and each succeeding yard 20 cts. more to the last; what did the last yard cost him? *Ans.* $20.20.

3. The first term of an ascending series is $\frac{2}{3}$, the number of terms 18, and the common diff. $\frac{3}{8}$, what is the last term? *Ans.* $7\frac{1}{24}$.

GEOMETRICAL PROGRESSION (477).

512. CASE III.—*Given the first term, the common ratio, and the number of terms, to find the sum of the terms.*

Ex. If the first term of a series of numbers in geometrical progression be 5, the ratio 3, and the number of terms 12, what is the sum of the terms?

OPERATION. $s = a\left(\frac{r^n - 1}{r - 1}\right) = 5 \times \frac{3^{12} - 1}{3 - 1} = 1328600$, *Ans.*

513. RULE.—*From the power of the ratio whose degree is the number of terms, subtract one; divide the remainder by the common ratio, less one, and multiply the quotient by the first term.*

EXAMPLES FOR PRACTICE.

1. If the first term be 4, the ratio 3, and the number of terms 7, what is the sum of the terms? *Ans.* 4372.

2. The first term in a geometrical progression is 1, the number of terms 7, and the common ratio 4, what is the sum of the terms? *Ans.* 5461.

3. If a body be put in motion by a force which moves it 8 miles in the first portion of time, 7 miles in the second equal portion, and so, in the ratio of $\frac{7}{8}$, for ever, how many miles will it pass over? *Ans.* 64 miles.

514. CASE IV.—*Given the first term, the last term, and the sum of the terms, to find the common ratio.*

Ex. The first term of a geometrical progression is 1, the last term is 177147, and the sum of all the terms is 265720; what is the common ratio?

OPERATION. $r = \frac{s - a}{s - l} = \frac{265720 - 1}{265720 - 177147} = 3$, *Ans.*

515. RULE.—*Divide the difference between the first term and the sum by the difference between the last term and the sum: the quotient will be the common ratio.*

EXAMPLES FOR PRACTICE.

1. The first term is 4, the last term 1372, and the sum of the terms 1600; what is the ratio? *Ans.* 7.

2. The first term of a geometrical progression is $\frac{1}{8}$, the last term is 32, and the sum of the terms is $63\frac{7}{8}$; what is the ratio? *Ans.* 2.

3. A debt of $4095 can be discharged in a year by monthly payments, in geometrical progression, of which the first payment is $1, and the last $2048; what will be the ratio of the series?

PROMISCUOUS EXAMPLES IN PROGRESSIONS.

1. A lady gave to a poor person on the first day of the year $.10; on the second, $.25; and each succeeding day $.15 more, than on the former: how much did this person receive on the last day of the year? *Ans.* $54.70.

2. For 7 days a captain distributed some money to his soldiers; on the first day he gave them $.40, and on the following days he multiplied that sum by a certain number: find that number, knowing that on the 7th day, they received $290. *Ans.* 3.

3. What sum must be paid for a thermometer, whose price is equal to the 1st. term of an arithmetical progression, of which the 15th. term is $7.50 and the ratio $.50? *Ans.* 50 cts.

4. How many loaves of 2 lb. each did a baker sell on the last day of a week, if, on the first day, he sold for $3.84 at the rate of 6 cents per lb.; and, if the sale of each day was triple that of the preceding day? *Ans.* 23328.

5. On the first year of his business, a confectioner cleared $650; on the 7th. year he cleared $473850; required the ratio of increase per year. *Ans.* 3.

LIFE INSURANCE.

516. Life Insurance is a contract in which a company stipulates to pay a certain sum of money on the death of the person insured, in consideration of payments made by the insured as specified in the policy.

NOTES.—1. The policy may continue in force during the life of the insured, when it is called a *Life Policy;* or only for a specified number of years, when it is called a *Term Policy.*

2. The insurance may be effected on the payment of a single premium at the time the policy is made; or the premium may be payable annually, semi-annually, or quarterly, according to agreement.

3. The insured may designate to whom the amount of the policy shall be paid.

517. Life Insurance Policies are of the following kinds: 1st. *An Endowment Insurance Policy*, that is, a contract in which an Insurance Company agrees to pay to the party insured a specified sum at a certain age, or to his heirs, should his death occur before that age, on condition that he shall pay an annual premium until the policy matures; 2nd. *A Non-forfeiting* life, or *Endowment Policy*, is one in which, even though the party insured should fail to pay his annual premiums after the first, the company agrees to pay an equitable amount of the sum insured on the maturity of the policy.

The *Expectation of Life* is the average number of years that persons of the same age may be expected to live, as determined by tables of mortality.

NOTES.—1. The annual premium must be such a sum as will, when put at interest, amount to the sum insured, *at the close of the expectation of life.* This sum is easily found upon the principle of Life Annuities.

2. Life Insurance Companies have tables showing the premium to be paid at any age to secure ad annuity of $100, during the remainder of life.

3. Since a payment is made at the issue of the policy, and another at the expiration of the first year, the number of payments on a policy will always be 1 more than the number of years.

4. There are two tables showing the Expectation of Life. One, called the Carlisle Table, the other, the Wigglesworth Table. The Exspectation of Life, according to the two tables named, is shown in the following

Age.	Carlisle Table.	Wigglesworth Table.	Age.	Carlisle Table.	Wigglesworth Table.	Age.	Carlisle Table.	Wigglesworth Table.
0	38.72	28.15	32	33.03	29.43	64	12.30	13.05
1	44.68	36.78	33	32.36	29.02	65	11.79	12.43
2	47.55	38.74	34	31.68	28.62	66	11.27	11.96
3	49.82	40 01	35	31.00	28.22	67	10.75	11.48
4	50.76	40.73	36	30.32	27.78	68	10.23	11.01
5	51.25	40.88	37	29 64	27.34	69	9.70	10.50
6	51.17	40.69	38	28.96	26.91	70	9.18	10.06
7	50.80	40.47	39	28.28	26.47	71	8.65	9.60
8	50.24	40.14	40	27.61	26.04	72	8.16	9.14
9	49.57	39.72	41	26.97	25.61	73	7.72	8.69
10	48.82	39.23	42	26.34	25.19	74	7.33	8.25
11	48.04	38.64	43	25.71	24.77	75	7.01	7.83
12	47.27	38.02	44	25.09	24.35	76	6.69	7.40
13	46.51	37.41	45	24.46	23.92	77	6.40	6.99
14	45.75	36.79	46	23.82	23.37	78	6.12	6.59
15	45.00	36.17	47	23.17	22.83	79	5.80	6.21
16	44.27	35.76	48	22.80	22.27	80	5.51	5.85
17	43.57	35.37	49	21.81	21.72	81	5.21	5.50
18	42.87	34.98	50	21.11	21.17	82	4.93	5.16
19	42.17	34.59	51	20.39	20.61	83	4.65	4.87
20	41.46	34.22	52	19.68	20.05	84	4.39	4.66
21	40.75	33.84	53	18.97	19.49	85	4.12	4.57
22	40.04	33.46	54	18.28	18.92	86	3.90	4.21
23	39.31	33.08	55	17.58	18.35	87	3.71	3.90
24	38.59	32.70	56	16.89	17.78	88	3.59	3.67
25	37.86	32.33	57	16.21	17.20	89	3.47	3.56
26	37.14	31.93	58	15.55	16.63	90	3.28	3.73
27	36.41	31.50	59	14.92	16.04	91	3.26	3.32
28	35.69	31.08	60	14.34	15.45	92	3.37	3.12
29	35.00	30.66	61	13.82	14.86	93	3.48	2.40
30	34.34	30.25	62	13.31	14.26	94	3.53	1.98
31	33.68	29.83	63	12.81	13.66	95	3.53	1.62

LIFE TABLE.

ANNUAL PREMIUM ON A POLICY OF $100.

Age at issue.	Payments during life.	Payments To cease at 65.	Payments To cease at 60.	Payments To cease at 50.	Age at issue.
14	$1.4707	$1.4999	$1.5238	$1.6150	14
15	1.5105	1.5422	1.5683	1.6681	15
16	1.5516	1.5861	1.6145	1.7240	16
17	1.5940	1.6316	1.6625	1.7826	17
18	1.6377	1.6786	1.7124	1.8444	18
19	1.6829	1.7275	1.7644	1.9096	19
20	1.7296	1.7782	1.8186	1.9785	20
21	1.7780	1.8310	1.8753	2.0516	21
22	1.8280	1.8859	1.9344	2.1292	22
23	1.8798	1.9431	1.9963	2.2118	23
24	1.9335	2.0027	2.0612	2.3000	24
25	1.9891	2.0648	2.1291	2.3944	25
26	2.0470	2.1300	2.2007	2.4959	26
27	2.1071	2.1981	2.2761	2.6054	27
28	2.1696	2.2695	2.3555	2.7238	28
29	2.2346	2.3444	2.4395	2.8525	29
30	2.3023	2.4230	2.5284	2.9928	30
31	2.3728	2.5058	2.6226	3.1466	31
32	2.4464	2.5930	2.7228	3.3163	32
33	2.5232	2.6851	2.8296	3.5044	33
34	2.6034	2.7824	2.9436	3.7142	34
35	2.6873	2.8856	3.0657	3.9503	35
36	2.7752	2.9951	3.1971	4.2182	36
37	2.8674	3.1117	3.3387	4.5251	37
38	2.9641	3.2361	3.4919	4.8807	38
39	3.0658	3.3692	3.6584	5.2981	39
40	3.1729	3.5120	3.8402	5.7959	40
41	3.2856	3.6654	4.0393		41
42	3.4046	3.8311	4.2588		42
43	3.5303	4.0106	4.5021		43
44	3.6632	4.2055	4.7735		44
45	3.8038	4.4181	5.0782		45
46	3.9530	4.6512	5.4235		46
47	4.1111	4.9075	5.8180		47
48	4.2782	5.1902	6.2726		48
49	4.4549	5.5038	6.8032		49
50	4.6417	5.8536	7.4317		50
51	4.8393	6.2470			51
52	5.0486	6.6935			52
53	5.2708	7.2061			53
54	5.5067	7.8017			54
55	5.7577	8.5048			55

ENDOWMENT ASSURANCE TABLE.

ANNUAL PREMIUM ON A POLICY OF $100.

Age at issue.	Policy due at 40.	Policy due at 45.	Policy due at 50.	Policy due at 55.	Policy due at 60.	Policy due at 65.	Age at issue.
14		$2.475	$2.113	$1.868	$1.704		14
15		2.587	2.197	1.935	1.759		15
16	$3.356	2.707	2.285	2.004	1.816	$1.694	16
17	3.545	2.835	2.379	2.077	1.876	1.746	17
18	3.752	2.937	2.478	2.153	1.939	1.799	18
19	3.978	3.122	2.585	2.234	2.004	1.855	19
20	4.228	3.283	2.698	2.320	2.073	1.914	20
21	4.504	3.458	2.819	2.410	2.145	1.974	21
22	4.812	3.648	2.949	2.506	2.220	2.038	22
23	5.156	3.855	3.089	2.608	2.300	2.104	23
24	5.544	4.083	3.239	2.717	2 384	2.174	24
25	5.985	4.333	3.402	2.832	2.473	2.247	25
26	6.489	4.611	3.578	2.956	2.567	2.323	26
27	7.082	4.920	3.770	3.088	2.666	2.404	27
28	7.752	5.265	3.979	3.231	2.772	2.489	28
29	8.558	5.654	4.208	3.384	2.884	2.578	29
30	9.526	6.096	4.461	3.549	3.004	2.672	30
31		6.601	4.740	3.728	3.132	2.772	31
32		7.185	5.051	3.923	3.268	2.877	32
33		7.866	5.398	4.135	3.415	2.989	33
34		8.673	5.789	4.368	3.573	3.108	34
35		9.641	6.232	4.624	3.743	3.235	35
36			6.739	4.906	3.928	3.370	36
37			7.325	5.221	4.128	3.515	37
38			8.008	5.572	4.347	3.670	38
39			8.816	5.967	4.586	3.837	39
40			9.787	6.415	4.849	4.017	40
41				6.927	5.139	4.212	41
42				7.518	5.462	4.424	42
43				8.207	5.822	4.655	43
44				9.022	6.227	4.908	44
45				10.000	6.686	5.185	45
46					7.210	5.491	46
47					7.813	5.830	47
48					8.515	6.208	48
49					9.343	6.630	49
50					10.332	7.105	50
51					11.536	7.645	51
52						8.265	52
53						8.983	53
54						9.826	54
55						10.831	55

NON-FORFEITING TABLE.

Annual Premiums payable for five, ten, fifteen, or twenty years, to secure $100, payable at the death of the insured. If the insured be living, and should fail to make any payment when due, the policy will not be forfeited (all the other conditions of the policy being complied with), but will be good for a sum payable at death proportioned to the number of premiums paid; that is, after one premium, for one-fifth, one-tenth, one-fifteen, or one-twentieth of the sum assured, and so on for each successive premium paid. Dividends of Return Premiums will be made during life, on the ordinary annual Life Rates.

Age.	Five Paym's	Ten Paym's	Fifteen Paym's	Twenty Paym's	Age.	Five Paym's	Ten Paym's	Fifteen Paym's	Twenty Paym's
21	$7.696	$4.294	$3.190	$2.656	36	$10.470	$5.914	$4.434	$3.726
22	7.848	4.386	3.260	2.716	37	10.702	6.050	4.546	3.816
23	8.020	4.482	3.332	2.778	38	10.936	6.190	4.648	3.906
24	8.192	4.582	3.412	2.844	39	11.178	6.336	4.758	4.002
25	8.374	4.690	3.494	2.914	40	11.422	6.478	4.870	4.096
26	8.556	4.798	3.574	2.986	41	11.664	6.612	4.972	4.186
27	8.758	4.914	3.666	3.062	42	11.886	6.738	5.062	4.276
28	8.962	5.030	3.750	3.138	43	12.110	6.862	5.164	4.366
29	9.154	5.138	3.838	3.210	44	12.332	6.988	5.266	4.462
30	9.316	5.234	3.908	3.274	45	12.566	7.122	5.366	4.564
31	9.488	5.330	3.980	3.338	46	12.808	7.262	5.478	4.676
32	9.660	5.430	4.060	3.408	47	13.062	7.410	5.610	4.800
33	9.842	5.540	4.152	3.480	48	13.336	7.574	5.752	4.936
34	10.258	5.658	4.246	3.558	49	13.648	7.762	5.914	5.092
35	14.400	5.782	4.334	3.640	50	13.984	7.972	6.106	5.274

EXAMPLES FOR PRACTICE.

1. What sum must a person pay annually to an Insurance Company, for a life policy of $2750, his age being 31 years at the issue of the policy?

OPERATION.

$2750 × .023728 = $65.252, *Ans.*

ANALYSIS. — We multiply the face of the policy, $2750, by the rate % found opposite 31 years in the Life Table, expressed decimally, and obtain $65.252, the annual premium required.

2. A person at the age of 23 insures his life for $1500; what is his annual premium? *Ans.* $28.197.

3. What is the annual payment on an endowment policy for $3500, payable at the age of 55, issued to a person at the age of 27 years?

4. A man 32 years old took an endowment assurance policy for $850, due at the age of 50, and died when 49 years old; how much more would his heirs have realized if he had taken a life policy for the same amount with payment to cease at 50? *Ans.* $265.40.

5. A gentleman obtained an insurance on his life at the age of 26, and died at the age of 45, the policy taken required annual payments of premium during life, and secured to his heirs $3659.24 more than the whole premium paid. What is the face of the policy?

Ans. $6195.80.

6. Leo Perry, 50 years old, effects an insurance for life for $7000, for which he pays an annual premium of $4.60 on each $100 insured. If he should die at the age of 80 years, how much less will be the amount of insurance than the payments, allowing the latter to be without interest? *Ans.* $2660.

ANNUITIES.

518. An **Annuity** is a fixed sum of money payable annually, or at the end of equal periods of time, to continue for a given number of years, for life, or for ever.

519. A **Certain Annuity** is one whose period of continuance is for a definite number of years.

520. A **Contingent Annuity** is one whose time of commencement, or ending, or both, is uncertain.

521. A **Perpetual Annuity**, or **Perpetuity**, is one which continues forever.

522. An **Annuity in Reversion** (whether certain, contingent, or perpetual) is one which begins at a specified future time, or on the occurrence of a specified future event.

523. An **Annuity in Arrears**, or **Forborne**, is one whose payments were not made when due. Interest is to be reckoned on each payment of an annuity in arrears, from its maturity, the same as on any other debt.

524. The **Amount**, or **Final Value**, of an annuity is the sum of all the payments, at compound interest, from the time each became due, to the end of the annuity.

525. The **Present Value** of an annuity, at compound interest, is the sum of the present values of all its payments; or the present worth of its final value. The present value, put out at compound interest, will amount, at the time of the expiration of the annuity, to its final value.

NOTES.—1. An annuity is said to be *deferred*, when it does not commence until after a certain period; *reversionary*, when it does not commence until after the occurrence of some specified future event, as the death of a certain person; and in *possession*, when it has commenced, or commences immediately.

2. The subject of annuities is one of great practical importance in the affairs of life. Its principal applications are leases, life-estates, pensions, rents, dowers, reversions, life-insurance, etc. The problems are readily solved by means of tables which give the present and final values of $1 for a given number of years, at the ordinary rates of interest.

Showing the amount of an annuity of $1 per annum, at compound interest for any number of years not exceeding fifty.

Years.	3 per cent.	3½ per cent.	4 per cent.	5 per cent.	6 per cent.	7 per cent.
1	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
2	2.030000	2.035000	2.040000	2.050000	2.060000	2.070000
3	3.090900	3.106225	3.121600	3.152500	3.183600	3.214900
4	4.183627	4.214943	4.246464	4.310125	4.374616	4.439943
5	5.309136	5.362466	5.416323	5.525631	5.637093	5.750739
6	6.468410	6.550152	6.632975	6.801913	6.975319	7.153291
7	7.662462	7.779408	7.898294	8.142008	8.393838	8.654021
8	8.892336	9.051687	9.214226	9.549109	9.897468	10.259803
9	10.159106	10.368496	10.582795	11.026564	11.491316	11.977989
10	11.463879	11.731393	12.006107	12.577893	13.180795	13.816448
11	12.807796	13.141992	13.486351	14.206787	14.971643	15.783599
12	14.192030	14.601962	15.025805	15.917127	16.869941	17.888451
13	15.617790	16.113030	16.626838	17.712983	18.882138	20.140643
14	17.086324	17.676986	18.291911	19.598632	21.015066	22.550488
15	18.598914	19.295681	20.023588	21.578564	23.275970	25.129022
16	20.156881	20.971030	21 824531	23.657492	25.670528	27.888054
17	21.761588	22.705016	23 697512	25 840366	28.212880	30.840217
18	23.414435	24.499691	25.645413	28.132385	30.905653	33 999033
19	25 116868	26.357180	27.671229	30.539004	33.759992	37.378965
20	26.870374	28.279682	29.778079	33.065954	36.785591	40.995492
21	28.676486	30.269471	31.969202	35.719252	39.992727	44.865177
22	30.536780	32.328902	34.247970	38.505214	43 392290	49 005739
23	32.452884	34.460414	36.617889	41 430475	46.995828	53 436141
24	34.426470	36.666528	39.082604	44 501999	50.815577	58 176671
25	36.459264	38.949857	41 645908	47.727099	54.864512	63.249030
26	38.553042	41.313102	44.311745	51.113454	59.156383	68.676470
27	40.709634	42.759060	47.084214	54.669126	63.705766	74.483823
28	42.930923	46.290627	49.967583	58.402583	68 528112	80.697691
29	45.218850	48.910799	52.966286	62.322712	73.639798	87.346529
30	47.575416	51.622677	56.084938	66.438848	79.058186	94.460786
31	50.002678	54.429471	59.328335	70.760790	84 801677	102.073041
32	52.502759	57.334502	62.701469	75.298829	90.889778	110.218154
33	55.077841	60.341210	66.209527	80.063771	97.343165	118.933425
34	57.730177	63 453152	69.857909	85.066959	104.183755	128.258765
35	60.462082	66.674013	73.652225	90.320307	111.434780	138.236878
36	63.271944	70.007603	77.598314	95.836323	119.120867	148.913460
37	66.174223	73.457869	81.702246	101.628139	127.268119	160.337400
38	69.159449	77.028895	85.970336	107.709546	135.904206	172.561020
39	72.234233	80.724906	90.409150	114 095023	145.058458	185.640292
40	75.401260	84.550278	95.025516	120.799774	154.761966	199.635112
41	78.663298	88.509537	99.826536	127.839763	165.047684	214.609570
42	82.023190	92.607371	104.819598	135.231751	175.950645	230.632240
43	85.483892	96.848629	110.012382	142.993339	187.507577	247.776496
44	89.048409	101.238331	115.412877	151.143006	199.758032	266.120851
45	92.719861	105.781673	121.029392	159.700156	212.743514	285.749311
46	96.501457	110.484031	126.870568	168.685164	226.508125	306 751763
47	100.396501	115.350973	132.945390	178.119422	241.098612	329.224386
48	104.408396	120.388297	139.263206	188.025393	256.564529	353.270093
49	108.540648	125.601846	145.833734	198.426663	272.958401	378.999000
50	112.796867	130.999910	152.667084	209.347976	290.335905	406.528929

Showing the present worth of an annuity of $1 per annum, to continue for any number of years not exceeding fifty.

Years.	3 p. cent.	3½ p. cent.	4 p. cent.	5 p. cent.	6 p. cent.	7 p. cent.	Years.
1	0.970874	0.966184	0.961538	0.952381	0.943396	0.934579	1
2	1.913470	1.899694	1.886095	1.859410	1.833393	1.808017	3
3	2.828611	2.801637	2.775091	2.723248	2 673012	2.624314	4
4	3.717098	3.673079	3.629895	3.545951	3.465106	3.387209	2
5	4.579707	4.515052	4.451822	4.329477	4.212364	4.100195	5
6	5.417191	5.328553	5.242137	5.075692	4.917324	4.766537	6
7	6.230283	6.114544	6 002055	5.786373	5.582381	5.389286	7
8	7.019692	6.873956	6.732745	6.463213	6.209744	5.971295	8
9	7.786109	7.607687	7.435332	7.107822	6.801692	6.515228	9
10	8.530203	8.316605	8.110896	7.721735	7.360087	7 023577	10
11	9.252624	9.001551	8.760477	8 306414	7.886875	7.498669	11
12	9.954004	9.663334	9.385074	8.863252	8.383844	7.942671	12
13	10.634955	10.302738	9.985648	9.393573	8.852683	8.357635	13
14	11.296073	10.920520	10.563123	9.898641	9.294984	8.745452	14
15	11.937935	11.517411	11.118387	10.379658	9.712249	9.107898	15
16	12.561102	12.094117	11.652296	10.837770	10.105895	9.446632	16
17	13.166118	12.651321	12.165669	11.274066	10.477260	9.763206	17
18	13.753513	13.189682	12.659297	11.689587	10.827603	10.059070	18
19	14.323799	13.709837	13.133939	12.085321	11.158116	10.335578	19
20	14.877475	14.212403	13.590326	12.462210	11.469421	10.593997	20
21	15.415024	14.697974	14.029160	12.821153	11.764077	10.835527	21
22	15.936917	15.167125	14 451115	13.163003	12.041582	11.061241	22
23	16.443608	15.620410	14.856842	13.488574	12.303379	11.272187	23
24	16.935542	16.058368	15.246963	13.798642	12.550358	11.469334	24
25	17.413148	16.481515	15.622080	14.093945	12.783356	11.653583	25
26	17.876842	16.890352	15.982769	14.375185	13.003166	11.825779	26
27	18.327031	17.285365	16.329586	14.643034	13.210534	11.986709	27
28	18.764108	17.667019	16.663063	14.898127	13.406164	12.137111	28
29	19.188455	18.035767	16.983715	15 141074	13.590721	12.277674	29
30	19.600441	18.392045	17.292033	15.372451	13.764831	12.409041	30
31	20.000428	18.736276	17.588494	15.592811	13.929086	12.531814	31
32	20 338766	19.068685	17.873552	15.802677	14.084043	12.646555	32
33	20.765792	19.390208	18.147646	16.002549	14.230230	12.753790	33
34	21.131837	19.700684	18.411198	16 192204	14.368141	12.854009	34
35	21.487220	20.000661	18.664613	16.374194	14.498246	12.947672	35
36	21.832252	20.290494	18.908282	16.546852	14.620987	13.035208	36
37	22 167235	20.570525	19.142579	16.711287	14 736780	13.117017	37
38	22.492462	20.841087	19.367864	16.867893	14.846019	13.193473	38
39	22.808215	21.102500	19.584485	17.017041	14.949075	13.264928	39
40	23.114772	21.355072	19.792774	17.159086	15.046297	13.331709	40
41	23.412400	21.599104	19.993052	17.294368	15.138016	13.394120	41
42	23.701359	21.834883	20.185627	17.423208	15.224543	13.452449	42
43	23 981902	22.062689	20.370795	17.545912	15.306173	13.506962	43
44	24.254274	22.282791	20.548841	17.662773	15.383182	13.557908	44
45	24.518713	22.495450	20.720040	17.774070	15.455832	13.605522	45
46	24.775449	22.700918	20.884654	17.880067	15.524370	13.650020	46
47	25.024708	22.899438	21.042936	17.981016	15.589028	13.691608	47
48	25.266707	23.091244	21.195131	18.077158	15.650027	13.730474	48
49	25.501657	23.276564	21.341472	18.168722	15.707572	13.766799	49
50	25.729764	23.455618	21.482185	18.255925	15.761861	13.800746	50

526. To find the *amount*, or *final value*, of an annuity certain, at compound interest, in *arrears*, or *forborne*.

Ex. An annuity of $400 a year remained unpaid 6 years; what is the amount due, at 6 % compound interest?

OPERATION.

$6.975319, amount of $1 for 6 years. (See Table).
400
$2790.1276; " $400 "

NOTE.—When the annuity draws *simple interest*, the amount is found as in *annual interest*.

527. RULE.—*Multiply the amount, or final value, of an annuity of* $1 *for the given rate and time, found in the table, by the given annuity, and the product will be the required amount.*

528. To find the *present value* of an annuity *certain*.

Ex. What is the present value of an annuity of $80, to continue 20 years, at 5 %?

OPERATION.

$12.46221, present value of $1.
80
$996.9768; " " $80.

NOTE.—Since the present value of an annuity is the *present worth* of its *amount*, or final value, the present value of an annuity may also be found by first finding the amount, and then the present worth of this amount.

529. RULE.—*Multiply the present value of an annuity of* $1, *for the given rate and time, by the given annuity.*

530. To find the *present value* of an annuity in *perpetuity*.

Ex. What is the present value of a perpetual leasehold, which yields an income of $840 a year, at 6 %?

OPERATION.

$840 ÷ .06 = $14000, present val.

ANALYSIS.—The present value must evidently be a principal which yields an annual interest of $840, at 6 per cent.

531. RULE.—*Divide the given annuity by the interest of* $1, *for one year.*

NOTE.—When an annuity is payable semi-annually, or quarterly, interest must be allowed on the half-yearly or quarterly payments to the close of the year.

532. To find the *present value* of an annuity in *reversion*.

Ex. What is the present value of an annuity of $500, deferred 10 years, and to continue 8 years, allowing 6 % compound interest?

OPERATION.

$10.827603,	present value of $1 for 18 years.					
7.360087,	"	"	"	10	"	
$ 3.467516,	"	"	"	8	"	deferred 10 years.
500						
$1733.758000,	"	"	$500		"	" "

ANALYSIS.—The present worth of an annuity of $1 for 18 years must be equal to its present worth for 10 years, plus its present worth for the 8 succeeding years. Hence, the present worth of an annuity of $1 for 8 years deferred 10 years, must equal its present worth for 18 years *minus* its present worth for 10 years. The present worth of $500 is evidently 500 times the present worth of $1.

533. RULE.—*Find from the table the present value of an annuity of* $1, *commencing at once and continuing till the* TERMINATION *of the annuity, and also, till the reversion* COMMENCES. *Multiply the difference of these present values, by the given annuity.*

NOTE.—If the annuity is *perpetual*, the present worth of $1, commencing at once, is found according to the rule (530).

534. To find the *annuity*, the present or final value, time and rate being given.

Ex. An annuity running 8 years, at 6 %, compound interest, is worth $623.70; what is the annuity?

OPERATION.

$623.70 ÷ $6.209744 = $100.43 + .

ANALYSIS. — Since $6.209744, at 6 % compound interest, for 8 years, yields an annuity of $1, $623.70 will yield an annuity equal to $623.70 ÷ 6.209744.

535. *Divide the present or final value of the given annuity by the present or final value of an annuity of* $1, *for the given rate and time.*

NOTE.—When the amount of an annuity, the time and rate, are given, the *annuity* may be found *by dividing the given amount by the amount of* $1 *for the given time and rate.*

EXAMPLES FOR PRACTICE.

1. What is the present value of an annuity of $1300, to continue 12 years, at 5 %? *Ans.* $11522.22 + .

2. A ground rent in the city of Quebec yields an annual income of $1926, at 6 % int. What is the value of the estate? *Ans.* $32100.

3. Find the final value of an annuity of $600, running 10 years, at 4 % compound interest. *Ans.* $7200.66 + .

4. The present value of an annuity, to be continued 14 years, at 6 %, compound interest, payable annually, is $5520; required the annuity. *Ans.* $593.86 + .

5. What is the present value of a leasehold of $900, deferred 6 yr., and to run 10 yr., at 5 % compound interest? *Ans.* $2804.43 + .

6. What is the amount of an annuity of $1000, forborne for 15 yr., at 3½ % compound interest? *Ans.* $19295.68.

7. A widow is entitled to $420 a year, payable semi-annually, for 18 years; what is the present value of her interest, at 7 %, compound interest? *Ans.* $4261.

8. A yearly pension, unpaid for 12 years, at 6 %, compound int., amounted to $9550.2762; what was the pension? *Ans.* $1139.12 + .

9. What sum should be paid for a perpetual annuity of $1500, payable semi-annually, interest being at 6 %? *Ans.* $25000.

10. A lease, whose rental is $600 a year, is left to a son and a daughter. The son is to receive the rent for 8 years and the daughter for the 12 yr. succeeding. What is the present value of the daughter's interest, allowing 5 % compound interest? *Ans.* $3599.39 + .

11. What will an annuity of $40, payable semi-annually, amount to, in arrears for 5 years, at 6 %? *Ans.* $458.55 + .

12. I wish to purchase an annuity which shall secure to my ward, at 5 % compound interest, $300 for 15 years. What must I deposit in the annuity office? *Ans.* $3113.89 + .

13. A laborer agreed to work for 1 year and 6 months at the rate of $25 payable monthly; he was paid only at the end of the 18 mo.; how much did he receive, being allowed 6 % simple interest per annum? *Ans.* $469.12½.

14. A merchant being desirous to secure a dowry for his son, deposits annually a sum which, placed at simple interest, commencing at his 12th year to his 23rd., amounts to $630, and that due for dowry, to $5580; find the value of the yearly deposit, and the rate %.

Ans. Deposit, $300; rate, 30 %.

15. A founder wishes to economise $360 in 5 years; what sum shall he have to deposit at the end of each year so as to have the required sum at the end of the 5th. year, comprising both capital and compound interest, at 5 % per annum? *Ans.* $65.15.

16. What is the amount of an annuity of $45, payable semi-annually, for 3 years, at 7 % compound interest? *Ans.* $294.75 + .

17. A servant leaves his yearly salary of $250 in the hands of his master, on condition that he will allow him 4½ % interest, per annum, to be added to his capital; find how much will be due the servant at the end of 15 years. *Ans.* $5196.01.

18. A marbler buys divers blocks of marble measuring altogether 14.850 cubic yards at $116 a cu. yard; he pays $122.60 in cash, and settles the remainder in 4 annuities; what is the amount of each annuity at 4½ % interest. *Ans.* $445.99.

19. Find the amount of an annuity of $225 for 5½ yr. payable every three months, interest at 1½ % also quarterly. *Ans.* $5729.62½.

20. An oil merchant bought 32 bbl. of superfine olive oil, for which he paid yearly $190 for 10 years. If money was worth 6 % per annum compound int., what was the cost of a barrel? *Ans.* $43.70.

21. A planter agrees to pay $598 in 13 payments, in such a manner that each succeeding payment shall be greater than the preceding one by $6; what will be his first and last payment? *Ans.* Last $82.

22. What must be paid for a life annuity of $560 a year, by a person aged 55 years, at 5 %, compound interest? *Ans.* $5794.32.

23. An officer who is 54 years old, wishes to secure an annuity of £575. The company to which he applies charges 5 % per aunum on the capital deposited: what capital must he deposit, if interest be calculated at 4½ % yearly, and if the probable duration of his life be 18 years and 1 month? *Ans.* £7341 12 5½.

24. A laborer, from the age of 16 to 60 years, spent yearly $5 for tobacco; at his death, he left $416.66⅔ to his children. How much would he have left them, had he dispensed himself from that useless habit, and placed every year that money at 6 % compound interest? *Ans.* $1415.45⅔.

25. A mechanic bought tools for the sum of $1120; what did he pay yearly, so as to cancel his debt in 3 years, at 5 % per annum compound interest? *Ans.* $411.27.

26. I have deposited $5000 at 5 % interest, on condition that I will withdraw only $150 yearly, and that the remainder of the interest will be added to the capital; what will be due me at the end of 12 years? *Ans.* $6591.71.

27. When dying, a father left a sum of money to his twin children, a boy and a girl 10 years old. The sum appointed for the boy was deposited at 7 %, simple interest, and that for the girl, at 6 % compound interest, payable semi-annually; the amount for each was $2000 when 21 years of age; what sum was appointed to each of them?

Ans. $1129.94 +, for the boy; $1043.78 +, for the girl.

28. A carpenter furnished and hung blinds to 35 windows each being 1⅔ yards wide and twice as high. He was paid at the rate of $3.37½ per square yard, but observe that $\frac{3}{10}$ of the height of the window must be added to make up for the heading of the blinds. He was paid in 3 instalments, the first in cash, and the others yearly; interest being at 5½ %; what was each payment? *Ans.* $232.40.

29. A person wishes to sell an 11 years annuity of $700, which is to open 9 years hence; how much will the purchaser pay in ready money, if he is allowed a yearly 6 % discount? *Ans.* $3267.41.

30. A roof of 128 square yards was overlaid with a coat of bituminous cement of ⅛ of an inch in thickness, at 18s. 1½d. a sq. yard. The contractor was paid in 6 instalments payable at the end of each year, except the last which was paid 5 mo. after the preceding one, at 5 % comp. int.; what was each payment? *Ans.* £19 13 8¾.

31. A man who is 45 years old, withdraws from business, having secured a yearly income of $5340 from stock bearing 4½ %; he then sells his stock at the rate of $93.25, and invests the amount in a life annuity. What shall be his yearly income; 1° if he has to pay ⅛ % brokerage for selling his stock; 2° if the company where he secures the annuity charges 9 % for management; 3° if the int. is calculated at 4½ %? The man is supposed to live yet 24 yr. 11 mo. *Ans.* $6995.38.

32. A debtor owes $20000 for which he pays int. at 7 %; what sum could he yearly join to his capital so as to cover both debt and int. in 20 yr., if 5 % int. is allowed on his yearly acct.? *Ans.* $2004.85.

33. A clerk receiving a salary of $56 per month, deposits weekly

$8 in a Savings Bank, at 3½ %; and continues the same for 20 years, commencing at the age of 25. What will be his capital at the end of the 20th year, including an income of $40 from stock bearing 4½%, which, at his request, the bank purchased for him from his deposits every second year, at the average rate of 93 %? *Ans.* $12352.94.

34. An engineer who earns on an average $2.30 a day, and works 25 days a month, spends yearly $170 for house-rent, $325 for family expenses, and $60 for sundry expenses. We want to know; 1° how much he may save yearly; 2° what will be the total value of his yearly savings by depositing them at 6 % compound interest, from his 30th to his 47th year; 3° what yearly income he will enjoy at the age of 64, if at 47 he converts the total value of his savings into an annuity bearing 4½% interest, supposing him to live to the age of 75 and 4 months? *Ans.* 1° $135; 2° 3808.74; 3° $943.68 + .

TEMPERATURE—THERMOMETERS.

536. Temperature is a term employed to denote the condition of a body in respect to heat, or cold; it also expresses the greater or less capacity of a body to excite in us the sensation of heat or cold.

NOTE.—Heat and cold are correlative terms; that is, as the former increases in a body, the latter decreases, and the converse. Temperature generally refers to the amount of sensible heat in a body; cold being regarded as the absence of heat.

537. A **Thermometer** is an instrument used to measure the temperature of bodies.

NOTES.—1. The construction of thermometers depends on the principle, which is universal, that bodies are expanded by increasing and contracted by decreasing their temperature. The thermometer commonly used for measuring temperatures neither extremely high nor extremely low consists of a glass tube having a small bore of uniform diameter, and at its end a bulb within which is mercury. There is also a scale which measures in degrees the length of the column of mercury, which by its expansion or contraction within the tube indicates the temperature to which the thermometer is exposed.

2. Thermometers are graduated by marking on the tubes, or attached plates, two points at which the mercury stands at fixed and easily ascertained temperatures, the lower being that of freezing water called the freezing point, and the higher that of boiling water, when the barometer stands at 29.92 inches (760 millim.).

538. In the *centigrade* thermometer, or that of *Celsius*, the freezing point is marked zero (0°), the boiling point, 100°; and the intermediate space is divided into 100 equal parts called degrees.

539. In *Fahrenheit's* thermometer the freezing point is marked 32°, and the boiling point, 212°, and the interval (180°) is divided into 180 equal parts.

NOTE.—The 0° of Fahrenheit's thermometer is the temperature of a mixture of snow and common salt.

540. In Reaumur's thermometer the freezing point is marked 0°, and the boiling point, 80°, the interval being divided into 80 equal parts or degrees.

The graduations of these instruments may be extended upward from the boiling point, or downward from the freezing point of water, by repeating in these directions the equal divisions between these points for each thermometer. But below the freezing point for mercury (— 39$\frac{4}{9}$° C., — 39° F., — 31$\frac{5}{9}$° R.), and above the boiling point of mercury (348$\frac{8}{9}$° C.; 660° F.; 279$\frac{1}{9}$° R.), the instruments would cease to indicate the temperature by the scales. Their indications are, in fact, unreliable for temperatures above 600° by Fahrenheit's scale (313$\frac{1}{3}$° C.; 250$\frac{2}{3}$° R.)

NOTE.—For high temperatures an instrument called a pyrometer is used. Wedgwood's pyrometer = 1077° on Fahrenheit's scale. For measuring temperatures below — 39° F., spirits of wine or alcohol instead of mercury is used for filling the thermometer.

Since the interval between the freezing and boiling points of water is divided into 100 equal parts in the Centigrade, 180 equal parts in Fahrenheit's, and 80 equal parts in Reaumur's thermometers, it follows that 1° F. = $\frac{5}{9}$° C. = $\frac{4}{9}$° R. But in Fahrenheit's scale the freezing point of water is marked 32°; whereas in the Centigrade and Reaumur's scales, this point is marked 0°. Hence,

541. To change a temperature as given by Fahrenheit's scale into the same as given by the Centigrade scale.

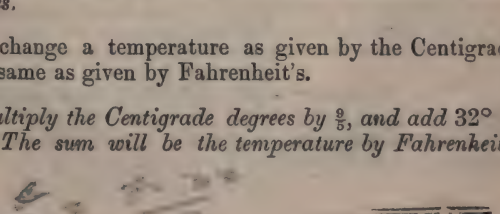
RULE.—*Subtract* 32° *from Fahrenheit's degrees, and multiply the remainder by* $\frac{5}{9}$. *The product will be the temperature in Centigrade degrees.*

542. To change from Fahrenheit's to Reaumur's scale.

RULE.—*Subtract* 32° *from Fahrenheit's degrees, and multiply the remainder by* $\frac{4}{9}$. *The product will be the temperature in Reaumur's degrees.*

543. To change a temperature as given by the Centigrade scale into the same as given by Fahrenheit's.

RULE.—*Multiply the Centigrade degrees by* $\frac{9}{5}$, *and add* 32° *to the product. The sum will be the temperature by Fahrenheit's scale.*



544. To change from Reaumur's to Fahrenheit's scale.

RULE.—*Multiply the degrees on Reaumur's scale by $\frac{9}{4}$, and add 32° to the product. The sum will be the temperature by Fahrenheit's scale.*

NOTE.—The above rules are applicable to all temperatures, observing that on changing degrees below 32 on Fahrenheit's scale to degrees on the Centigrade, or on Reaumur's scale, the *minus* sign must be prefixed to the result; and that when we change degrees below the zero on the Centigrade, or on Reaumur's scale, to degrees on the Fahrenheit scale, we subtract the degrees on either of the first scales after they are converted into degrees on Fahrenheit's scale from 32, if the degrees thus converted be less than 32, and give the remainder the *plus* sign; but if the degrees on either of the first two scales changed into degrees on the Fahrenheit scale be greater than 32, we subtract the 32 and prefix the *minus* sign to the result.

545. The degrees on the Centigrade, Fahrenheit's, and Reaumur's scales, corresponding to temperatures differing by 10° Centigrade between the freezing and boiling points of water, are given in the following table:

Centigrade.		Fahrenheit.		Reaumur.
0° Zero	=	32° Freez'g.	=	0° Zero.
10°	=	50°	=	8°
20°	=	68°	=	16°
30°	=	86°	=	24°
40°	=	104°	=	32°
50°	=	122°	=	40°
60°	=	140°	=	48°
70°	=	158°	=	56°
80°	=	176°	=	64°
90°	=	194°	=	72°
100°	=	212°	=	80°

By means of the rules (541, 542, 543, 544,) the student can readily extend this table above and below these limits.

EXAMPLES FOR PRACTICE.

1. What temperature by Fahrenheit's scale corresponds to 176° Centigrade? *Ans.* $348\frac{4}{5}$°.

2. When the temperature of a body by Reaumur's thermometer is 78°, what is it by Fahrenheit's? *Ans.* $207\frac{1}{2}$°.

3. What temperature by Reaumur's thermometer answers to $83\frac{1}{2}$° Centigrade?

4. What temperature by Centigrade's scale corresponds to 45° Fahrenheit? *Ans.* $7\frac{2}{9}$°.

5. When Fahrenheit's thermometer indicates —13°, what should the Centigrade and Reaumur's indicate?

Ans. Centigrade, —25°; Reaumur, —20°.

A METER.....	*is a measure of length:*		
	= 39.37 inches.....	An inch	= .0254+ meter.
	= 3.28+ feet.....	A foot	= .3048+ meter.
	= 1.0936 + yards.....	A yard	= .9144+ meter.
A KILOMETER...	*is 1000 meters:*		
	= 3280 feet 10 inches.....	A yard	= .000914 kilom.
	= .621+ of a mile.....	A mile	= 1.609+ kilom.
AN ARE.....	*is a measure of surface:*		
	= 119.6 sq. yards.....	A sq. yard	= .834+ centiare
	= 3.953+ sq rods.....	A sq. rod	= .2529+ are.
A HECTARE.....	= 2.471 acres.....	An acre	= .4046+ hect.
	= .00386+ sq. mile.....	A sq. mile	= 258.99+ hect.
A CENTIARE.....	*is a square measure:*		
	= 1550 sq. inches.....	A sq. inch	= .000645+ cent.
	= 10.763+ sq. ft.....	A sq. foot	= .0929+ cent.
A STERE.....	*is a solid measure:*		
	= a cubic meter.....	A cu. inch	= .0000163+ stere
	= .2759 of cord of wood.....	A cord	= 3.62+ stere.
	= 1.3079+ cubic yards.....	A cu. yard	= .764+ stere.
	= 35.3144+ cubic feet.....	A cu. foot	= .0283+ stere.
A LITER.	*is a measure of capacity:*		
	= a cubic decimeter.....	A cu. foot	= 28.32 liters.
	= .908+ of a quart, Dry M.	A qt., D. M.	= 1.1013+ liters.
	= 1.056+ quarts Liquid M.	A qt., W. M.	= .946+ liter.
	= .2641+ gal., Liquid M...	A gallon	= 3.78+ liters.
	= 33.81 fluid oz., Apoth. M.	A fluid oz.	= .02958+ liter.
	= 61.02+ cubic inches.	A cu. inch	= .0163+ liter.
A HECTOLITER..	= 2.83+ bushels.....	A bushel	= .352+ hectol.
	= 3.531714 cubic feet.....	A cu. foot	= .283+ hectol.
A GRAM.....	*is a measure of weight:*		
	= 15.432 grains, Troy Wt...	A grain	= .0648+ gram.
	= 0.35+ ounce, Av. Wt.....		
A KILOGRAM....	= 2.204+ lb. Av. Weight...	An avoir. lb.	= .4536+ kilog.
	= 35.27+ oz. Av. Weight...	An avoir. oz.	= .0283+ kilog.
	= 2.679 lb. Troy Weight.....	A Troy lb.	= .373+ kilog.
A TONNEAU.....	= 2204.6 lb. Av. Weight.....	An avoir. lb.	= .000453+ ton.
	= 1.102+ tons.....	A ton	= .907+ tonneau
A GRADE.....	*is a measure of angles:*	A right angle	= 100 grades.
	= .9 of a degree	A degree	= 1.11 grades.

NOTES.—1. The following are very nearly equivalents; viz., 2½ centimeters, 1 inch; 3 decimeters, 1 foot; 5 meters, 1 rod; 1610 meters, 1 mile; 40 ares, 1 acre; 4 liters, 1 gallon; 35 liters, 1 bushel; 30 grams, 1 ounce, avoirdupois weight; 1 kilogram, 2⅕ lb., av. weight.

2. The equivalents in the above table are calculated from the fundamental units, whose equivalents are copied in the tables on 126, 127, 128, from the schedule tables accompanying the law authorizing the use of the system in the United States.

MENSURATION.

DEFINITIONS.

546. Mensuration treats of the measurement of lines, surfaces and solids.

547. A **Point** is that which has place, or position, but not magnitude.

548. A **Line** has length without breadth or thickness, and may be straight or curved.

549. A **Surface** is that which has length, or breadth, without height, or thickness. There are three kinds of surfaces; viz., *plain*, *convex*, or *curved*, and *concave*.

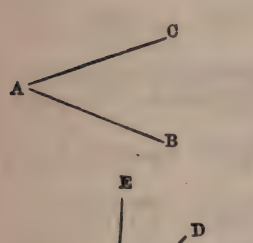
550. A **Plane Surface** is one, every point of which is touched by a straight line, extended over and upon it.

551. A **Curved Surface** is one that has length and breadth without thickness, and is constantly changing its direction.

552. A **Concave Surface** is the reverse of the curved, and constitutes the interior surface of a hollow sphere.

553. A **Solid, Volume,** or **Body,** is that which has length, breadth, and thickness. Length, breadth, and thickness, are called dimensions. Hence, a solid has three dimensions, a surface two, and a line one.

ANGLES.



554. An **Angle** is the divergence of two straight lines from a common point; as the angle A. Also read A B C. The two straight lines are called the *sides* of the angle, and the common point of intersection, the *vertex*.

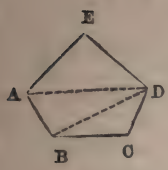
555. A **Right Angle** is an angle formed by a straight line and a perpendicular to it, and contains 90°; as the angles A B E and E B C.

556. An **Acute Angle** is one less than a right angle; as the angles E B D, D B C.

557. An **Obtuse Angle** is one greater than a right angle; as the angle A B D.

558. **Parallel** lines are those that lie in the same direction; they are everywhere equally distant from each other; as A B and C D.

POLYGONS.



559. A **Polygon** is a figure bounded by straight lines; as A B C D E.

560. A **Diagonal** of a figure is a line which joins two of its opposite angles; as A B and B D.

561. The **Perimeter** of a polygon is the sum of all its sides; as A B C D E.

562. The **Base** of a figure is the side on which it is supposed to stand; as B C.

563. The **Altitude** is a line drawn from the opposite side, or angle, perpendicular to the base.

564. The **Area** of a figure is the number of square units in its surface.

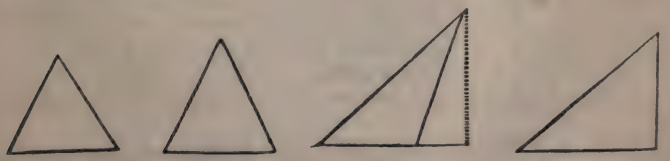
565. Polygons of three sides are called *triangles;* of 4, *quadrilaterals;* of 5, *pentagons;* of 6, *hexagons:* of 7, *heptagons;* of 8, *octagons;* of 9, *nonagons;* of 10, *decagons;* of 11, *undecagons;* of 12, *dodecagons,* etc.

TRIANGLES.

566. There are several kinds of triangles, namely:

1. An **Equilateral Triangle,** the three sides of which are equal.
2. An **Isosceles Triangle,** two sides of which are equal.
3. A **Scalene Triangle,** the three sides of which are unequal.
4. A **Right-angled Triangle,** which has one right angle.

In the right-angled triangle, the side opposite the right angle, is called the *hypothenuse.*



Equilateral. Isosceles. Scalene. Right-angled.

NOTE.—The dotted lines represent the altitude of the triangle.

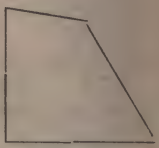
QUADRILATERALS.

567. There are three kinds of quadrilaterals, namely:

1. The **Parallelogram,** which has its opposite sides parallel.
2. The **Trapezoid,** which has only two of its sides parallel.
3. The **Trapezium,** which has none of its sides parallel.

Parallelogram.

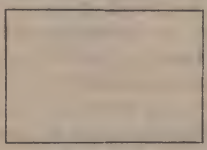
Trapezoid.



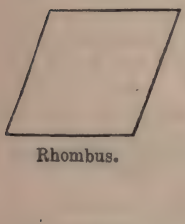
Trapezium.

568. There are four kinds of parallelograms, namely:

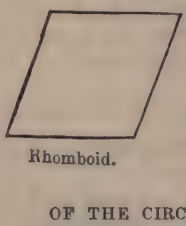
Square.



Rectangle.



Rhombus.



Rhomboid.

1. The **Square,** whose sides are equal, and whose angles are right angles.

2. The **Rectangle,** which is any right-angled parallelogram.

3. The **Rhombus,** or **Lozenge,** whose sides are equal, and whose angles are not right angles.

4. The **Rhomboid,** whose opposite sides are equal, but its angles are not right angles, and its length exceeds its breadth.

OF THE CIRCLE.

569. A **Circle** is a plane figure bounded by a line, every part of which is equally distant from a point within called the *centre*, as A G H B C E D.

The **Circumference** of a circle is the line that bounds it. It is divided into 360 parts called *degrees*.

570. An **Arc** is any portion of the circumference; as A D, A G.

571. A **Radius** is a line drawn from the centre to the circumference; as O A, or O C.

572. A **Diameter** is a line which passes through the centre, and is terminated by the circumference; as A B.

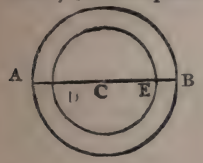
573. A **Chord** is a straight line joining the extremities of an arc; as D C.

574. A **Secant** is a line which meets the circumference in two points, and lies partly within and partly without the circle; as G H.

575. The **Segment** of a circle is the portion included by an arc and its chord; as the space included by the arc D E C and the chord D C.

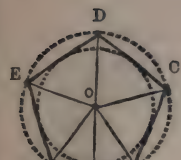
576. The **Sector** of a circle is the portion included by two radii and the intercepted arc; as the space O B C O.

577. A **Zone** is the space between two parallel chords of a circle; as the space A G H B A.



578. A **Circular Ring** is the space included between the circumference of two concentric circles; as the space between the rings A B and D E.

579. A **Lune**, or **Crescent**, is the space contained between the arcs of two intersecting circles.



580. A **Regular Polygon** is one whose sides and angles are *equal;* as the pentagon A B C D E.

581. An **Irregular Polygon** is one whose sides and angles are *unequal.*

582. An **Inscribed Polygon** is one whose vertices of all the angles are in the circumference; as A B C D E, and whose radius of the circle is A O.

583. A **Circumscribed Polygon** is one whose all sides are tangent to the circumference, and whose radius of the circle is O F.

MENSURATION OF SURFACES.

Problem I.

To find the area of any parallelogram.

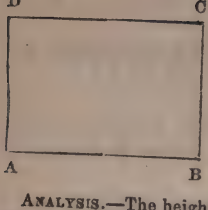
584. Rule.—*Multiply the base by the altitude, and the product will be the area.*

Ex. 1. What is the area of a square A B C D, whose side is 4 yards?

Operation. $4 \times 4 = 16$ square yards, *Ans.*

2. Find the area of the squares whose sides are: 1st 26.30 yd.; 2nd 324.48 yd.; 3rd 638.7 yd.; 4th 1476.004 yd.; 5th 3684.0132 yd.

Ans. 1st 691.69 sq. yd.; 2nd 105287.2704 sq. yd.; 3rd 407937.69 sq. yd., etc.



3. Find the area of a rectangle A B C D, of which the base A B = 7 yards, and the altitude A D = 4 yards.

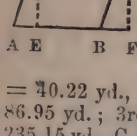
OPERATION. $7 \times 4 = 28$ sq. yd., *Ans.*

4. What is the height, or altitude, of a rectangle whose base is 14 yards, and area, 112 square yards?

ANALYSIS.—The height required should be such, as being multiplied by the base, the product will equal 112: hence, that height should be equal to $112 \div 14 = 8$ yards, *Ans.*

5. Find the area of each of the following rectangles, whose bases and altitudes are: 1st A B = 46.70 yd., B C = 15.45 yd.; 2nd A B = 146.24 yd., B C = 75.20 yd.; 3rd A B = 206.75 yd., B C = 147.24 yd.; 4th A B = 467.35 yd., B. C = 250.75 yd.

Ans. 1st 721.51½ sq. yd.; 2nd 10997.248 sq. yd., etc.



6. What is the area of the rhombus A B C D of which the base A B is 12 feet, and altitude E D, 4 feet?

OPERATION. $12 \times 4 = 48$ sq. feet, *Ans.*

7. Find the area of the rhombus whose bases and altitudes are as follows: 1st A B = 40.22 yd., E D = 32.75 yd.; 2nd D C = 105.75 yd., C F = 86.95 yd.; 3rd A B = 145.20 yd., E D = 127.54 yd.; 4th D C = 235.15 yd., C F = 180.35 yd.; 5th A B = 375.75 yd., E D = 295.85 yd.

Ans. 1st 1317.205 sq. yd.; 2nd 9194.9625 sq. yd.; 3rd 18518.8080 sq. yd., etc.

8. Required the area of a room whose length is 15 feet 6 inches, and width, 7 ft. 8 in. *Ans.* 118 sq. ft. 10 sq. in.

9. What is the difference between the area of a floor 50 feet square, and that of two others, each 25 feet square? *Ans.* 1250 feet.

10. Find the bases of rectangles containing each 19208 sq. yd., their altitudes being respectively, 1st 100 yd., 2nd 224 yd., 3rd 352.80 yd., 4th 705.60 yd., 5th 940.80 yd.

Ans. 1st. 192.08 yd.; 2nd. 85.75 yd., etc.

11. How many boards will be required to floor a room 16 yards long by 8 yards wide, if each board is 3.90 yards long by .32 yard wide? *Ans.* 102.56 boards.

12. A side-walk 35 ft. 3 in. long by 2 ft. 9 in. wide is to be overlaid with a mixture of bitume and sand. What will be the cost at $2.92½ a square foot? *Ans.* $283.54+.

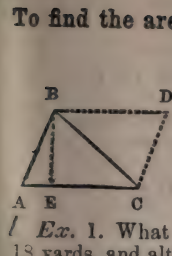
13. There is a square whose area is 3600 yd.; what is the side of a square, and the breadth of a walk along each side and each end of the square, which shall take up just one half of the whole?

Ans. 42.42+ yd., side of the square; 8.78+ yd., breadth of the walk.

14. A piece of land in the form of a parallelogram is 264 yd. long, and its width is $\frac{5}{12}$ of its length; how many bushels of wheat will be required to sow it, if it takes 1½ bu. per 1000 sq. yd.? *Ans.* 43.56 bu.

Problem II.

To find the area of a triangle, when the base and altitude are known.



585. Rule.—*Multiply the base by the altitude, and half the product will be the area.*

Or,

Multiply the base by half the altitude, and the product will be the area.

Ex. 1. What is the area of the triangle A B C, whose base A C is 18 yards, and altitude B E, 10 yards?

Operation. $18 \times 10 \div 2 = 90$ sq. yards, *Ans.*

2. Find the area of triangles whose bases and altitudes are as follows: 1st A C = 66.20 yd., B E = 74.24 yd.; 2nd A C = 94.70 yd., B E = 137.09 yd.; 3rd A C = 109.21 yd., B E = 75.75 yd.; 4th A C = 245.67 yd., B E = 123.45 yd. *Ans.* 1st 2457.34$\frac{2}{3}$ sq. yd.; 2nd 6491.2115 sq. yd., *Ans.*

3. Find the area of a triangle whose base is 76.5 feet, and altitude 92.2 feet. *Ans.* 3526.65 sq. ft.

4. The triangular gable of a certain building has a base of 50 feet, and an altitude of 18 feet; how many square feet of boards will cover it? *Ans.* 450 sq. ft.

5. A field of a triangular form whose base is 530.40 yards, and altitude, 248.50 yards, was sold at the rate of \$28.75 a 100 sq. yd.; how much did it cost? *Ans.* \$18946.88$\frac{1}{4}$.

6. Find the area of a piece of land divided into 3 triangles, measuring as follows: the base of the first is 15$\frac{1}{4}$ toises, altitude 12$\frac{3}{20}$ to.; the base of the 2nd is 25$\frac{1}{10}$ to., altitude 14$\frac{3}{5}$ to.; the base of the 3rd is 18$\frac{9}{20}$ to., altitude 8$\frac{2}{5}$ to. *Ans.* 0 arp. 39.26 + sq. per.

Problem III.

To find the area of a triangle when the three sides are known.

586. Rule.—I. *Add the three sides together and take half their sum.*

II. *From this half sum take each side separately.*

III. *Multiply together the half sum and each of the three remainders, and then extract the square root of the product, which will be the required area.*

Note.—If a triangle be equilateral, its area equals *the square of the sides multiplied by the decimal* .4330127.

Ex. 1. Find the area of a triangle whose sides are 9, 15, and 20 yd.

Operation. $(9 + 15 + 20) \div 2 = 22$, half sum; $22 - 9 = 13$,

1st rem.; 22 — 15 = 7, 2nd rem.; 22 — 20 = 2, 3rd rem. Then, to obtain the product, we have 22 × 13 × 7 × 2 = 4004; $\sqrt{4004}$ = 63.2771 sq. yards, the area, *Ans.*

2. How many square yards of plastering are there in a triangle, whose sides are 15, 20, and 25 feet? *Ans.* 16⅔ sq. yd.

3. The sides of a triangular field are 49 chains, 50.25 chains, and 25.69 chains; what is its area? *Ans.* 61.4979 acres.

4. What is the area of an isosceles triangle, whose base is 30, and each of the equal sides 22.5? *Ans.* 251.55.

5. How many square yards in a triangle whose sides are 8 ft. 5 in., 10 ft. 4 in., and 15 feet? *Ans.* 4.649 sq. yd.

6. How many arpents are there in a triangular field whose sides are 15 to. 3 ft., 24½ to., and 36 to. 5 ft.? *Ans.* 0 arp. 15.063 sq. per.

7. There is a triangle, the longest side of which is 15.6 feet, the shortest side 9.2 feet, and the other side 10.4 feet. What are the contents? *Ans.* 46.139 + feet.

8. How many acres in a triangle whose three sides are 570, 630, and 1147½ yards? *Ans.* 20 A. 3 R. 4.72 per.

9. What is the area of a triangular meadow, each side measuring 35 perches? *Ans.* 530.44 + per.

10. Find the area of each of the following equilateral triangles whose sides are : 1st 15 ft. 8 in.; 2nd 18 ft. 10' 8''; 3rd 81 yards?

Ans. 1st 106.28 + sq. ft.; 2nd 154.49 + sq. ft.; 3rd 2840.99 + square yards.

Problem IV.

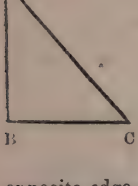
To find the hypothenuse of a right-angled triangle when the base and perpendicular are known.

587. Rule.—I. *Square each of the sides separately.*

II. *Add the squares together.*

III. *Extract the square root of the sum, which will be the hypothenuse of the triangle.*

Note.—Were it to find the base of a triangle whose area and altitude are given, *divide the area by the altitude, and double the quotient, the result will give the base.*



Ex. 1. In the right-angled triangle A B C, we have, A B = 20 feet, and B C = 15 feet, to find A C.

Operation. $20^2 = 400$; $15^2 = 225$; 400 + 225 = 625; $\sqrt{625} = 25$ ft., A C.

2. The height of a mast planted on the brink of a pond, is 144 ft., and the breadth of the pond 84 yards; what is the length of a line which would reach from the top of the mast to the opposite edge of the pond? *Ans.* 290.24 + ft.

3. What is the base of a triangle whose area is 1 acre and 3 roods, and altitude $5\frac{1}{2}$ perches? *Ans.* $101\frac{8}{11}$ per.

4. The area of a triangular field is 7 arpents, and its altitude is 4 yards; find the base in yards. *Ans.* 14371.897 yd.

5. A ladder 50 feet long, will reach to a window 30 feet from the side-walk on one side of a street, and by turning the ladder over to the other side of the street, it will reach a window 40 feet from the side-walk; what is the breadth of the street? *Ans.* 70 feet.

6. The side roofs of a house of which the eaves are of the same height, form a right angle at the top. Now, the length of the rafters on one side is $12\frac{1}{2}$ feet, and on the other $17\frac{1}{4}$ feet; what is the breadth of the house? *Ans.* $21\frac{1}{2}$ feet.

7. What would be the width of the house, in the last example, if the rafters on each side were $12\frac{1}{2}$ feet? *Ans.* 17.67 ft.

8. What would be the width, if the rafters on each side were $17\frac{1}{2}$ feet? *Ans.* 24.78 ft.

Problem V.

When the hypothenuse and one side of a right-angled triangle are known, to find the other side.

588. Rule.—*Square the hypothenuse and also the other given side, and take their difference; extract the square root of this difference, and the result will be the required side.*

Ex. 1. In the right-angled triangle A B C, Prob. IV, there are given A C = 25 feet and A B = 20 feet, find the side B C.

Operation. $25^2 = 625$; $20^2 = 400$; $625 - 400 = 225$; $\sqrt{225} = 15$ feet, *Ans.*

2. The hypothenuse of a triangle is 106 feet, and the perpendicular 90 feet; what is the base? *Ans.* 56 feet.

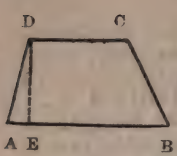
3. The height of a wall on the brink of a river is 50 feet, and a line of 160 feet in length will just reach from the top of it to the opposite bank; what is the breadth of the river? *Ans.* 151.98 feet.

4. The roof of a house whose side walls are each 36 feet high, forms a right angle at the top; if one of the rafters be 10 feet long, and its opposite yoke fellow 12, what is the breadth of the building, the length of the prop set upright to support the ridge of the roof, and the part of the floor at which it must be placed? *Ans.* Breadth of the building 15.6204 ft., length of the prop 43.68 ft., etc.

Problem VI.

To find the area of a trapezoid.

589. Rule.—*Multiply the sum of the parallel sides by the perpendicular distance between them, and then divide the product by two: the quotient will be the area.*



Ex. 1. What is the area of the trapezoid A B C D, having given A B = 34 yards, D C = 26 yards, and D E = 20 yards?

OPERATION. $(34 + 26) \times 20 = 1200$; $1200 \div 2 = 600$ sq. yd., *Ans.*

2. Required the area of trapezoids whose perpendicular heights and bases are: 1st H = 16 ft., B = 24 ft. and 36 ft.; 2nd H = 20.15 yd., B = 34.25 yd. and 62.49 yd.; 3rd H = $36\frac{1}{2}$ ft., B = $75\frac{7}{10}$ ft. and $85\frac{4}{5}$ ft.; 4th H = $55\frac{1}{2}$ yd., B = $106\frac{1}{4}$ yd. and $134\frac{9}{20}$ yd.: 5th H = $70\frac{1}{4}$ ft., B = $145\frac{3}{4}$ ft. and $109\frac{1}{4}$ ft. *Ans.* 1st 480 sq. ft.; 2nd 974.6555 sq. yd.; 3rd 2923.15 sq. ft., etc.

3. What is the area of a trapezoid, the parallel sides of which are 12.41 and 8.22 chains, and the perpendicular distance between them 5.15 chains? *Ans.* 5 A. 1 R. 9.956 per.

4. The parallel sides of a piece of land having the form of a trapezoid, are 2482 and 1644 links, and their perpendicular distance is 1030 links; find its area. *Ans.* 21 A. 0 R. 39.824 per.

5. A field in the form of a trapezoid whose parallel sides are 75.28 and 60.72 yards, and the perpendicular distance 46 yards, was sold for $18768; what shall be the cost of another field of the same kind having a rectangular form, whose base is 115 yards, and altitude 75 yards? *Ans.* $51750.

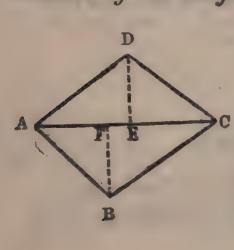
PROBLEM VII.

To find the area of a quadrilateral.

590. *Measure the four sides of the quadrilateral, and also one of the diagonals: the quadrilateral will thus be divided into two triangles, in both of which all the sides will be known. Then, find the areas of the triangles separately, and their sum will be the area of the quadrilateral.*

Or again,

Let fall on the diagonal two perpendiculars drawn from the vertex of the opposite angles; multiply the sum of those perpendiculars by the diagonal, half of the product will be the area.



Ex. 1. Suppose that in the quadrilateral A B C D, the diagonal A C = 88, the perpendicular D E = 27, and B F = 25; what is the area?

OPERATION. $27 + 25 = 52$; $52 \times 88 \div 2 = 2288$, *Ans.*

2. In the quadrilateral A B C D, the side A B = 12 feet, the side B C = 15 ft., the side C D = 10 ft., the side

A D = 18 ft., and the diagonal A C = 22 ft.; what is the area?
Ans. 174.02 sq. ft.,

3. What is the area of a quadrilateral whose diagonal is 40.25 feet, and the 2 perpendiculars 12.25, and 15.05 ft.? *A.* 549.4125 sq. ft.

4. Required the area of a quadrilateral whose diagonal is 108 feet 6 inches, and the perpendiculars 56 feet 3 inches and 60 feet 9 inches. *Ans.* 6347.25 sq. ft.

5. Find the area of each of the following quadrilaterals: 1st diagonal, 65, perpendiculars, 28 and $33\frac{1}{2}$; 2nd perpendiculars, 18 and 16, diagonal, 42; 3rd diagonal, 100, perpendiculars, 35 and 30.
Ans. 1st 1998.75; 2nd 714; etc.

6. In the quadrilateral A B C D, A B = 40 yards, D C = 36 yd., B F = 34 yd., D E = 35.10 yd.; also, F E = 8 yd.; find the area of the quadrilateral. *Ans.* 1280.42+ sq. yd.

7. Suppose that in the quadrilateral A B C D, on account of some obstacle, we could measure only A B, D C, B F, D E, and F E, which measure respectively 25, 22, 20, 21, and 7 yards; what is the area of the quadrilateral? *Ans.* 585.275 sq. yards.

Problem VIII.

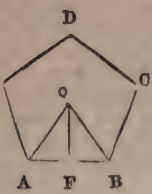
To find the area of a regular polygon.

591. *Multiply the perimeter of the figure by half the perpendicular let fall from the centre on one of the sides, and the product will be the area.* Or,

Square the side of the polygon, then multiply the square so found, by the tabular area set opposite the polygon of the same number of sides, and the product will be the area.

The following Table shows the areas of regular polygons of any number of sides, from *three* to *twelve*, the side of each being unity, or 1; it also shows the length of the radius of the inscribed circle.

Number of sides.	Names.	Areas.	Radius of inscribed circle.
3	Triangle....	0.4330127	0.2886751
4	Square	1.0000000	0.5000000
5	Pentagon....	1.7204774	0.6881910
6	Hexagon	2.5980762	0.8660254
7	Heptagon....	3.6339124	1.0382617
8	Octagon.....	4.8284271	1.2071068
9	Nonagon	6.1818242	1.3737387
10	Decagon.....	7.6942088	1.5398418
11	Undecagon ..	9.3656404	1.2028437
12	Dodecagon...	11.1961524	1.8660254



Ex. 1. Required the area of the regular pentagon A B C D E, each of whose sides A B, B C, etc., is 12 feet, and the perpendicular O F, 9 feet.

OPERATION. $12 \times 5 \times \frac{9}{2} = 270$ ft., *Ans.*

2. Find the area of each of the following regular hexagons: 1st side, = 20, perpendicular = 15; 2nd perpendicular = $12\frac{1}{2}$, one of the sides = 18; 3rd side, = 36, perpendicular = 27.

Ans. 1st 900, 2nd 675, 3rd 2916.

3. Required the area of each of the following regular polygons: 1° of a pentagon whose side is 30, and the perpendicular, 24 feet; 2° of a heptagon whose side is 16, and the perpendicular, $12\frac{1}{4}$ feet; 3rd of an octagon whose perpendicular is 20, and each side, 22 feet. *Ans.* 1st 1800 feet, etc.

4. What is the area of the following regular polygons: 1st of a hexagon whose side is 25.40 chains; 2nd of a nonagon whose side is 30.55 chains; 3rd of a dodecagon whose side is 28.30 chains?

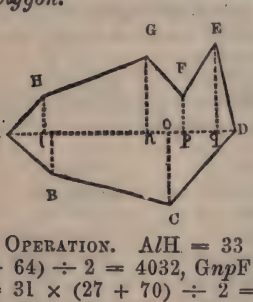
Ans. 1st 1676.174841 sq. ch.; etc.

5. How many pavements in the shape of a regular hexagon, the side of which is 3 inches, are required to pave a room $6\frac{1}{2}$ yards long by $4\frac{3}{4}$ yards wide? *Ans.* 1.31 + pavements.

PROBLEM IX.

To find the area of an irregular polygon.

592. RULE.—*Divide the polygon into triangles and trapezoids; find the area of each separately according to the Prob.* II *and* VI*; the sum of these areas will be the whole area of the polygon.*



Ex. 1. What is the area of the irregular polygon A B C D E F G H measuring as follows: A*l* = 33 yd., *ln* = 84 yd., *np* = 28 yd., *pq* = 31 yd., *q*D = 13 yd.; A*m* = 41 yd., *mo* = 96 yd., *o*D = 52 yd.; H*l* = 32 yd., G*n* = 64 yd., F*p* = 27 yd., E*q* = 70 yd.; *m*B = 32 yd., *o*C = 61 yd.?

OPERATION. A*l*H = $33 \times 32 \div 2 = 528$, H*ln*G = $84 \times (32 + 64) \div 2 = 4032$, G*np*F = $28 \times (64 + 7) \div 2 = 1274$, F*pq*E = $31 \times (27 + 70) \div 2 = 1503.5$, E*q*D = $13 \times 70 \div 2 = 455$, A*m*B = $41 \times 32 \div 2 = 656$, B*mo*C = $96 \times (32 + 61) \div 2 = 4464$, *o*CD = $52 \times 61 \div 2 = 1586$. Hence, the area of the polygon represented by A B C D E F G H = $528 + 4032 + 1274 + 1503.5 + 455 + 656 + 4464 + 1586 = 14498.5$ sq. yd., or 2.995 + acres, *Ans.*

2. Suppose the same irregular polygon A B C D E F G H to measure as follows; A*m* = 10 ft. 3 in., *mo* = 32 ft. 6 in., *o*D = 28 ft.; A*l* = 6 ft. 4 in., *ln* = 21 ft. 9 in., *n*P = 12 ft., P*q* = 11 ft. 6 in., *q*D = 4 ft. 10 in.; B*m* = 10 ft., *co* = 15 ft. 2 in.; H*l* = 10 ft. 9 in., G*n* = 16 ft., FP = 4 ft. 5 in., E*q* = 18 ft. 8 in.; what is its area?

Ans. 1297.82+ sq. ft.

PROMISCUOUS EXAMPLES IN RECTILINEAL SURFACES.

1. A hall is 15 yards long, 7 yards wide, and 4 yards high; allowing 30 sq. yards for the windows, what must be paid for painting it, at the rate of 36 cts. per square yard for the walls, and 60 cts. for the ceiling? *Ans.* $115.56.

2. Some paper 15 inches long and 12 inches wide, costs 16 cts. a quire; what will a quire of the same quality cost which is 18½ inches long and 13 inches wide? *Ans.* $.21$\frac{7}{15}$.

3. The panelling of a room is 12½ toises long and ¾ toises high: what must be paid for it, knowing that the carpenter's work costs 18s. 4d. a sq. toise, and the painting 5s. 3d.? *Ans.* £11 1 1¼.

4. What is the width of the room, whose surface is 72 sq. yd., knowing that if it were square the same would be 81 sq. yd.? *Ans.* 8 yd.

5. What is the area of a triangle, whose sides are 20, 30, and 42 yards? *Ans.* 276.66 yd.

6. The hypothenuse of a triangle is 45 feet, and its perpendicular 25 ft.; what is its base? *Ans.* 37.416 feet.

7. How many sq. yd. in a quadrilateral figure, the diagonal being 50 ft. and the two perpendiculars, 20 and 24 ft.? *Ans.* 122⅔ sq. yd.

8. The walls of a room are 40 square toises in area; how many rolls of paper-hangings are required to cover them, the rolls being 32 ft. long and 18 inches wide; and what must be paid for the whole at $1.75 a roll? *Ans.* 30 rolls; $52.50.

9. What is the surface of a sheet of paper ⅔ yd. long and ⅜ yd. wide? *Ans.* ¼ sq. yd., or 2¼ sq. ft.

10. The sides of three squares are 3, 4, and 5 ft.; what is the length of the side of a square which is equal to all three? *Ans.* 7.07+ ft.

11. A court-yard is 15 yards long and 6 yards wide; how many paving-tiles are required to cover it, knowing that each tile is 10 in. long by 7 in. wide? *Ans.* 1666⅔ paving-tiles.

12. A side-walk 6½ yards long and 1½ yards wide, is to be paved with stones, each stone has a surface of 70 square inches; what will be the cost of the whole pavement, at the rate of $18.50 per hundred stones? *Ans.* $32.53.

13. What is the area of a garden in the shape of a rhombus, the length of which is 45 yd. and the breadth 25 yd.?

14. A square yard of a floor costs $2.80, the whole floor costs $114, the breadth is 5¼ yeards; what is the length? *Ans.* 7$\frac{7}{15}$ yards.

15. A ladder 18½ feet in length stands upright against a wall, how far must the bottom of it be drawn out from the wall, so as to lower the top 8 inches? *Ans.* 4 ft. 11 in.

16. What must be paid for the plastering of a wall 55.50 yd. long and 5.25 yd. high, at the rate of $2.10 the sq. yd.? *Ans.* $611.88+.

17. What cost a piece of cloth $12\frac{1}{2}$ yards long and $1\frac{1}{2}$ yards wide, at the rate of \$1.90 a yard in length? *Ans.* \$23.75.

18. What is the area of a trapezoid, whose diagonal is 45.10 yards long and the two perpendiculars, 15.80 and 20 yards?

Ans. 807.29 sq. yards.

19. A man plastered three ceilings each 7.35 yards long by 5.40 yards wide, and painted 6 doors each 2.05 yards high by 1.05 yards wide; what sum must be yet paid him, if he charges \$1.22 a sq. yard for the ceiling, and \$0.36 a sq. yard for the doors, having been paid already \$22.40 on account?

20. Find the side of an equilateral triangle equal in area to a square whose side is 8 feet. *Ans.* 12.15 + ft.

21. Find the area of a piece of land comprising three trapezoids, and one triangle; the parallel sides of the first trapezoid are 36 and 54 yards, altitude 19.50 yards; those of the second are 110 and 75 yards, altitude 126 yards; those of the third 186 and 141 yards, altitude 219 yards; the base of the triangle is 69 yards, altitude 36 yards. *Ans.* 10.244 + acres.

22. A field whose parallel sides are 630 and 436 yards, altitude 80 yd., is let for \$200 a yr.; how much is it per acre? *Ans.* \$22.70 + .

23. A room 12 yards long by 7 yards broad was floored with boards 3 yards in length; the waste made on employing those boards was $\frac{1}{8}$ of their gross surface, and they cost \$.25 per sq. yard, gross surface. The work was done in 12 days at \$1.10 a day, and the nails used amounted to \$2.50. Find the whole cost of the floor. *Ans.* \$39.70.

24. A man wishes to plant 1815 trees at an equal distance from one another, so as to form a rectangle whose length is to its breadth as 5 is to 3; how many trees should he plant on each line?

Ans. 55 and 33.

25. The $\frac{5}{8}$ of the cost of a barn gate being paid, there still remains $\frac{4}{5}$ of that cost plus \$23.40 to be paid. Suppose the barn to have two gates each 3 yards in width and 5.40 yards in length; what cost the square yard? *Ans.* \$4.

26. Some earth was brought and levelled upon a field whose area equals that of a regular heptagon, the side of which measures 42 yd.; what cost the work at $1\frac{1}{2}$d. a sq. yd.? *Ans.* £40 1 3 + .

27. What will be the cost of roofing a building with sheet-iron at \$1.22 a sq. yard, if the roof comprises two equal triangles whose bases are 9.40 yd., and altitudes 6.32 yd., and also two equal trapezoids whose parallel sides are 25.48 and 16.08 yd., their altitudes being the same as those of the triangle? *Ans.* \$392.92 + .

28. The breadth of a field in the form of a parallelogram is to its length, as 5 is to 18; what are the dimensions of this field which, sowed in wheat, produced $28\frac{2}{5}$ bushels per acre, and $345\frac{13}{10}$ bushels in all? *Ans.* 460.24 yd. in length; 127.84 yd. in width.

29. An individual has a property forming a trapezoid whose parallel sides are 465 and 806 yards, altitude 550 yd. In the centre stands a square pond whose side is 45 yd. Find 1° the whole area of the field; 2° that of the pond; 3° that of the cultivable part.

Ans. 1° 349525 sq. yd.; 2° 2025 sq. yd.; 3° 347500 sq. yd.

30. The roof of a building comprises 1° two equal trapezoids whose parallel sides are 22 and 8.70 yd., altitudes 9.25 yd.; 2° two equal triangles whose bases are 16.20 yd., altitudes 7.20 yd.; also 4 hips each being 10.80 yd. long by 0.33 yd. broad, plus 1 row of slates along the gutters; the slates, which cost $9 a thousand, are 0.298 by 0.217 yd., and are overlaid 0.198 yd. in their length. A slater can roof 8 sq. yd. a day for $1.10; each slate requires two nails at $0.15 per lb., there being 285 nails in a pound; and 12 cents per 4 sq. yd. are paid for sundry expenses. Find the whole cost of that roof.

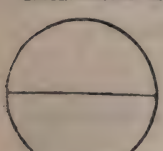
Ans. $265.22½.

Problem X.

To find the circumference of a circle, the diameter being given.

593. Rule.—*Multiply the diameter by* 3.1416, *and the product will be the circumference.*

Nota.—3.1416 is the circumference of a circle whose diameter is 1.



Ex. 1. What is the circumference of a circle whose diameter is 18 yards?

Oper. 3.1416 × 18 = 56.5504 yd., *Ans.*

2. Required the circumference of circles whose diameters are 1° 26 yd.; 2° 46.25 yd.; 3° 59.75 yd.; 4° 67.75 yd.; 5° 224 ft. 6 in. *Ans.* 1° 81.6816 yd.; 2° 145.299 yd.; 3° 187.7106 yd., etc.

3. What are the circumferences of circles whose radii are 1° 42.25 yd.; 2° 67.24 yd.; 3° 117.70 yd.; 4° 149.70 yd.; 5° 160 ft. 9 in.? *Ans.* 1° 265.4652 yd.; 2° 422.482368 yd., etc.

Problem XI.

To find the diameter of a circle, the circumference being known.

594. Rule.—*Divide the circumference by* 3.1416, *and the quotient will be the diameter.*

Ex. 1. What is the diameter of a circle whose circumference is 25.1328 yards?

Operation. 25.1328 ÷ 3.1416 = 8 yards, *Ans.*

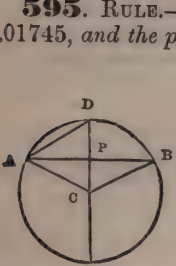
2. Required the diameters of circles whose circumferences are 1° 37.70 yd.; 2° 59.69 yd.; 3° 76.34 yd.; 4° 126.45 yd.; 5° 206¼ yd.; 6° 30 ft. 8′ 10″. *Ans.* 1° 12.0002 yd.; 2° 18.9998 + yd.; 3° 24.2997 + yd., etc.

3. What are the radii of circles whose circumferences are 1° 69.25 yd.; 2° 152.12 yd.; 3° 227.20 yd.; 4° 380.17 yd.; 5° 45⅓ yd.; 6° 50 ft. 6′ 3″? *Ans.* 1° 11.02145 yd.; 2° 24.2105 + yd., etc.

Problem XII.

To find the length of a circular arc, when the number of degrees which it contains, and the radius of the circle are known.

595. Rule.—*Multiply the number of degrees by the decimal* .01745, *and the product arising, by the radius of the circle.*



Ex. 1. Suppose the arc A B to contain 120 degrees, and the radius A C be 10 feet; what is the length of the arc?

Oper. .01745 × 120 × 10 = 20.94, *Ans.*

2. What is the length of an arc containing 25°, the diameter of the circle being 15 ft.?

Ans. 3.2718 ft.

3. Required the length of each of the following arcs: 1st 12° 10', diameter 20; 2nd 10° 15', diameter 68; 3rd 57° 17' 44½", diameter 25; 4th 60°, radius 14.

Ans. 1st 2.123 +; 2nd 6.0813 +, etc.

Problem XIII.

To find the length of the arc of a circle, the chord and radius being given.

596. Rule.—I. *Find the chord of half the arc.*

II. *From* 8 *times the chord of half the arc, subtract the chord of the whole arc, divide the remainder by* 3, *and the quotient will be the length of the arc, nearly.*

Ex. 1. If the chord A B, fig. of Prob. XII., equals 30 feet, and the radius A C be 20 feet; what is the length of the arc A D B?

Operation. First draw D C perpendicular to the chord A B; it will bisect the chord at P, and the arc of the chord at D. Then A P = 15 feet. Hence, $\overline{AC}^2 - \overline{AP}^2 = \overline{CP}^2$, that is, 400—225=175; and $\sqrt{175} = 13.228 = C\,P$.

Then D C —CP = 20 —13.228 = 6.772 = D P.

Again, $A\,D = \sqrt{\overline{A\,P}^2 + \overline{P\,D}^2} = \sqrt{225 + 45.859984}$.

Hence, A D = 16.457 = chord of the half arc.

Then, $\frac{16.457 \times 8 - 30}{3} = 33.885$ = arc A D B, *Ans.*

2. If the chord A D of half the arc A B D, fig. of Prob. XII, be 30 feet, and the chord A B of the whole arc, 50 feet; what is the length of the arc?

Ans. 63⅓ feet.

3. The chord of an arc is 12 feet, and the chord of half the arc is 7; what is the length of the arc? *Ans.* $14\frac{3}{4}$ feet.

4. The chord A B of the whole arc, fig. of Prob. XII, equals 48.74, and the chord A D of half the arc equals 30.25; what is the length of the arc? *Ans.* 64.42.

Problem XIV.

To find the area of a circle, the diameter, or the circumference, or both, being given.

597. Rule.—*Multiply the square of the diameter by* .7854. Or,

Multiply the square of the circumference by ·07958. Or,

Multiply the circumference by half the radius and the product will be the area.

Ex. 1. What is the area of a circle whose diameter is 12 yards?

Operation. $.7854 \times 12^2 = .7854 \times 144 = 113.0976$ sq. yd., *Ans.*

2. Find the area of a circle whose circumference is 12 yards.

Oper. $.07958 \times 12^2 = .07958 \times 144 = 11.4591$ sq. yd., *Ans.*

3. Required the area of a circle whose circumference is 37.70 yd., and the radius 6 yards.

Operation. $37.70 \times \frac{6}{2} = 113.10$ sq. yd., *Ans.*

4. Required the areas of the circles whose circumferences are 1° 10.75 yd.; 2° 5.4978 yd.; 3° $3\frac{1}{4}$ miles; 4° $35\frac{1}{2}$ toises; 5° $25\frac{1}{4}$ yd.; 6° $4\frac{1}{8}$ miles. *Ans.* 1° 91.9646 sq. yd.; 2° 2.405+ sq. yd.; 3° .9748 sq. mi.; 4° 100.2906+ sq. to., etc.

5. Find the areas of the circles whose diameters are 1° 7 yd.; 2° 13.27 yd.; 3° 24 ft. 7 in.; 4° $40\frac{1}{2}$ yd.; 5° 180.40 yd.

Ans. 1° 38.4846 sq. yd.; 2° 138.3033 sq. yd.; 3° 474.6488 sq. ft., etc.

6. What are the areas of the circles whose dimensions are as follows: 1° circumf. 21.9912 yd., radius $3\frac{1}{2}$; 2° diameter 15, circumf. 47.124; 3° radius 25, circumf. 157.08; 4° circumf. 32.9868, diameter $10\frac{1}{2}$; 5° radius 16, circumf. 105.55776; 6° circumf. 6.2832, diameter 2? *Ans.* 1° 38.4846 sq. yd.; 2° 176.715; 3° 1963.50; 4° 86.59035, etc.

Problem XV.

Given a circle, to find a square which shall have an equal area.

598. Rule.—I. *The diameter* × .8862 = *side of an equivalent square.*

II. *The circumference* × .2821 = *side of an equivalent square.*

Ex. 1. The diameter of a circular field is 650 yards, what would be the side of a square field of an equal area?

OPERATION. $650 \times .8862 = 576.03$ sq. yd., *Ans.*

2. The circumference of a circular fishpond is 200; what is the side of a square of an equal area?

OPERATION. $200 \times .2821 = 56.42$, *Ans.*

3. Find the sides of squares of equal areas to circles whose circumferences are 1° 250 yd.; 2° 300 yd.; 3° 412.50 yd.; 4° 135.75 yd.; 5° 40 ft. 10 in. *Ans.* 1° 70.525 yd.; 2° 84.63 yd.; 3° 116.36625 yd., etc.

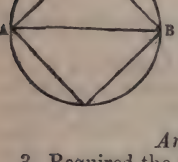
4. What are the sides of squares of equal areas to circles whose diameters are 1° 25 yd.; 2° 30 ft.; 3° 75.10 yd.; 4° 45 ft. 8 in.; 5° 20.65 yd.? *Ans.* 1° 22.155 yd.; 2° 26.5860 ft.; 3° 66.5536+ yd., etc.

PROBLEM XVI.

Given the diameter, or the circumference, of a circle, to find the side of the inscribed square.

599. RULE.—I.—*The diameter* $\times$ .7071 = *side of the inscribed square.*

II. *The circumference* $\times$ .2251 = *side of the inscribed square.*



Ex. 1. The diameter A B of a circle is 300; what is the value of A C, the side of the inscribed square?

OPERATION. $300 \times .7071 = 212.13$, *Ans.*

2. What are the sides of the inscribed squares, if the diameters of the circle are 1° 312; 2° 400; 3° 150.20; 4° 225.80 yd.; 5° 170 ft. 8 in.?

Ans. 1° 320.6152. 2° 282.84; 3° 106.206+, etc.

3. Required the sides of the inscribed squares of which the circumferences of the circle are 1° 718 yd.; 2° 180.40 yd.; 3° 368.10; 4° 130.70 yd.; 5° 800.20. *Ans.* 1° 161.6218 yd.; 2° 40.608+ yd.; 3° 82.85931, etc.

PROBLEM XVII.

To find the area of a sector of a circle.

600. RULE.—I. *Find the length of the arc by Problem* XII.

II *Multiply the arc by one half the radius, and the product will be the area.*

Ex. 1. What is the area of the sector A D B C, Prob. XII, whose arc is 95, and the radius of the circle 20 feet?

OPERATION. $95 \times \frac{20}{2} = 950$ feet, *Ans.*

2. Required the area of a sector, of which the arc is 26°, and the radius of the circle 24 feet.

OPERATION. $.01745 \times 26 \times 24 = 10.8888$, length of the arc; $10.8888 \times \frac{24}{2} = 130.6656$ sq. ft., *Ans.*

3. What is the area of a sector, of which the arc is 79 and the radius of the circle 47 inches? *Ans.* 1856.5 sq. in.

4. Find the area of a sector whose radius is 25 and the arc one of 145° 18'. *Ans.* 792.337.

5. What is the area of a semicircle in which the radius is 17? *Ans.* 453.8745.

6. What is the area of a sector, of which the arc is 73° 44' 30", and the radius of the circle $12\frac{1}{4}$? *Ans.* 100.5306+.

PROBLEM XVIII.

To find the area of the segment of a circle.

601. RULE.—I. *Find, by the last Problem, the area of the sector which has the same arc with the segment.*

II. *Find the area of the triangle formed by the chord of the segment and the two radii drawn to its extremities.*

III. *If the segment is greater than the semicircle, add the two areas together; but if it is less, subtract them, and the result in either case, will be the area required.*

Or use the following

RULE.—I. *Multiply half the radius by the difference between the arc which is the base of the segment, and the half of the chord that would subtend an arc doubled that of the given segment.*

II. *If the segment is greater than a semicircle, multiply half the radius by the sum of the arc and of the chord that would subtend an arc equal to double the given arc less* 360°, *the result will be the area required.*

Ex. 1. Let the chord A B, in the diagram of Prob. XII, be 24, and the radius C A, be 20; what is the area of the segment A D B?

OPERATION. $\sqrt{C A^2 - A P^2} = \sqrt{20^2 - 12^2} = 16$, the measure of C P. Also C D − C P = P D = 20 − 16 = 4, the measure of P D. Then, $\sqrt{A P^2 + P D^2} = \sqrt{12^2 + 4^2} = \sqrt{160} = 12.64911$ = A D; and finally, $\frac{(A D \times 8) - 24}{3} = \frac{(12.64911 \times 8) - 24}{3} =$ 25.7309, the measure of the arc A D B (Prob. XIII).

25.7309 × 10, half radius, = 257.309, area of the sector A D B C; and A P × C P = 12 × 16 = 192, area of triangle C A B; and the area A D B C — area C A B = area of segment A D B; that is, 257.309 — 192 = 65.309, area required. It is also obvious, that the area of the sector A D B C subtracted from that of the whole circle A D B E, will leave the area of the sector A E B C.

2. Required the area of a segment whose arc is 220 degrees, the radius of the circle being 20 yards.

Operation. The double of 220° less 360° = 440° — 360° = 80°. The arc rectified of 220° = 3.1416 × 40 × $\frac{220}{360}$ = 76.79 yd. The chord of 80° (*see* Table of chords) = 20 × 1.2856 = 25.71 yd. $20 \times \left(\frac{76.79 + 25.71}{2}\right)$ = 1025 sq. yd., area of the segt., *Ans.*

3. What is the area of the segment of a circle whose radius is 10, and the chord of the arc 16 yards?

Operation. The chord in the table for the arc of the segment = 16 ÷ 10 = 1.6; 1.6 = 106° 20' = 106.333+; (3.1416 × 20 × 106.333) ÷ 360 = 18.5586. The chord of an arc doubled, or [360° — (106° 20' × 2)] = 147° 20' = 1.9193; 1.9193 × 10 = 19.193; $\frac{1}{2}$ × [18.5586 — 19.193 ÷ 2)] = 44.81+ sq. yd., area of the segment, *Ans.*

4. Find the area of a segment; the radius of the circle being 10, and the chord of the arc 12 yards. *Ans.* 16.326 sq. yd.

5. What is the area of the segment whose height is 27 and the diameter of the circle 75? *Ans.* 1432.31+.

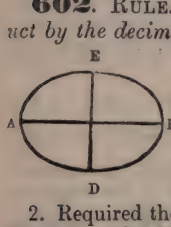
6. Required the area of a segment whose arc is 90°, and the radius of the circle 6. *Ans.* 28.27+.

7. In a circle whose radius is 50 feet, what are the areas of the segments whose chords are 1° 17 ft.; 2° 24 ft.; 3° 45.75 ft.; 4° 69.20 ft.; 5° 70 ft.? *Ans.* 1° 8.1075 sq. ft.; 2° 23.3125 sq. ft.; 3° 170.75 sq. ft., etc.

Problem XIX.

To find the area of an ellipse, the two axes being given.

602. Rule.—*Multiply the two axes together, and their product by the decimal* .7854, *the result will be the required area.*



Ex. 1. What is the area of a garden in the form of an ellipse whose transverse axis A B is 40 yards, and the conjugate axis D E is 25 yards?

Operation. 40 × 25 × .7854 = 785.40 sq. yd., *Ans.*

2. Required the areas of the ellipses whose axes are 1° 5 and 4 yd.; 2° 12.14 and 7.25 yd.; 3° 17.15 and 7.29 yd.; 4° 45.21 and 34.18 yd.; 5° 70.40 and 45.65 yd. *Ans.* 1° 15.708 sq. yd.; 2° 69.1269 sq. yd.; 3° 98.1934 sq. yd., etc.

Problem XX.

Given the area of an ellipse and one of its axes, to find the other axis.

603. Rule.—*Divide the given area by* .7854, *and that quotient by the given axis, the result will be the required axis.*

Ex. 1. The area of an ellipse is 400 square feet, and one of its axes is 25 feet; what is the length of the other axis?

Oper. $400 \div .7854 \div 25$, or $400 \div (.7854 \times 25) = 20.37+$ ft., *Ans.*

2. The area of an elliptical piece of land is 6 acres, and one of its axes is 40 rods; what is the length of the other axis?

Ans. 30.55 + rods.

3. What are the conjugate axes of ellipses whose area is 72.24 sq. yd., and the transverse axes are 1° 24 yd.; 2° 28 yd.; 3° 36 yd.; 4° 40 yd.; 5° 56.25 yd.? *Ans.* 1° 3.83 yd.; 2° 3.28 yd.; 3° 2.55 yd., etc.

Problem XXI.

To find the circumference of an ellipse, the two axes being given.

604. Rule.—*Multiply the sum of the two axes by* 1.5708, *and the product will give the circumference, nearly.*

Ex. 1. What is the circumference of an ellipse, the longer axis of which is 20 and the shorter 16?

Operation. $(20 + 16) \times 1.5708 = 56.5488$, nearly, *Ans.*

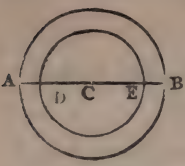
2. Required the circumferences, or perimeters, of ellipses whose axes are 1° 12 and 10 yd.; 2° 18 and 15 yd.; 3° 15 and 12 yd.; 4° 24 and 20 yd.; 5° 30.50 and 24.30 yd. *Ans.* 1° 34.55 + yd.; 2° 51.83 + yd.; 3° 42.41 + yd., etc.

Problem XXII.

To find the area of a circular ring, or of the space included between two concentric circles.

605. Rule.—*Multiply the sum of the two diameters by their difference, and the product arising, by* .7854 *for the area of the ring.* Or,

Square the diameter of each ring, subtract the square of the less from that of the greater, and multiply the difference of the squares by the decimal .7854, *the product will be the area.*



Ex. 1. The diameter A B is 20, and D E is 12; what is the area of the ring?

OPERATION. $20 + 12 = 32$, the sum; $20 - 12 = 8$, difference; $32 \times 8 \times .7854 = 201.0624$, area of the ring, *Ans.*

2. If the diameters are 16 and 10, what will be the area included between the circumferences? *Ans.* 122.5224.

3. What are the areas of the rings whose diameters are 1° 24 and 30 yards; 2° 36 and 52 ft.; 3° 60.30 and 90.50 yd.; 4° 114.36 and 151.40 yd.; 5° 178.90 and 290.50? *Ans.* 1° 254.4696 sq. yd.; 2° 1105.8432 sq. ft.; 3° 3576.8372 sq. yd., etc.

PROMISCUOUS EXAMPLES IN CIRCULAR SURFACES.

1. What is the area of a circular pond whose radius is 12 yards? *Ans.* 452.3904 sq. yd.

2. Required the area of a circular pond whose perimeter is 75 yards. *Ans.* 447.1279 sq. yd.

3. A circular basin occupies $\frac{1}{5}$ the area of a garden in the form of a square, the side of which is 45 yards; what is the radius of the basin? *Ans.* 11.354 yd.

4. What is the area of the semi-circular top of a window whose width is $2\frac{3}{4}$ yards? *Ans.* 2.9697 sq. yd.

5. The wheels of a carriage turn 2200 times over the space of 5 miles; what is their diameter? *Ans.* 3.8165 + ft.

6. What is the diameter of a circular parterre which occupies the fourth of the area of a rectangular garden, whose base is 24 and height 16 yards? *Ans.* 11.054 yd.

7. Find the area of a dial whose diameter is $4\frac{1}{2}$ feet. *Ans.* 15.9043+ sq. feet.

8. The diameter of the large wheels of a locomotive engine, is 1.66 yd.; what is the distance in miles and leagues, run over by that engine, if each of its wheels revolved 1695875 times? *Ans.* 5024.993+ miles, or 1674.996 leagues.

9. Required the area of an elliptical flower-garden whose axes are 36 and 27 yards. *Ans.* 763.4088 sq. yd.

10. What is the length of an arc of 60°, in a circle whose radius is 14 feet? *Ans.* 14.65 + feet.

11. Required, in arpents, the area of a circular piece of land whose radius equals the minimum radius of the curves adopted for the construction of rail-ways, that radius being 800 toises. *Ans.* $2234\frac{2}{7\,5}$ arp.

12. What is the area of an elliptical basin inscribed in a rectangle whose base is 30 and height 20 yards? *Ans.* 471.24 sq. yd.

13. What are the areas of the circles circumscribed to 1° an equilateral triangle whose side is 6 yd.; 2° a square whose side is 7 yd.; 3° a pentagon whose side is 9 yd.; 4° a hexagon whose side is 10

yd.; 5° an octagon whose side is 18 yd.? *Ans.* 1° 37.6099 sq. yd.; 2° 76.9770 sq. yd.; 3° 183.854286 sq. yd., etc.

14. The chord of an arc is 30 and its height 7 feet; what is the length of the arc? *Ans.* 34.13+ ft.

15. What are the radii of the circles equal in area to ellipses whose axes are 1° 26 and 12 yd.; 2° 30 and 24 yd.; 3° 45 and 36 yd.; 4° 52 and 42 yd.; 5° 62.20 and 46.40 yd.? *Ans.* 1° 8.83 yd.; 2° 13.41 yd.; 3° 20.12 yd., etc.

16. What weight will a solid cast iron column securely support, whose diameter is .12 yd. in diameter and 3.80 yd. in height, if each hundredth sq. yd. of transversal section can support 666 pounds? *Ans.* 75323 lb.

17. Required the area of the small circle of the earth corresponding to the 49° of latitude north, knowing that the value of each degree of longitude, in that latitude, is 80018.43 yards.

18. The diameters of two concentric circles are 45 and 30; what is the area of the ring formed by those circles? *Ans.* 883.575.

19. How many rings 2 inches in radius, can be drawn from a tin sheet 22 inches long by 15 inches wide? *Ans.* $20\frac{5}{8}$.

20. The diameter of the bottom of a basket is .46 yd., and the circumference of its top is 2.262 yd.; what difference is there between the area of the lower base and that of the upper base? *Ans.* 0.240977 sq. yd.

21. From a zinc sheet 28 inches long by 25 inches broad, how many rings can be drawn whose diameters are $2\frac{1}{2}$ and $3\frac{1}{2}$ in.? *Ans.* $57\frac{1}{7}$.

22. The circumference of a circle is 314.16 yd.; what is the radius of a concentric circle half the area? *Ans.* 35.35 yd.

23. What is the area of the ring of a circular parterre, knowing that the exterior diameter of the parterre is 18.40 yd., and the breadth of the ring 80 yd.? *Ans.* 44.233728 sq. yd.

24. What is the side of a square equal in area to a circle whose diameter is 4? *Ans.* 3.544+.

25. The exterior diameter of a circular pond is $15\frac{1}{2}$ yards, the breadth of the ring $\frac{9}{10}$ yd.; required 1° the area of the ring; 2° what will be paid to have it paved in flag-stones, at the rate of \$4.16 a sq. yd. *Ans.* \$171.72.

26. A circular garden whose diameter is 26.5 yd., is enclosed by a grassy ring 6.4 yd. broad; how many bundles of grass, each weighing 56.64 lb., can be gathered from this ring, knowing that when the grass dries up, it loses 56 % of its weight, and gives 2056 lb. of hay per acre? *Ans.* $17\frac{43}{100}$.

27. The radius of a circular pond is 12 yd.; what must be the width of a grassy ring around the pond, that contains the same area? *Ans.* 4.97 yd.

28. A triangular meadow whose sides are 5420, 6280, 3400 yd., encloses an elliptical pond whose diameters are 195 and 348 yd. Find the worth of the hay produced by that meadow, if 5056 lb. are cut per acre, and sold at 4 cents per bundle of 10 lb. *Ans.* \$38246.29+.

29. A man has a cistern whose diameter is 3 ft. $10\frac{4}{5}$ in.; its edge, which is $23\frac{3}{5}$ in. broad, is to be covered with tin-plate at the cost of \$2.10 a sq. yd. Find the cost. *Ans.* \$8.36+.

30. The floor of a room cost \$69.30, at the rate of \$4.50 a sq. yd. Find the length of each of the diameters of an ellipse equal in area to that floor, if one of the diameters is $\frac{12}{17}$ of the other.

31. A man gets a yard-gate made whose top is semicircular; the square part of the gate is 4.5 yd. wide by 6.2 yd. high, and the diameter of the top part is equal to the breadth of the gate. He pays the carpenter at the rate of \$9 a sq. yd.; the painter, 85 cts. a sq. yd. for bronzing the outside, and 60 cts. a sq. yd. for the inside; the smith \$25 for irons. Find the whole cost. *Ans.* \$399.65.

32. The flooring of a room cost \$15, which was at \$0.37 a sq. yd. for the wood, and \$0.65 a sq. yd. for the work. What must be the axes of an ellipse equal in area to that floor, if they are to each other as 3 is to 5? *Ans.* 5.586 and 3.351 yd.

33. How many bushels of peas, reckoning $25\frac{4}{5}$ bu. per arpent, can be gathered from a field whose area is equal to that of the segment of a circle whose radius is 326 yd., and the chord of the segment 139.75 yd.? *Ans.* 4.413+ bu.

34. A man, owning a circular piece of land containing 10 acres, sold the central portion of 25 perches in diameter. What part does he own still? *Ans.* 6 acres 3 per. 26+ sq. yd.

35. The area of a field is equivalent to that of the zone of a circle whose radius is 380 yd.; the chords of the zone are 284 and 328 yd. The field produces 3500 lb. of hay per acre: how many bundles of $11\frac{3}{5}$ lb. each can be gathered? *Ans.* $185\frac{4}{5}$ bundles.

36. How many bundles of bean-stalk were gathered from a field whose area is equal to that of a segment, the arc of which is 170°, being part of a circle whose radius is 245 yards; if each acre produced 1760 lb., and the bundles weighed $13\frac{1}{2}$ lb.? *Ans.* 2258.23+ bundles.

37. A field in the form of a sector whose central angle is 86°, and the radius of the circle 87.56 yd., was planted with oats. What sum was spent for sowing that field, knowing that it required 5 bushels of oats per acre at the cost of 70 cts. a bushel?

38. The area of a field is equal to that of a triangle, whose angle opposite its base, stands at the centre of a circle, the radius of which is 248 yd. This field, being planted with vetch, produced 1840 lb. forage per acre; how many bundles, each weighing $7\frac{1}{2}$ lb., were gathered?

MENSURATION OF SOLIDS.

DEFINITIONS.

606. The **Mensuration of Solids** is divided into two parts. 1st, The mensuration of the surfaces of solids; and 2nd, The mensuration of their solidities.

607. A **Solid** is a magnitude which has length, breadth, and thickness.

608. A **Polyhedron** is a body or solid contained by many sides or planes.

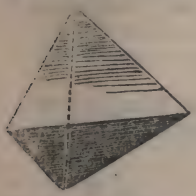
609. The *edge* of a polyhedron is the line formed by the common intersection of two adjacent faces.

610. **Polyhedrons** are divided into *regular* and *irregular polyhedrons*.

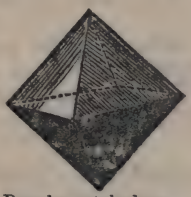
611. A **Regular Polyhedron** is a solid whose all faces are regular polygons equal to one an other, and whose solid angles are also equal to one an other.

612. A solid angle is the space comprised between several planes which cut one an other at one point.

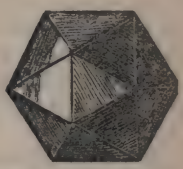
613. An **Irregular Polyhedron** is a solid whose all faces are not regular polygons equal to one an other, and whose solids angles are unequal.



Regular tetrahedron.



Regular octahedron.



Regular icosahedron.

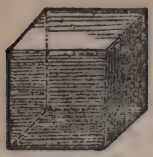
614. Regular polyhedrons are five in number. Three are made of equilateral triangles; viz., the *tetrahedron*, the *octahedron*, the *icosahedron;* one of two squares, the *hexahedron* or *cube;* and one of pentagons, the *dodecahedron*.

615. The regular **Tetrahedron** is a solid figure comprehended under four equilateral and equal triangles.

616. The regular **Octahedron** is a solid contained by eight equilateral and equal triangles.

617. The regular **Icosahedron** is a solid consisting of twenty

equal and similar triangular pyramids, whose vertices meet in the centre of a sphere supposed to circumscribe it.



Regular hexahedron.

Regular dodecahedron.

618. The regular **Hexahedron** or cube, is a solid whose surface present 6 equal squares.

619. The regular **Dodecahedron** is a solid contained under twelve equal and regular pentagons, or having twelve equal bases.

620. The principal *irregular* polyhedrons are the *prism* and the *pyramid*.

621. A Prism is a figure whose bases, or ends, are any similar, equal, and parallel plane figures, and whose sides are parallelograms.

622. A prism whose axis is perpendicular to the base, is called a *right* prism.

Oblique prism, A B height.

Right Parallelop.

623. An *oblique* prism is one whose axis is not perpendicular to the base.

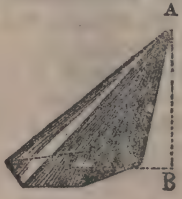
624. The height, or altitude of a prism, or solid, is a line drawn from its vertex, perpendicular to its base; but in an oblique one, the altitude is the perpendicular of a right-angled triangle, whose hypothenuse is the axis.

625. A triangular, quadrangular, pentagonal, hexagonal, etc., prism, is one whose base is a triangle, a quadrilateral, a pentagon, a hexagon, etc.

626. A **Parallelopipedon** is a prism, whose base is a parallelogram.

627. A **Pyramid** is a solid figure contained by several triangles, whose bases are all in the same plane, and which have one common vertex.

Regular pyramid.



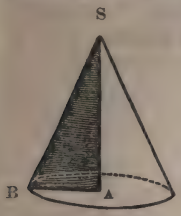
Oblique pyramid.

Frustum of a pyramid.

628. The **Frustum** of a solid is the part that remains after cutting off the top by a plane parallel to the base.

529. A **Cylinder** is a long, circular body, of uniform diameter, and its extremities forming equal parallel circles.

630. A **Cone** is a solid body having a circle for its base and tapering uniformly to a point called the *vertex*.



631. The *axis* of the cone is the perpendicular drawn from its top to the centre of its base; as A S.

632. The *generant* or side of the cone is the hypothenuse, which, by the revolution of the right-angled triangle, describes the lateral surface of the cone; as B S.

633. A *right* cone is one whose axis is perpendicular to the plane of its base, and its sides equal.

634. An *oblique* or *scalene* cone, is one whose axis is inclined to the plane of its base, and its sides unequal.

635. The *frustum* of a cone, is the portion remaining when an upper section is removed.

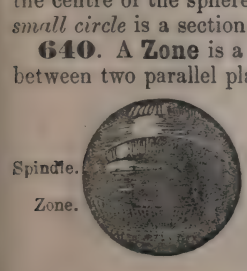
636. A **Sphere** is a solid, bounded by a curved surface, every part of which is equally distant from a point within, called the centre.

637. The *radius* of a sphere is a line drawn from the centre to any part of the surface.

638. The *axis* or *diameter* of a sphere is a line passing through the centre, and terminated by the surface.

639. A *great circle* of the sphere is any section passing through the centre of the sphere, and dividing it into equal parts; and a *small circle* is a section which divides it into unequal parts.

640. A **Zone** is a part of the surface of a sphere included between two parallel planes, which form its *bases*.



641. A **Circular Spindle** is a solid, the figure or shape of which is marked by the revolution of the arc of a circle about its chord, which remains stationary.

Spherical Wedge.

Segment with two bases.

642. A **Spherical Sector** is a solid generated by the revolution of a sector of a circle about one of its radii.

643. A **Spherical Segment** is a portion of the sphere cut off by any plane. The plane is the *base* of the segment; the perpendicular distance from the centre of the base to the convex surface, is the *height* of the segment.

644. A **Spherical Wedge** is the portion of a sphere comprehended between the halves of two great circles.

645. A **Spheroid,** or **Ellipsoid,** is a figure produced by the revolution of a semi-ellipsis about one of its axes, that axis remaining fixed. When it revolves about its transverse axis, the figure is said to be *prolate;* and when about its conjugate axis, it is called *oblate.*

Problem I.

To find the surface or area of a prism.

646. Rule.—*Multiply the perimeter of the base by the altitude, and to the product add the area of the bases: the sum will be the surface.*

Ex. 1. What is the surface of a rectangular prism whose base is 3 by 4 yd., and altitude 5 yd.?

Operation. The perimeter of the base is $(4 \times 2) + (3 \times 2) = 14$; $14 \times 5 = 70$ yd., convex surface; $70 + (4 \times 3 \times 2) = 94$ yd., *Ans.*

Ex. 2. Required the entire surface of a pentagonal prism, when each side of the base is 10 feet and the height 30.

Operation. $10 \times 5 \times 30 = 1500$ sq. ft., convex surface; $10^2 \times$ tabular number, or $100 \times 1.720477 = 172.0477$, area of one base.

Then, convex surface	=	1500.	square feet.
lower base	=	172.0477	" "
upper base	=	172.0477	" "
Entire surface		1844.0954	" "

3. What is the surface of a triangular prism, whose base is an equilateral triangle, each side of which measures 20 inches, and its height 25 feet? *Ans.* 127 sq. ft. 58 sq. in.

4. What is the wall-surface of a square room, whose sides are each 16 feet long and 10 feet high? *Ans.* $71\frac{1}{9}$ sq. yd.

5. A rectangular prism is 3 yd. long, 5 yd. broad, and 8 yd. high: what is 1° the convex surface of the prism; 2° the whole surface of its two bases; 3° the side of a cube whose entire surface equals that of the prism? *Ans.* 1° 128 sq. yd.; 2° 30 sq. yd.; 3° 5.13 sq. yd.

6. What extent of surface is an oblique prism, the ends of which are hexagonals, each side measuring 10 inches, the height of the prism 20 feet, and the perimeter of a section perpendicular to the sides, $4\frac{1}{2}$ feet? *Ans.* 93.60843+ sq. ft.

7. Required the entire surface of an octagonal prism, the side of whose base is 15 and altitude 12 feet. *Ans.* 3612.7921 sq. ft.

8. What must be paid for lining a rectangular box with tin, at the rate of 2s. 6d. a yard, the inner dimensions of the box being $5\frac{1}{3}$ feet in length, 4 ft. 5 in. in breadth, and $3\frac{1}{4}$ ft. in depth? *Ans.* £1 11 $4\frac{3}{4}$.

Problem II.

To find the solidity of a prism.

647. Rule.—*Multiply the area of the base by the perpendicular height, and the product will be the solidity.*

Ex. 1. The base of a prism is a hexagon, each side measuring 3 feet, and its height 15 feet. What is the solidity?

Operation. $3^2 \times 2.5980762 \times 15 = 350.7402+$ solid feet, *Ans.*

2. What is the solidity of a prism, the base of which is an equilateral triangle, each side of which measures 6 inches, and the length of the prism 18 inches? *Ans.* 280.59+ cu. in.

3. How many cub. ft. in a block of stone, whose length is 4 ft. 3 in., breadth 3 ft. 8 in., and depth 3 ft. 9 in.? *Ans.* $58\frac{7}{8}$ cub. ft.

4. How many perches of masonry in an octagonal prism, the side of which is 3 ft. 4 in., and the height 10 ft.? *Ans.* 21.676 per.

5. How many gallons of water, wine measure, will a cistern contain, whose dimensions are the same as in the third example? *Ans.* $437\frac{1}{4}$.

6. The three sides of the base of an oblique prism measure respectively 4, 5, and 6 yards, and the altitudes of those three sides of the prism are 7, 8, and 9 yd.; what is its solidity? *Ans.* 79.36 cu. yd.

Problem III.

I. To find the surface of a regular pyramid.

648. Rule.—*Multiply the perimeter of the base by half the slant height, and the product will be the convex surface: to this add the area of the base, if the entire surface is required.*

II. To find the slant height of a regular pyramid from its superficial area, and the side of its base.

649. RULE.—*From the whole area subtract the area of the base, and divide the remainder by one half the perimeter of the base.*

III. To find the side of the base of a regular pyramid from its superficial area and its slant height, the area of the base not being included.

650. RULE.—*Divide the given area by half the slant height, and that quotient again by the number of sides.*

Ex. 1. What is the entire area of a triangular pyramid, the slant height of which is 10 feet, and each side of the base 4 feet?

OPERATION. $4 \times 3 = 12$, perimeter of the base; $12 \times \frac{10}{2} = 60$ sq. ft., area of convex surface; $4^2 \times .4330127 = 6.9282$, the area of the base; $60 + 6.9282 = 66.9282$ sq. ft., entire surface, ***Ans.***

Ex. 2. The area of a regular triangular pyramid is 31.732052 sq. ft., and the side of its base 2 feet; what is its height?

OPERATION. $2^2 \times .433013 = 1.732052$, area of base; and $31.732052 - 1.732052 = 30$, the upright area. Then $30 \div 3$, half the perimeter of the base $= 10$ feet, ***Ans.***

Ex. 3. The superficial area of the sides of a regular triangular pyramid, is 30 square feet, and its height 10 feet; what is the linear measure of its sides?

OPERATION. $30 \div 5 = 6$, and $6 \div 3 = 2$ feet, ***Ans.***

4. The slant height of a regular pentagonal pyramid is 40 feet, and each side of the base 15 feet; what is the convex surface, and also the entire surface? ***Ans.*** 1887.107415 sq. ft.

5. What is the convex surface of a regular triangular pyramid, the slant height being 20 ft., and each side of the base 5 ft.? ***Ans.*** 150 ft.

6. The area of the sides of a regular hexagonal pyramid is 360 square feet, and its slant height 36 feet; what is the linear measure of its sides? ***Ans.*** $3\frac{1}{3}$ feet.

7. What is the total area of a regular heptagonal pyramid, whose slant height is 21 feet, and the measure of its sides 18 inches? ***Ans.*** 118.4263 sq. ft.

8. The area of a regular heptagonal pyramid is 463.93 square feet, and the side of its base 6 feet; what is its slant height? ***Ans.*** 95.17+ ft.

Problem IV.

To find the surface of the frustum of a regular pyramid.

651. Rule.—*Multiply half the sum of the perimeters of the two bases by the slant height of the frustum, and to the product, add the area of the two ends, the result will be the entire surface.*

Ex. 1. What is the superficial area of the frustum of a hexagonal pyramid, the slant height of which is 18 yards, the measure of the side of the larger base 6 yards, and of the smaller 4 yards?

Operation. $6 \times 6 = 36$, perimeter of the base, and $4 \times 6 = 24$, upper perimeter. Then $(36 + 24) \div 2 = 30$, and $30 \times 18 = 540$ yd., area of the sides. Again. $6 \times 2.5980762 = 15.5884572$, area of lower base, and $4 \times 2.5980762 = 10.3923048$, area of upper base. Then, $540 + 15.5884 + 10.3923 = 565.9807$ square yards, *Ans.*

2. The slant height of the frustum of a quadrangular pyramid is 16 feet, the measure of the side at the base 12 ft., and at the top 4 ft.; what is its whole surface? *Ans.* 672 sq. ft.

3. What is the convex surface of the frustum of a heptagonal pyramid whose slant height is 50 feet, each side of the lower base 7, and of the upper base 4 feet? *Ans.* 1925 sq. ft.

Problem V.

I. To find the solidity of a pyramid.

652. Rule.—*Multiply the area of the base by one third of the height, the product will be the solidity.*

II. To find the side of a regular triangular, quadrangular, pentangular, etc., pyramid, from its solidity and height.

653. Rule.—*Divide the solidity by one third the perpendicular height and the result by the corresponding tabular number, and extract the square root of the quotient.*

III. To find the height of a regular pyramid from the side of the base, and its solidity.

654. Rule.—*Divide the solidity by the corresponding tabular number, and also by the square of the side of its base, and multiply the result by* 3.

Ex. 1. What is the solidity of a triangular pyramid, the height of which is 20 feet, and each side of the base 4 feet?

OPERATION. $4^2 \times .4330127 \times \frac{20}{3} = 46.188$ cub. ft., *Ans.*

Ex. 2. If the solidity of a reg. octagonal pyramid be 2433.5273088 solid feet, and its height 42 feet; what is the measure of one of its equal sides?

OPERATION. $2433.5273088 \div \frac{42}{3} = 173.8233792$; $173.8233792 \div 4.8284272$ (*See* Table) $= 36$, and $\sqrt{36} = 6$ ft., side of the base required.

Ex. 3. A regular octagonal pyramid contains 2433.5273088 solid feet, and one of its equal sides measures 6 feet; what is its height?

OPERATION. $2433.5273088 \div 4.8284272 = 504$, and $(504 \div 6^2) \times 3 = 42$ feet, *Ans.*

4. Find the solidity of a regular pentagonal pyramid, its height being 15 feet, and each side of its base $2\frac{1}{2}$ feet? *Ans.* 53.7649 sq. ft.

5. How many cubic yards in a triangular pyramid, the height of which is 3.55 yards, and the three sides of its base 1.5, 1.9, and 2.6 yards? *Ans.* 1.6669 cu. yd.

6. A regular pentagonal pyramid contains 45.879297 solid yards, and its sides measure 6 feet; what is its height? *Ans.* 60 feet.

7. How many solid yards are there in a pentagonal pyramid, the side of which, at the base, measures 6 feet, and its height 60 feet? *Ans.* 45.8794 + cu. yd.

8. What is the measure of one of the sides of a regular pentagonal pyramid, containing 4678.56 solid feet, and having a height of 54 feet? *Ans.* 12.29 + feet.

9. An octagonal stone monument has a perpendicular height of 45 feet, and the linear measure of its side is 5 feet 10 inches. Also, each side of the inner cavity measures at the base 4 feet 11 inches, and its perpendicular height 41 feet. How many yards of stone does the monument contain? *Ans.* 32.1973 + cub. yd.

PROBLEM VI.

To find the solidity of the frustum of a pyramid.

655. RULE.—*Multiply the areas of the two bases together, and extract the square root of the product. This root will be the area of a base which is a mean between the other two. Take the sum of the areas of the three bases, and multiply it by one third of the altitude; the product will be the solidity.*

Ex. 1. If the length of a frustum of a square pyramid be 18 feet, the side of its greater base 27 inches, and that of its less 15 inches; what is the volume?

OPERATION. 27 in. = 2.25 ft., 15 in. = 1.25 ft.; $2.25^2 = 5.0625$, $1.25^2 = 1.5625$; $5.0625 \times 1.5625 = 7.91015625$; $\sqrt{7.91015625} = 2.8125$; $2.8125 + 5.0625 + 1.5625 = 9.4375$; $9.4375 \times \frac{18}{3} = 56.625$ cub. ft., *Ans.*

2. What is the solidity of a regular pentagonal frustum, whose altitude is 8 feet, each side of the lower base 18 inches, and each side of the upper base 6 inches? *Ans.* 14.9106 cub. ft.

3. What is the solidity of a frustum of pyramid, whose height is 2 yd., and the two ends are regular hexagons the sides of which are .70 and .20 yd.? *Ans.* 1.160484 cu. yd.

4. How many cubic feet in a square piece of timber, the areas of the two bases being 504 and 372 inches, and its length $31\frac{1}{2}$ feet? *Ans.* 95.44 + cub. ft.

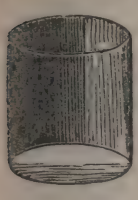
Problem VII.

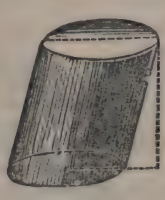
I. To find the surface of a cylinder.

656. Rule.—*Multiply the circumference of the base by the altitude, and the product will be the convex surface; and to this, add the areas of the two bases, when the entire surface is required.*

II. To determine the area of surface in a cylindrical ring.

657. Rule.—*To the thickness of the ring add the inner diameter, and multiply the sum by the thickness of the ring, and that product by* 9.8696.





Ex. 1. What is the entire surf. of the cylinder in which the diameter of the base is 10 feet, and the altitude 24 feet?

Operation. $3.1416 \times 10 = 31.416$, circumference of the base; $31.416 \times 24 = 753.984$, convex surf. Also, $10^2 \times .7854 = 78.54$, area of the base. Then, $753.984 + (78.54 \times 2) = 911.064$ sq. ft., *Ans.*

Ex. 2. The thickness of a cylindrical ring is 2 inches, and its inner diameter 6 inches; what is its area?

Operation. $(2 + 6) \times 2 = 16$, and $16 \times 9.8696 = 157.9136$ square inches, *Ans.*

3. Required the convex surface of a cylinder 13 feet long, the circumference of whose base is 57 inches. *Ans.* $61\frac{3}{4}$ sq. ft.

4. The radius of the base of a cylinder is .35 yd., the height is twice the diameter of the base; what is the whole surface of the cylinder? *Ans.* 3.84846 sq. yd.

5. Required the area of a ring 3 inches in thickness, and the inner diameter 8 inches. *Ans.* 325.6968 sq. in.

6. What is the concave surface of a semi-circular arch, whose diameter is 15 feet, and length 60 feet? *Ans.* 1413.72 sq. ft.

7. How much must be paid for the painting of the wall and ceiling of a circular room, whose diameter is 30 and height 15 feet, at $2.50 a sq. yd.? *Ans.* $589.05.

Problem VIII.

I. To find the solidity of a cylinder.

658. Rule.—*Multiply the area of the base by the altitude, and the product will be the solidity.*

Ex. 1. If a cylinder measure 8 feet in diameter at its base, and 18 feet in length; how many solid feet does it contain?

Operation. $8^2 \times .7854 = 50.2656$, area of the base. Then, $50.2656 \times 18 = 904.7808$ cub. ft., *Ans.*

Ex. 2. If the thickness of a cylindrical ring is 2 inches, and its diameter 6 inches, what is its solidity?

Oper. $(6 + 2) \times 2^2 = 32$; $32 \times 2.4674 = 78.9568$ solid in., *Ans.*

3. What is the capacity of a circular basin, the radius of whose base is 5 yards, and altitude 2 yards? *Ans.* 157.08 cub. yd.

4. What is the solidity of a cylinder whose base equals 2.15 sq. yd., and altitude 1.46 yd.? *Ans.* 3.139 cub. yd.

5. What is the solidity of a circular ring, 4 inches in thickness and 18 inches in diameter? *Ans.* 868.5248 cub. in.

6. A cast-iron rod is $1\frac{1}{2}$ inches in diameter, and 15 feet in length; what is its solidity in cubic inches? *Ans.* 318.087 cub. in.

7. Required the solidity of a cylinder whose altitude is 1.50 yd., and the circumference of whose base 3.08 yd. *Ans.* 1.132391 cub. yd.

8. The area of the base of a cylinder is 4 sq. yd., and the perpendicular distance between the two bases is 8 yards; what is its solidity?

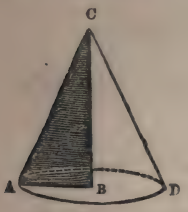
Problem IX.

I. To find the entire surface of a cone.

660. Rule.—*Multiply the perimeter or the circumference of the base by half of the slant height, and to the product add the area of the base.*

II. To find the height or diameter of a cone, one of them and its solidity being given.

661. RULE.—*Divide the solidity by .7854; then, if the* DIAMETER *be required, by one third the altitude also, and extract the square root of the quotient; but if the* ALTITUDE *be required, by the square of the diameter, and multiply the quotient by 3.*



Ex. 1. What is the entire surface of the cone whose vertex is C, the radius A B of its base being 5 feet, and the side C A, 40 feet?

OPERATION. $3.1416 \times (5 \times 2) = 31.416$, circumf. of base. $31.416 \times \frac{40}{2} = 628.32$, convex surface; $10^2 \times .7854 = 78.54$; $628.32 + 78.54 = 706.86$ sq. ft., *Ans.*

Ex. 2. What is the diameter of the base of a cone, if its solidity be 24 feet, and its altitude 12 feet?

OPERATION. $\sqrt{(24 \div .7854 \div \frac{12}{3})} = 2.764$ feet, nearly, *Ans.*

Ex. 3. If the solidity of a cone be 36 feet, and its diameter at the base 3 feet; what is its altitude?

OPER. $36 \div .7854 \div 3^2 = 5.0929$; $5.0929 \times 3 = 15.278$ ft., *Ans.*

4. Required the entire surface of a cone whose side is 36 and the diameter of its base 18 feet. *Ans.* 1272.348 sq. ft.

5. If the solidity of a cone be 72 feet, and its altitude 30 feet; what is its diameter? *Ans.* 3.027+ feet.

6. The circumference of the base of a cone is 9.50, and the slant height is 20.75; what is the entire surface? *Ans.* 105.744+

7. What will it cost to tin a circular steeple, the base of which is 16 feet in diameter, and the slant height 48 feet, at 75 cts. per square yard? *Ans.* $100 53 +.

8. Find the convex surface of a cone, whose slant height is 40 feet, and the circumference at its base 12 feet.

9. If the solidity of a cone be 3684 feet, and its diameter 30 feet; what is its altitude? *Ans.* 15.635+ feet.

PROBLEM X.

To find the solidity of a cone.

662. RULE.—*Multiply the area of the base by the altitude; and divide the product by 3, the quotient will be the solidity.*

Ex. 1. What is the solidity of a cone, the diameter of whose base is 4 feet, and altitude 5 feet?

OPERATION. $4^2 \times .7854 = 12.5664$ square feet, area of the base; $(12.5664 \times 5) \div 3 = 20.944$ cub. ft., *Ans.*

Ex. 2. What is the solidity of a cone whose side is 2.5 yards, and the radius of its base 1.5 yards?

Find first the altitude of the cone. The altitude, radius, and side of the cone, form a right-angled triangle whose hypothenuse is the side of the cone; let h be the altitude, we have

$h^2 = 2.50^2 - 1.50^2 = 6.25 - 2.25 = 4$ yd.; hence, $h = \sqrt{4} =$ 2 yd. $3.1416 \times 1.5^2 = 7.0686$ sq. yd., area of the base; $7.0686 \times \frac{2}{3} = 4.7124$ cub. yd., *Ans.*

3. The circumference of the base of a cone is 40 ft., and the altitude 60 ft.; what is its solidity? *Ans.* 2546.56 cub. ft.

4. What is the solidity of a circular pyramid, the diameter of which at the base measures 4 ft., and its height 18 ft.? *Ans.* 75.3984 cu. ft.

5. What is the solidity of a cone whose height is 1.35 yd., and the area of the base 3.40 sq. yd.? *Ans.* 1.530 cub. yd.

6. Required the solidity of a cone whose altitude is 1.23 yd., and the circumference of its base 1.98 yd. *Ans.* 0.127913 cub. yd.

7. What is the solidity of a cone, the side of which is 5 yards, and altitude 4 yards? *Ans.* 37.6992 cub. yd.

Problem XI.

To find the surface of the frustum of a cone.

663. Rule.—*Add together the circumferences of the two bases; and multiply the sum by half the slant height of the frustum; the product will be the convex surface, to which add the areas of the bases, when the entire surface is required.*

Ex. 1. What is the entire surface of the frustum of a cone, the slant height of which is 10 feet, and the circumferences of the bases 8 and 6 feet?

Operation. $(8 + 6) \times \frac{10}{2} = 70$ sq. ft., convex surface; $8^2 \times .07958 = 5.09312$, lower base, $6^2 \times .07958 = 2.86488$, upper base; $70 + 5.09312 + 2.86488 = 77.958$ sq. ft., entire surface, *Ans.*

2. What is the convex surface of the frustum of a cone, the side being .7 yd., and the radii of the bases .3 and .95 yd.?

Ans. 2.7489 sq. yd.

3. What is the concave surface of a tub whose diameter of the bottom is 2.10 yd., that of the top 2.30 yd., and slant height 3.84 yd.?

Ans. 26.5402 sq. yd.

4. There is a frustum of a cone, whose slant height is 12 feet, the circumference of the base 18 feet, and that of the upper end 9 feet; what is its whole surface? *Ans.* 194.22+ sq. ft.

Problem XII.

To find the solidity of the frustum of a cone.

664. Rule.—*Find the sum of the areas of the two ends, and*

of a geometrical mean between them ; multiply the same by one third the altitude, and the product will be the solidity.

Ex. 1. If the diameters of the two bases of the frustum of a cone be 24 and 20 feet, and the altitude 30 feet ; what is its solidity ?

OPERATION. $24^2 \times .7854 = 452.39$, area of the lower base; and $20^2 \times .7854 = 314.16$, area of upper base ; and $\sqrt{452.39 \times 314.16} = 376.99$, the geometrical mean ; then, $(452.39 + 314.16 + 376.99) \times \frac{30}{3} = 11435.4$ cub. ft., *Ans.*

2. What is the solidity of the frustum of a cone, the altitude being .90 yd., the area of the lower base 2.25 sq. yd., and of the upper 1.21 sq. yd. ? *Ans.* 1.533 cub. yd.

3. How many cubic feet in the frustum of a cone, whose altitude is 28 feet, and the diameters of the bases 22 and 18 feet ?

4. Required the solidity of the frustum of a cone, the altitude being 6.75 yd., the circumference of the lower base 1.445 yd., and of the upper .628 yd. *Ans.* .606975 + cub. yd.

5. What is the height of the frustum of a cone, the convex surface of which is 84 sq. ft., knowing that the area of the upper base is 3 sq. ft., and of the lower base 12 sq. ft. ? *Ans.* 12 feet.

PROBLEM XIII.

I. To find the area of a wedge.

665. RULE.—*Find the area of the head*, which is a parallelogram, *of the two sides*, which are parallelograms, *and of the two ends*, which are triangles ; *the sum of these several areas will be the required area.*

II. To find the area of a prismoid or frustum of a wedge.

666. RULE.—*Find the area of each of the sides and ends of the prismoid, separately ; the amount of these several areas will be the area required.*

Ex. 1. The back of a wedge is 10 inches long and 2 broad, and each face 12 inches long ; what is its area ?

OPERATION. $10 \times 2 = 20$ sq. in., area of the head, and $12 \times 10 \times 2 = 240$ sq. in., area of the two sides ; $\sqrt{12^2 - 1^2} = 11.96$, nearly, the perpendicular distance from the head of the wedge to the central point of its thickness or head ; and $11.96 \times 2 = 23.92$, area of the two ends. Then $20 + 240 + 23.92 = 283.92$ sq. in., *Ans.*

Ex. 2. The length and breadth of the base of a prismoid are 10 and 2 inches, the length and breadth of the section cut off are 10 and 1 inches, and the length from the base to the upper section 10 in. ; what is the area ?

OPERATION. $10 \times 2 = 20$ sq. in., area of the base; $10 \times 1 =$

10 sq. in., area of the section cut off; and $10 \times 10 \times 2 = 200$ sq. in., area of both faces. Then $\frac{2-1}{2} = .5$ in., one half the differ. between the thickness of the base and the section cut off; and $\sqrt{10^2 - .5^2} = 9.98$ in., the perpendicular distance between the base and upper section; and $(2+1) \times 9.98 = 29.94$ sq. in., area of the two ends. Then $20 + 10 + 200 + 29.94 = 259.94$ sq. in., *Ans.*

3. The back of a wedge is 8 in. long and 4 in. broad, and each face 18 in. long; what is the area in sq. ft.? *Ans.* 2.7191 + sq. ft.

4. The length and breadth of the back of a wedge are 10 and 4 in., the length and breadth of the upper section 5 and 2 in., and the length of each face 20 in.; what is the whole surf.? *Ans.* 3.26 + sq. ft.

5. The perpendicular height of a wedge is 20 inches, the thickness of the head 3 inches, and its length 5 inches; what is its entire area? *Ans.* 274.85 sq. in.

Problem XIV.

I. To find the solidity of a wedge.

667. Rule.—*Multiply the sum of twice the length of the base and the length of the edge by the breadth of the base, and that product by one sixth the height of the wedge, the result will be the solidity.*

II. To find the solidity of a prismoid or frustum of a wedge.

668. Rule.—*Multiply the sum of the areas of the two ends, and of four times the area of a section parallel to, and equally distant from, the two ends, by $\frac{1}{6}$ the height of a prismoid.*

Ex. 1. The length of the base of a wedge is 36 inches, its breadth 12 inches, the length of the edge 60 inches, and its height 18 inches; what is its solidity?

Operation. $(36 \times 2 + 60) \times 12 \times 3 = 4752$ solid inches, or 2.75 solid feet, *Ans.*

Ex. 2. The dimensions of a rectangular prismoid are as follows: length and breadth of the base 10 and 6 inches; of the face parallel to the base 6 and 4 inches; and the perpendicular height 40 inches. What is its solidity?

Operation. $10 \times 6 = 60$ sq. in., area of the base; $6 \times 4 = 24$ sq. in. of opposite section. Then $(10 + 6) \div 2 = 8$, the length of the central section, and $(6 + 4) \div 2 = 5$, the breadth of the central section. Then $(8 \times 5) \times 4 = 160$ sq. in., or four times the area of the central section; $60 + 24 + 160 = 244$, and $244 \times \frac{40}{6} = 1626+$ solid inches, *Ans.*

3. What is the solidity of a stone pillar, the base measuring 3 ft. by 2 ft. 6 in.; the top 2 ft. by 1 foot 6 inches; and the perpendicular height being 8 feet? *Ans.* 40 cub. ft. 1152 cub. in.

4. If the length of the base of a wedge be 24 inches, its breadth 7 inches, its edge 32 inches, and its height 33 inches; what is its solidity? *Ans.* 3080 cub. inches.

Problem XV.

I. To find the surface of a sphere or globe.

669. Rule.—*Find the area of a circle of the same diameter as the sphere, and multiply the same by* 4. Or,

Multiply the diameter by the circumference of the sphere, the product will be the surface.

II. To find the diameter of a sphere from its surface.

670. Rule.—*Divide one fourth the area by* .7854, *and extract the square root of the quotient.*

III. To find the surface of a spheroid or ellipsoid.

671. Rule.—*Multiply the product of the two diameters by* .7854, *and that product by* 4, *the result will be the surface.*

IV. To find the convex surface of a segment or zone of a sphere.

672. Rule.—*Multiply the circumference of the sphere of which the segment or zone forms a part, by the height of the segment or zone.*

Ex. 1. What is the surface of a globe 50 inches in diameter?

Operation. The surface of a great circle is $.7854 \times 50^2 = 1963.50$ sq. in. Hence, the surface of the globe is $1963.5 \times 4 = 7854$ sq. inches, *Ans.* Or, $50 \times 3.1416 = 157.08$, the circumference of a great circle; $157.08 \times 50 = 7854$ sq. in., surface of the globe, *Ans.*

Ex. 2. If the area of the surface of a sphere be 24 square feet; what is its diameter?

Operation. $(24 \div 4) \div .7854 = 7.6394$, and $\sqrt{7.6394} = 2.76$ feet, *Ans.*

Ex. 3. If the longer diameter of an ellipsoid be 6 feet, and the shorter 5 feet; what is its surface?

Operation. $(6 \times 5) \times .7854 = 23.562$; $23.562 \times 4 = 94.248$ sq. ft., *Ans.*

Ex. 4. If the diameter of a sphere be 50 inches, what is the convex surface of a segment of the same 10 inches high?

OPERATION. $50 \times 3.1416 = 157.08$, circumference of the circle, and $157.08 \times 10 = 1570.8$ sq. in., area required, *Ans.*

5. What is the surface of a sphere, the circumference of whose great circle is 4.84 yd.? *Ans.* 7.4506 sq. yd.

6. The diameter of a sphere is 21 inches; what is the surface of a zone whose height is $4\frac{1}{2}$ inches? *Ans.* 296.8812 sq. in.

7. If the surface of a sphere be 6.16 square yards, what is its diameter? *Ans.* 1.40 yd.

8. The longer diameter of an ellipsoid is 18 feet, and the shorter 15 feet; what is its surface?

9. Required the surface of the segment of a sphere, comprised between two parallel plans at a distance of 1.25 yd. from each other, the radius of the sphere being 3.50 yd. *Ans.* 27.489 sq. yd.

10. The radius of a sphere is 3.08 yd.; required 1° the circumference of a great circle; 2° the surface of that sphere.

Ans. 1° 19.352 yd.; 2° 119.2098 sq. yd.

11. The area of a zone is 2.85 sq. yd.; required the entire surface of the sphere, the height of the zone being .45 yd.?

Ans. 12.742 sq. yd.

12. Required in miles the surface of the two frigid zones, allowing 327.15657 miles for the height of each of them, and 3955.82936 miles for the radius of the sphere.

NOTA.—To find the surfaces of irregular solids, or bodies, the following process is followed:—If the solids are composed of plane faces, *find the area of each face, and add them together for the whole surface of the solid;* if composed of circular faces, *divide these into a number of faces infinitely great, so that each might be considered a plane. Then proceed as above to obtain the entire surface.*

PROBLEM XVI.

I. To find the solidity of a sphere.

673. RULE.—*Multiply the surface by one third of the radius, and the product will be the solidity.* Or,

Multiply the cube of the diameter by the decimal .5236, *and the product will be the solidity.*

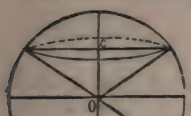
II. To find the diameter of a sphere from its solidity.

674. RULE.—*Divide the solidity by* .5236, *and extract the cube root of the quotient.*

III. To find the solidity of a spheroid or ellipsoid.

675. RULE.—*Multiply the longer axis by the square of the shorter one, and the product by the decimal* .5236, *the result will be the required solidity.*

IV. To find the solidity of the segment of a sphere.



676. RULE.—1. *From three times the diameter of the sphere, deduct twice the height of the segment; multiply the remainder by the square of the height, and that product by* .5236; *the last product will be the solidity.*

677. RULE.—2. *To three times the square of the radius of the segment's base, add the square of its height; multiply this sum by the height, and the product by* .5236; *the last result will be the solidity.*

V. To find the solidity of a zone of a sphere.

678. RULE.—1. *To the sum of the squares of the radii of the two ends, add one third the square of the distance between them,* (that is, of the height of the zone,) *multiply the result by the height of the zone, and again by* 1.5708, *and the last result will be the solidity.*

679. RULE.—2. For the middle zone of a sphere: *From the square of the diameter of the sphere of which the zone is a part, subtract one third the square of its height, and multiply the remainder by the height, and also by* .7854.

VI. To find the solidity of a spherical sector.

680. RULE.—*Multiply the spherical surface by one third the radius of the sphere.*

Ex. 1. If the diameter of a globe be 12 inches, how many solid inches does it contain?

OPERATION. $3.1416 \times 12 = 37.6992$, circumference of the globe; $37.6992 \times 12 = 452.3904$; $452.3904 \times \frac{2}{3} = 904.7808$ cub. in.; or $12^3 \times .5236 = 904.7808$ cub. in., *Ans.*

Ex. 2. What is the diameter of a sphere containing 6856 solid ft.?

OPERATION. $\sqrt[3]{6856 \div .5236} = 23.54+$ feet, *Ans.*

Ex. 3. If the longer axis of an ellipsoid be 3 feet, and the shorter one 2 feet; what is its solidity?

OPERATION. $(2^2 \times 3) \times .5236 = 6.2836$ cub. ft., *Ans.*

Ex. 4. If the diameter of a sphere be 20 feet, what is the solidity of a segment of the same, the height of which is 5 feet?

OPERATION. By Rule 1; $(20 \times 3) - 10 = 50$; $50 \times 5^2 = 1250$; $1250 \times .5236 = 654.5$ cub. ft., *Ans.*

Ex. 5. What is the solidity of the temperate zone, the upper radius being 1586.57282526 miles; the lower radius 3648.86750538 miles; and the height 2062.2655 miles?

OPER. $[(1586.57282526)^2 + (3648.86750538)^2 + \frac{1}{3}(2062.2655)^2 \times 2062.2655 \times 1.5708 = 55877778668$ cubic miles, *Ans.*

Ex. 6. The diameter of a sphere is 15 feet. What is the solidity of a sector of the same, the circular base of which is $1\frac{1}{2}$ feet distant from the central section?

OPERATION. $15 \times 3.1416 \times 6 = 282.744$ sq. ft., the convex surface of the sector. Then $15 \div 2 = 7\frac{1}{2}$, radius of the circle; $282.744 \times (7\frac{1}{2} \div 3 = 2\frac{1}{2}) = 706.86$ cub. ft., *Ans.*

7. Required the diameter of a cannon-ball weighing 80 lb., knowing that a cubic foot of cast-iron weighs $450\frac{1}{4}$ lb. *Ans.* 0.6973 ft.

8. If the diameter of the base of the segmentof a sphere be 30 feet, and the height of the same 5 feet; what is its solidity?

Ans. 1832.6 cub. ft.

9. What is the solidity of a sector of a sphere 24 feet in diameter, the circular base of which is 4 feet distant from the central section?

Ans. 1809.5616 cub. ft.

10. The surface of a sphere is 55.44 square yards; what is its solidity? *Ans.* 38.863 + cub. yd.

11. What is the solid content of a spheroid, the longer axis of which is 16 feet, and the shorter 12 feet?

12. What is the solidity of the torrid zone, the diameter of the earth being 7957.75 miles, and the height of the zone 8173.14565052 miles? *Ans.* 149455081137 cub. miles.

13. The diameter of a sphere is 24 feet, what is its solid contents?

Ans. 7238.2464 cub. ft.

14. What is the solidity of a spherical segment whose height is 2 feet, and the diameter of the sphere 10 feet? *Ans.* 54.4544 cub. ft.

15. Required the solidity of the middle zone of a sphere, the top and bottom diameters being each 4 feet, and its height 6 feet?

Ans. 188.496 cub. ft.

16. The height of a spherical segment is 8 inches, and the radius of its base 14 in.; what is its solidity? *Ans.* 2731.0976 cub. in.

17. If the solidity of a sphere be 4.62 cub. yd., what is 1° its diameter; 2° the circumference of its great circle; 3° its whole surface?

18. Required the volume of a spherical sector, the circular base of which is .25 yd. distant from the central section, and the diameter of the sphere .84 yd. *Ans.* 2.216712 cub. yd.

19. The height of a spherical segment is .42 yd., its surface 1.6632 sq. yd.; what is 1° the radius of the sphere; 2° the solidity of the spherical sector? *Ans.* 1° .63 yd.; 2° .349272 cub. yd.

20. What is the solidity of a zone whose greater diameter is 25 ft., the less 20 feet, and the height between them 12 feet?

Ans. 5734.9908 cub. ft.

PROBLEM XVII.

To find the solidity of any regular polyhedron.

681. RULE.—*Multiply the surface of the polyhedron by one third the radius of the inscribed sphere, and the product will be the solidity.*

This rule is evident from the fact, that any regular polyhedron might be divided into as many regular pyramids as it has bases, having their vertices in the centre of the inscribed sphere.

NOTE.—To find the solidity of any irregular body, such as a stone, a chain, &c., *immerse the body in a vessel with sufficient water to cover it, the volume of the water displaced will represent the solidity of the body immersed.*

Or, *weigh the body and divide it by the weight of a cubic foot or cubic inch of the same material; the quotient will show the solidity of the body in cubic feet or inches.*

Ex. 1. What is the solidity of a stone which, immersed in water, displaces 12½ lb. of it?

OPERATION. The volume of one pound of water is 27.7274 cu. in.; therefore the solidity of the stone is 27.7274 × 12½ = 346.5925 cub. inches, *Ans.*

Ex. 2. A vessel holding 5 gallons is filled with water; an object is immersed and then taken out; the water remaining in the vessel is 3½ gallons. What is the solidity of the object?

OPERATION. 5 — 3.5 = 1.5; a gallon contains 277.274 cub. in.; hence 277.274 × 1.5 = 415.911 cu. in., *Ans.*

NOTE.—The solidity of certain bodies may be found by dividing them into pyramids or other solids whose contents may easily be reckoned. The easiest division consists in taking for the vertex of the pyramid, the vertical angle of the solid, and for base the side opposite. An easy process to find the height of all kinds of pyramids, is to produce the plane which is taken as the base, by means of a thin plane board, and a ruler is placed on the top of the pyramid parallel to that board; the shortest distance between the ruler and the plane will evidently give the height of the pyramid.

MISCELLANEOUS EXAMPLES IN SOLIDS.

1. The solidity of a regular hexagonal prism is 71.1126 cub. yd., and the side of the hexagon 2.34 yards; required 1° the area of the base; 2° the altitude of the prism

Ans. 1° 14.2261 sq. yd.; 2° 4.998 yd.

2. What is the solid contents of a spherical segment, the diameter of whose base is 12 ft., and its altitude 5 ft.? *Ans.* 348.194 cu. ft.

3. A room 9.25 yd. long, 4.85 yd. wide, and 4.80 yd. high, is to be papered; the rolls of paper are 12 yd. long by .5 yd. wide. Allowing 12.25 sq. yd. for the apertures in the walls, how many rolls will be required, and what will be the cost at 75 cts. per roll?

Ans. 20.51⅝ rolls, and $15.38 +.

4. Find the convex surface of a cylinder, the radius of whose base is 2.8 yd., and its altitude ⅜ of the circumf. *Ans.* 185.707 + sq. yd.

5. A cable 3 feet long and 9 inches in circumference weighs 22 lb.; what is the weight of another cable 6 feet in length and 1 foot in circumference? *Ans.* 96.1875 lb.

6. What is the weight of a square brick pillar whose side is 0.75 yards, and height 4.75 yards, if 1 cubic yard of brick masonry weighs 36 cwt.? *Ans.* 96.1875 lb.

7. The slant height of a regular hexagonal pyramid is 8 yd., and the side of its base 6 yards; what is its whole surface?

Ans. 237.5307 sq. yd.

8. A man had a wall built for $136, which was $3.20 a cub. yd. What is the height of that wall, knowing that it is 14.5 yd. long and 70 thick? *Ans.* 4.18 yd.

9. A rectangular basin is 12 to. long, 2.5 to. wide, and 1.5 to. deep; how many barrels of 31½ gal. each does it hold, there being 231 cu. in. in a gallon? *Ans.* 2811.9 + bbl.

10. The convex surface of regular triangular pyramid is 45 sq. yd., the slant height is 6 yd.; required the length of one of its side-edges.

Ans. 6.50 yd.

11. The lower base of a pile of stone is 26 by 12 yd., the upper one 16 by 8 yd., and the pile is 3 yd. high; find its cubic contents.

Ans. 639.84 cub. yd.

12. What is the convex surface of a right cone, the radius of whose base is 1.4 yd. and its side, the ¾ of the circumference of the base?

Ans. 29.0167 + sq. yd.

13. I desire to get a cylindrical tub made whose depth will be 3 ft.; what must be its diameter that it may hold twice as much as a similar tub whose depth is 5 ft., and diameter 3½ ft.? *Ans.* 6.38 ft.

14. What is the slant height of the frustum of a cone, whose convex surface is 12.26 square yards, and the radii of the two bases 1.71 and 2.2 yards? *Ans.* .998 yd.

15. How many cords in a pile of wood whose length, breadth, and height are respectively 15.5, 4, and 7.25 yd.? *Ans.* 94.81 + .

16. What must be the radius of a cylindrical basin holding 110045 gallons, its depth being 5 yd.? *Ans.* 5.889 + yd.

17. How many solid feet in the frustum of a pyramid whose bases are regular octagons, the sides of which are respectively 21 and 9 in., and the perpendicular distance of the bases 15 ft.?

Ans. 119.20 + cub. ft.

18. What is the surface of the base of a quadrangular prism, whose altitude is 1.15 yd., and its solidity 4.25 cu. yd.? *Ans.* 3.6956 sq. yd.

19. Find the solidity of a beam whose length, breadth, and thickness, are respectively 12.75, 0.35, and 0.25 yd. *Ans.* 1.115625 cu. yd.

20. A man gets a cemented cistern made in the ground that will hold 3000 gal.; what will be its depth, the length and breadth being respectively 1.8 and 1.75 yd.? *Ans.* 4.715 yd.

21. What is the solidity of a regular hexagonal pyramid whose altitude is 3.6 yd., and the side of the base 3.6 yd.?

Ans. 40.405651 cub. yd.

22. What is the convex surface of the frustum of a triangular pyramid whose bases are parallel, knowing that the sides of the lower base are 2, 3, and 4 ft.; the corresponding sides of the upper base 0.95, 1.20, and 2.10 ft.; and the height of the three trapezoids 5, 6, and 6.45 feet? *Ans.* 40.4537 sq. ft.

23. What are the dimensions of a barn whose capacity is 810 cu.

yd. excluding the loft, if its breadth is $\frac{4}{7}$ of its length, and its height up to the loft $\frac{2}{5}$ of its length?

24. What is the altitude of a cone whose solidity is 3.077 cu. yd., and the radius of its base 0.35 yd.? *Ans.* 23.985 yd.

25. What is the length of a great circle of a sphere which measures 12 sq. yards?

26. A gentleman wishes to have a circular fish-pond made in his garden that will cover an area of half an acre; what will be its diameter? *Ans.* 55.50 yd.

27. What is the surface of a segment serving as a base to a spherical sector whose solidity is 1.545 cub. yd., the radius of the sphere being 1.5 yd.? *Ans.* 5.15 sq. yd.

28. What is the surface of the earth, its radius being about 3955.865 miles? *Ans.* 196649933.55625464 sq. mi.

29. Find the solidity of a log 9.25 yd. long, the circumference of its ends being 1.5 and 0.55 yd. *Ans.* 0.77338062 cub. yd.

30. What must be the thickness of a hollow sphere whose internal and external surfaces are 3 and 3.12 yards? *Ans.* 0.1 yd.

31. A basin holds 1093.75 cubic yards of water; its length is to its breadth as 7 to 5, and to its depth as 5 to 3; find its dimensions.

32. Required the convex surface of the frustum of a regular heptagonal pyramid whose bases are parallel, and one side of each measuring respectively 1.64 and 1 yard, the slant height being 2.25 yd. *Ans.* 20.79 sq. yd.

33. A heap of sand rests on a rectangular base of 4.25 by 2.15 yd., and assumes a wedge form at the top whose length is 3.4 yd.; if its altitude is .8 yd., how many solid yards in the heap? *Ans.* 3.411333 cub. yd.

34. A cubic foot of copper was drawn into a wire of $\frac{1}{10}$ in. in diameter; no metal being lost, how long was the wire?

35. One pound of lead was drawn into a wire 168 yd. long, and sold for 60 cts.; allowing 1 cu. in. of lead to weigh 6.569 oz., and the wire perfectly cylindrical, what was its diameter?

36. A sphere, whose surface is 78.54 sq. yd. is divided by a plane cutting the radius at its $\frac{2}{5}$ from the centre; what is the convex surface of each part? *Ans.* 1st 23.5620 sq. yd.; 2nd 54.978 sq. yd.

37. What is the weight of a copper spherical shell 0.985 inch in thickness, if the external diameter of the sphere is 1.35 yd., and the weight of a cubic foot of copper 524$\frac{3}{4}$ lb.

38. A gentleman wants an elliptical fish-pond to be digged in his court-yard; the great axis of that fish-pond is to measure 15 yd., the small axis 12.75 yd., and the depth 4.50 yd., not including the thickness of the wall which is .45 yd. throughout. Required 1° the cost for digging it, if the contractor charges \$1.75 per cubic yard; 2° the cost of the masonry, at \$15.45 per cub. yd.; 3° the whole cost; and 4° the capacity of the pond.

39. A vessel in the shape of a frustum of a cone whose radius of the lower base is 5$\frac{1}{2}$ inches, the diameter of the upper base 13 inches, and its depth 16 inches, is filled with nitric acid. What is the value of that acid, if it be worth 16 cts. a quart? *Ans.* \$5.02.

40. Supposing the Moon to be a perfect sphere, what is her surface, her diameter being to that of the Earth as 3 is to 11, and the diameter of the Earth being 7912 miles? *Ans.* $14626758\frac{6}{11}$ sq. mi.

41. A founder wishes to cast a semi-spherical boiler whose internal diameter shall be $6\frac{3}{4}$ feet, its thickness $2\frac{7}{10}$ in. Required the weight of cast-iron it will take, if we allow 10 % waste in melting, knowing that the specific weight of cast-iron is 7.206. *Ans.* 8608.36+ lb.

42. The interior space of a blast furnace consists of two conic frustums uniting at their larger base whose diameter is $\frac{1}{3}$ of the height of the furnace. The altitude of the upper frustum is $\frac{2}{3}$ of the height of the furnace, its less diameter is $\frac{2}{3}$ of the greater. The altitude of the lower frustum is $\frac{5}{24}$ of the height of the furnace, its less diameter is $\frac{1}{3}$ of the greater. If the furnace is 15 yards high, what is its interior capacity? *Ans.* 41.921 cub. yd.

43. A fountain in the form of the frustum of a cone is filled with water. Required 1° how many gallons it contains, if the circumferences of its bases are 16.95 and 15.86 yards, and its depth 5.35 yd.; 2° in how many hours it will be emptied, if the water is let out by three pipes, of which the 1st empties $1\frac{1}{2}$ gal. in 1 minute; the 2nd, 11 gal. in 8 min.; the 3rd, 13 gal. in $\frac{1}{10}$ hr.; 3° what time would each pipe take to empty the whole fountain by itself?

TABLE OF CHORDS.

In the table of chords, the radius of any circle is represented by 1; and in decimal of the radius, is represented the length of chords that subtend arcs of 1', 2', 3', &c., up to an arc of 180°, which is itself a semi-circumference.

Any chord which is not a diameter, subtends two arcs, one of which is less, and the other greater than a semi-circumference; but their sum equals the circumference.

In all problems treating of arcs, the smaller arc is always implied, unless otherwise mentioned.

1st Rule.—To obtain the chord of any arc greater than a semi-circumference, *subtract the degrees of the given arc from* 360°, *and find in the table the chord that corresponds with the difference.*

Ex.—What is the chord of an arc of 310°?

360° — 310° = 50°. In the table, the chord of 50° is 0.8452.

2nd Rule.—To find the length of any chord in any given circle, *multiply the radius of the given circle by the chord indicated in the table.*

Ex.—How long is the chord of an arc of 24° in a circle whose radius is 25 yd.?

The chord of 24° in the table is 0.4158; 25 × 0.4158 = 10.395 yards, *Ans.*

3rd RULE.—To find the radius of any circle, *divide the given chord by the corresponding chord in the table.*

Ex. What must be the radius of a circle in which a chord of 12 yards subtends an arc of 20° 10' ?

The chord of 20° 10' in the table is 0.3502 ; 12 ÷ 0.3502 = 34.209 yards, *Ans.*

4th RULE.—To obtain the degrees of any arc, *divide the chord by the radius of the circle, and find in the table the degrees corresponding with the quotient.*

Ex.—What are the degrees of an arc whose chord is 4.24 yd., if the radius of the circle is 20 yd. ?

4.24 ÷ 20 = 0.2120 ; 0.2120 in the table indicates 12° 10', *Ans.*

The following table gives the chords for every 10' which is practically sufficient; but, by a proper approximation, its use may be extended to any number of degrees and minutes.

TABLE OF CHORDS.

D.	0'	10'	20'	30'	40'	50'
0°	0	0,0029	0,0058	0,0087	0,0116	0,0145
1	0,0175	0,0204	0,0233	0,0262	0,0291	0,0320
2	0,0349	0,0378	0,0407	0,0436	0,0465	0,0494
3	0,0523	0,0553	0,0582	0,0611	0,0640	0,0669
4	0,0698	0,0727	0,0756	0,0785	0,0814	0,0843
5	0,0872	0,0901	0,0931	0,0960	0,0989	0,1018
6	0,1047	0,1076	0,1105	0,1134	0,1163	0,1192
7	0,1221	0,1250	0,1279	0,1308	0,1337	0,1366
8	0,1395	0,1424	0,1453	0,1482	0,1511	0,1540
9	0,1569	0,1598	0,1627	0,1656	0,1685	0,1714
10	0,1743	0,1772	0,1801	0,1830	0,1859	0,1888
11	0,1917	0,1946	0,1975	0,2004	0,2033	0,2062
12	0,2091	0,2120	0,2148	0,2177	0,2206	0,2235
13	0,2264	0,2293	0,2322	0,2351	0,2380	0,2409
14	0,2437	0,2466	0,2495	0,2524	0,2553	0,2582
15	0,2611	0,2639	0,2668	0,2697	0,2726	0,2755
16	0,2783	0,2812	0,2841	0,2870	0,2899	0,2927
17	0,2956	0,2985	0,3014	0,3042	0,3071	0,3100
18	0,3129	0,3157	0,3186	0,3215	0,3244	0,3272
19	0,3301	0,3330	0,3358	0,3387	0,3416	0,3444
20	0,3473	0,3502	0,3530	0,3559	0,3587	0,3616
21	0,3645	0,3673	0,3702	0,3730	0,3759	0,3782
22	0,3816	0,3845	0,3873	0,3902	0,3930	0,3959
23	0,3987	0,4016	0,4044	0,4073	0,4101	0,4130
24	0,4158	0,4187	0,4215	0,4244	0,4272	0,4300

D.	0'	10'	20'	30'	40'	50'
26	0,4499	0,4527	0,4556	0,4584	0,4612	0,4641
28	0,4838	0,4867	0,4895	0,4923	0,4951	0,4979
30	0,5176	0,5204	0,5233	0,5261	0,5289	0,5317
32	0,5513	0,5541	0,5569	0,5598	0,5625	0,5652
34	0,5847	0,5875	0,5903	0,5931	0,5959	0,5986
36	0,6180	0,6208	0,6236	0,6263	0,6291	0,6319
38	0,6511	0,6539	0,6566	0,6594	0,6621	0,6649
40	0,6840	0,6868	0,6895	0,6922	0,6950	0,6977
42	0,7167	0,7195	0,7222	0,7249	0,7276	0,7303
44	0,7492	0,7519	0,7546	0,7573	0,7600	0,7627
46	0,7815	0,7841	0,7868	0,7895	0,7922	0,7948
50	0,8452	0,8479	0,8505	0,8531	0,8558	0,8584
54	0,9080	0,9106	0,9132	0,9157	0,9183	0,9209
58	0,9696	0,9722	0,9747	0,9772	0,9798	0,9823
62	1,0301	1,0326	1,0351	1,0375	1,0400	1,0425
66	1,0893	1,0917	1,0941	1,0965	1,0990	1,1014
70	1,1472	1,1495	1,1519	1,1543	1,1567	1,1590
74	1,2036	1,2060	1,2083	1,2106	1,2129	1,2152
78	1,2586	1,2609	1,2632	1,2654	1,2677	1,2699
82	1,3121	1,3143	1,3165	1,3187	1,3209	1,3231
86	1,3640	1,3661	1,3682	1,3704	1,3725	1,3746
90	1,4142	1,4163	1,4183	1,4204	1,4224	1,4245
94	1,4627	1,4647	1,4667	1,4686	1,4706	1,4726
98	1,5094	1,5113	1,5132	1,5151	1,5170	1,5189
100	1,5321	1,5340	1,5358	1,5377	1,5395	1,5414
104	1,5760	1,5778	1,5796	1,5814	1,5832	1,5849
108	1,6180	1,6197	1,6214	1,6231	1,6248	1,6265
112	1,6581	1,6597	1,6613	1,6629	1,6645	1,6662
116	1,6961	1,6976	1,6991	1,7007	1,7022	1,7038
120	1,7320	1,7335	1,7350	1,7364	1,7378	1,7393
124	1,7659	1,7673	1,7686	1,7700	1,7713	1,7727
128	1,7976	1,7989	1,8001	1,8013	1,8026	1,8039
132	1,8271	1,8283	1,8294	1,8306	1,8318	1,8330
136	1,8544	1,8554	1,8565	1,8576	1,8587	1,8598
140	1,8794	1,8804	1,8814	1,8824	1,8833	1,8843
144	1,9021	1,9030	1,9039	1,9048	1,9057	1,9065
148	1,9225	1,9233	1,9241	1,9249	1,9257	1,9265
152	1,9406	1,9413	1,9420	1,9427	1,9434	1,9441
156	1,9563	1,9569	1,9575	1,9581	1,9587	1,9593
160	1,9696	1,9701	1,9706	1,9711	1,9716	1,9721
164	1,9805	1,9809	1,9813	1,9817	1,9821	1,9825
168	1,9890	1,9893	1,9896	1,9899	1,9902	1,9905
170	1,9924	1,9926	1,9929	1,9931	1,9934	1,9936
172	1,9951	1,9953	1,9955	1,9957	1,9959	1,9961
174	1,9973	1,9974	1,9975	1,9977	1,9978	1,9980
176	1,9988	1,9989	1,9990	1,9991	1,9992	1,9992
179	1,9999	1,9999	1,9999	1,9999	1,9999	1,9999
180	2,0000					

Culling and Measuring of Timber, Masts, Spars, Deals, Staves and other articles of a like nature.

(From the Consolidated Statutes of Canada, Cap. 45.)

Deals.—A Quebec Standard Deal is 12 feet long 11 inches broad and $2\frac{1}{2}$ inches thick, and contains 2. ft. 3 in. 6 pts. cubic. One Hundred Quebec Standard contain 229 ft. 2 in. cubic, or 4.29.50 loads, and are equivalent to 2750 feet superficial board measure of one inch thick.

One Quebec Standard is 100 pieces of 12 ft. by 11 in. by $2\frac{1}{2}$ in., and is equal to 1 hd. 1 qr. 16 pcs. of St. Petersburg Standard; and 240 Quebec Standard Deals are equal to 11 loads.

One St. Petersburg Standard Hundred is equal to 120 pcs. of 12 ft. by 11 in. by $1\frac{1}{2}$ in., and is equal to 72 Quebec Standard, and equal to $3\frac{3}{10}$ loads of Timber.

A Load of Deals is 600 square feet by one inch in thickness, equal to 50 cubic feet; or 300 square feet of 2 inches, or 400 of $1\frac{1}{2}$ inch.

A Load is equal to 21 deals 1 foot $10\frac{1}{2}$ inches, Quebec Standard, and equal to $36\frac{1}{2}$ St. Petersburg Standard deals.

RULE.—To convert Quebec Standard Hundred into St. Petersburg Standard :—*Add two-thirds of the quantity, and divide the sum by* 120. *If there should be any remainder, divide it by* 30, *for quarters.*

Staves.—One Standard Stave is $5\frac{1}{2}$ feet long, $1\frac{1}{2}$ inch thick, and 5 inches broad.

One Mille, or 1200 Standard Staves, is equal to 343 feet 9 inches, or $6\frac{43}{60}$ loads.

One hundred and seventy-five Standard Staves are equal to 50 feet 1 inch $6\frac{3}{4}$ parts, or 1 load 1 inch $6\frac{3}{4}$ parts.

One Mille West India Staves, 1200 pieces, is equal to $87\frac{1}{2}$ feet, or $1\frac{37}{60}$ loads of timber.

Owing to the variations in breadth and thickness of Staves, it is customary to allow one Mille, Quebec Standard, to be equal to 13 loads.

Lathwood.—One cord of Lathwood is 8 feet long and 4 feet high, English measure.

CUSTOMARY ALLOWANCE FOR FREIGHT AND BROKEN STOWAGE.

Deals.—A Hundred St. Petersburg Standard, at twice the charged rate for timber per load.

Staves.—A Mille Standard, at six times the rate charged for timber per load. A Mille West India, at twice the rate charged for timber per load.

Lathwood.—A fathom of Lathwood, at the same rate as charged for timber per load.

FREIGHT AND SHIPPING.

To find Freight measurements, or cubical contents of packages.

RULE.—*Multiply length, breadth and thickness together ;* for surfaces, *length and breadth only.*

For Stowage.—97 quarters of Wheat, or 140 barrels of Flour, or 80 barrels of Ashes, are considered equal.

For Grain.—42 cubic feet equal 1 ton of shipping. One bushel is equal to 60 lbs. 2218½ cu. in. are equal to an Imperial bushel. 8 bushels are equal to one quarter=17745 cu. in., or $10\frac{27}{100}$ cu. ft. Therefore, 1 ton will take $4\frac{1}{10}$ quarters, 1 bushel being = 60 lbs. ; 1 quarter = 480 lbs. ; 1 ton = 1968 lbs. A ship of 200 tons measurement can, therefore, carry 820 quarters ; but it can generally carry much more.

CUBIC OR SOLID MEASURE.

42	solid feet	equal 1	ton of shipping.
40	solid feet, round or unhewn	" 1	ton or load.
50	solid feet, hewn or squared timber	" 1	ton or load.
50	cubic feet	" 1	barrel of flour.
8	barrels	" 1	ton.
5	quarters	" 51⅓	cubic feet.
5	quarters	" 1	load.

SQUARE MEASURE (*see p.* 118, 119, 120).

ENGLISH.				FRENCH.	
36801.7	Square	Feet	equal 1	Square	Arpent.
0.845	"	Acres	" 1	"	"
2.471	"	"	" 1	"	Hectare.
1	"	Foot	" 0.0929	"	Metre.

1 " Yard ..... " 1.8361 " "
3.955 " Perches ..... " 1 Acre.

The side of a square acre is 69½ yards in length, and is often quoted by French-Canadians as a unit of length for short distances.

1 French foot is equal to 12$\frac{16}{17}$ English Inches.
104 " lbs. are " to 112 " Pounds.
1 Canadian Minot is " to 1.054 Imperial Bushel.

The following rules for Timber Calculations may be found useful by the trade.

To Reduce Square Timber, of different sizes, to an Average Square.

RULE.—*Add the lengths (in feet) together; reduce the cubic contents of the whole to parts; divide the product by the total lineal feet; the square root of the quotient will be the average square, in inches.*

To find the Cubic Contents of Round Timber.

RULE.—*Square the diameter; multiply the product by* 11 *and divide by* 14; *multiply the result by the length of the log; then reduce the product to feet, inches, and parts, dividing by* 12 *and by* 12 (*or by* 144).

TABLE OF SPECIFIC GRAVITIES.

In estimating the weights or specific gravities of bodies, rain-water is generally taken as the Standard. Experiment has shown, that a cubic foot of rain-water weighs 62½ pounds Avoirdupois, or 1000 ounces. It hence follows that a cubic inch weighs 0.03616898148 of a pound. If, therefore, the specific gravity of a body be multiplied by the above decimal, the product will be the weight of a cubic inch of that body in pounds Avoirdupois, which may be changed to Troy weight by being multiplied by 175 and the product divided by 144, since 144 lbs. Avoirdupois = 175 lbs. Troy.

The weight of a cubic foot of rain-water is taken as the unit.—viz. 1000; and the weight of a cubic foot of any one of the following articles is in ounces Avoirdupois.

Woods (Dry).		Liquids, Metals, &c.	
Ash	845	Alcohol, pure, 60°	794
Apple	793	Beer	1034
Box-wood	1031	Brandy	924
Beech	852	Blood (human)	1054
Birch	567	Bees-wax	965
Butternut	376	Brass, cast	8396
Cedar	561	Brick, fire	2201
Cherry	715	Coal (Anthracite)	1436
Chesnut	610	Coal (Newcastle)	1270
Cocoa	1040	Coke	1000
Cork	240	Copper, cast	8788
Cypress	644	Earth, common	2194
Ebony (American)	1331	Glass, window	2642
Elm	570	Gold, 22 carats	17486
Fir, White	512	Granite (Scotch)	2625
Hackmatack	592	Guttapercha	980
Hazel	860	Honey	1450
Hemlock	368	Iron, cast	7207
Holly	760	Ivory	1825
Lignum vitæ	1333	Lead, cast	11352
Lime	804	Lime, hydraulic	2745
Logwood	913	Marble (Vermont)	2650
Mahogany (Honduras)	560	Milk	1032
Maple	750	Petroleum	878
Maple, bird's eye	576	Plaster of Paris	1176
Oak (Canadian)	872	Platinum, native	16000
Oak (English)	932	Quicksilver	13568
Pear	661	Salt	2130
Pine, White	554	Sand, common	1670
Pine, Red	590	Silver, pure cast	10474
Pine, Yellow	461	Soap, Castile	1071
Pine, Pitch	660	Starch	950
Plum	785	Steel Plates	7806
Poplar	383	Tallow	941
Spruce	500	Tin, pure	7291
Tamarack	383	Turpentine	870
Walnut, Grey	671	Water, common	1000
Walnut, Black	550	Water, sea	1026
Willow	585	Zinc, rolled	7191

BOOK-KEEPING.

DEFINITIONS.

1. Book-Keeping is a systematic record of business transactions, or the art of keeping accounts.

Every person, engaged in business for himself, should keep a book of some kind in which to record all his business transactions. The mechanic, the farmer, the professional man, etc., should keep an account with every person with whom they deal. For no one should trust transactions of a pecuniary nature to his memory alone.

2. All business transactions consist in an exchange of values.

3. There are two methods of Book-keeping in general use, distinguished as *Single* and *Double* entry.

4. The **Double Entry** is conceded to be greatly superior to the Single Entry, particularly from its more excellent tests for determining the correctness of the work.

5. Single Entry embraces only the accounts of persons, and consists of *but one debit*, or *one credit*.

6. Double Entry is derived from the fact that every business transaction must be entered to two or more Ledger accounts, as two or more persons or things are affected thereby.

7. Two books appear indispensable in Single Entry; viz., the Day Book and Ledger.

8. The three main books used in Double Entry are the Day Book, Journal, and Ledger. The Day Book and Journal are sometimes combined in one.

9. The number and character of the auxiliary books depend somewhat on the nature and extent of the business, but more on the amount and kind of information desired. Those most in use are the Cash Book, Bill Book, Invoice Book, Sales Book, the Commission Sales Book, etc.

10. The **Day Book** is that in which are entered the business transactions in the date and order of their occurrence.

This book should be plain, concise, and unequivocal in its statements. As the records in it are supposed to be made when the transactions and all the circumstances connected therewith are fresh in the mind, it is the only book allowed in court, in cases of litigation.

11. The **Journal** is a book in which the business transactions recorded in the Day Book are prepared to be entered in the Ledger, by ascertaining the proper debits and credits involved in each transaction. This process is called *journalizing*.

12. The **Ledger** is the book of *results*,—the final book of entry.

In this book, under appropriate heads, called accounts, are arranged all the facts necessary for a full and satisfactory statement of the business; including, not only an exhibition of the present resources and liabilities, but a distinct record of particular gains and losses. The process of transferring to the Ledger is called *posting*.

13. The **Cash Book** is that which shows all the sums of money which we receive or pay, with a short explanation relating to each sum.

The entries, in this book, are made immediately on receiving or paying the sums. If properly kept, it will, at any time show the amount of cash on hand. It is customary, in most business houses, to close up the Cash Book at the end of each business day, and bring the balance down as a basis for the next day's transactions.

14. The **Bill Book** is that which shows a description of all the notes or acceptances in our favor or against us, with their dates, credits, when due, and amounts.

The notes or acceptances in our favor are entered under the head of *Receivable*, and those against us under *Payable*.

15. The **Invoice Book** is that in which are copied all bills of goods bought, and all invoices of goods received into our possession.

From the Invoice Book the entries pass into the Day Book, either daily, weekly, or monthly. This book is sometimes made of coarse paper, and the original invoices pasted into it.

16. The **Sales Book** gives a full description of all goods sold or passed from our hands, or out of our possession.

From this book the amounts are tranferred to the Day Book, either daily, weekly, or monthly. At the time the purchaser selects his goods, they are described in the Sales Book—quantity, quality, and price; and from this book we make out his bill.

17. The **Commission Sales Book** contains a minute description of the merchandise sold by us for others.

The entries in this book are drawn from the common Sales Book, and from it we make the *Accounts of Sales* that we may have to remit to those for whom we have sold.

18. An **Account** is a statement of facts pertaining to some person, species of property or cause, which enters into the transaction, producing a debit or credit, and designated by a name, which appears upon the Ledger.

19. Every account has two sides, a *Debtor* and a *Creditor*; each containing the results of separate transactions.

20. In every transaction the sum of debits and credits must be equal.

Each Ledger account, by the use of these terms, is made to show an important result of itself.

21. A **Resource** is any kind of value belonging to the concern.

22. A **Liability** is any debt owing by the concern.

23. **Cash** is the title to designate money.

The Cash account in the Ledger is debited with all receipts of cash, and credited with all disbursements. The difference between the two sides must, at any time, exhibit a resource of the exact amount of cash on hand. The credit side of Cash account cannot exceed the debit, as more cash cannot be paid out than has been received.

24. **Bills Receivable** are written obligations of whatever form, in our possession, for which a certain specified amount is to be *received*.

The Bills Receivable account is debited with notes received, and credited with those disposed of, or in any manner canceled. The excess, if any, must be on the debit side, and will indicate that portion of our resources consisting in notes.

25. **Bills Payable** are written obligations of the concern, for which a specified amount is to be *paid*.

Under this head are placed, on the credit side, our notes and acceptances issued, and on the debit side, such of them as have been redeemed. The difference, if there be any, must exhibit our outstanding notes, or our liability in unredeemed paper.

26. **Merchandise** is a term which usually implies all property purchased or owned by the concern for purposes of traffic, and *remaining in store*.

Merchandise generally embraces all such property, unless the merchant, being curious to know his gains or losses on a particular kind, opens a separate account with that particular kind, under its own special title. This account, or any of its correlative titles, is debited with the cost of the property represented, and credited with its returns.

27. **Real Estate** relates to such property as houses and lands, and the account is similar in its objects and teachings to that of Merchandise.

28. **Bank Stock, Railroad Stock,** etc., are not accounts dissimilar to Merchandise and Real Estate, inasmuch as stocks of all kinds are bought and sold at their *market* value, rather than the value written on their face.

29. **Shipment** or **Adventure** is but another name for Merchandise, and is used to distinguish between property *in* store and *out* of store.

When property is sent away to be sold by an agent for us, we should distinguish it from our merchandise in store by giving it a significant name, such as "Shipment to Halifax," or "Shipment to A," our agent, or "Adventure" to the place sent. All such accounts are debited with their entire cost, and credited with their proceeds, the difference being a gain or loss.

30. **Personal Accounts**, that is, accounts representing personal indebtedness, and designated by the proper names of such

persons as sustain relations of debtor and creditor to the concern, are capable of showing either resources or liabilities.

Personal Accounts are debited with such sums as, from time to time, the persons may become indebted to the concern, or the concern has paid them, and credited with what they have paid the concern, or the concern may have become indebted to them.

31. Stock, used as a Ledger title, means simply the proprietor of the business, or the *stock*-holder.

There would be no valid objection to using the proprietor's name instead; but as no real good would result from the change, authors, teachers, and practical accountants, accept the term which custom has suggested.

This account is usually the first opened in the Ledger, and is important to show the net investment. It is generally credited with the whole investment, and debited with such liabilities as the concern assumes to pay for the proprietor. The difference is the net investment, or what the concern owes the proprietor.

From the foregoing remarks, we derive the *seven* principles which follow, and we believe that every student who will thoroughly familiarize himself with them, will have no difficulty in deciding upon the proper debits and credits involved in any business record which he may be called upon to make.

PRINCIPLES.

1.—*The Proprietor.*

The person or persons investing in the business should be *credited*, under some title, for all such investments, and also for his or their share of the gain; on the other hand, he or they should be *debited* for all liabilities assumed by the concern for him or them, for all sums withdrawn from the business by him or them, and for such loss as he or they are entitled to share.

2.—*Cash.*

Cash account should be *debited* for all cash receipts, and *credited* for all cash disbursements.

3.—*Merchandise.*

Merchandise, and all species of property, bought upon speculation, should be *debited*, under some appropriate head, for the cost of the property represented, and *credited* with its proceeds.

4.—*Bills Receivable.*

Bills Receivable account should be *debited* with other people's notes, acceptances, and other written obligations when they become ours, and *credited* when they are paid, or otherwise disposed of.

5.—*Bills Payable.*

Bills Payable account should be *credited* with our notes, acceptances, or written promises to pay, when they are issued, and *debited* when they are paid or redeemed.

6.—*Persons.*

Personal accounts, such as the name of persons, banks, or other institutions, competent to sue or be sued, should be *debited* under their proper titles when they become indebted to us, or we get out of their debt, and *credited* when we become indebted to them, or they get out of our debt.

7.—*Expense, etc.*

All expenses, of whatever name, should be *debited* for the outlay, and all causes, of whatever kind producing us value, should be *credited* under some name, for the amount thus produced.

The foregoing principles are all embraced in the following simple

FORMULA.

Debit what *costs* the concern value; and *Credit* what *produces* the concern value.

BOOK-KEEPING

BY

DOUBLE ENTRY.

SET I.

(INITIATORY.)

DAY BOOK, JOURNAL, AND LEDGER.

REPRESENTING THE BUSINESS OF A SINGLE PROPRIETOR.

WITH EXPLANATIONS FOR JOURNALIZING, STATEMENTS, ETC.

INSTRUCTIONS FOR SET I.

The following set comprises the most simple transactions in business; the main purpose being to illustrate the foregoing principles, and to initiate the student more fully into the processes of Book-keeping. The general instructions given in connection with this set, will apply with equal force to the succeeding ones. They should, therefore, be properly heeded.

The transactions are first recorded historically in the Day Book, in the order of their occurrence; from thence transferred to the Journal, and from thence to the Ledger. In journalizing a transaction, the first thing to be considered is, the person or thing affected; next, in what manner affected; and lastly, the proper application of the principle. The check-mark (√) is made opposite the Day Book entry, *immediately* upon its being journalized.

Transferring to the Ledger is called *posting*. If the transactions are properly journalized, the labor of posting is simply mechanical. It requires great care, however, and constant watchfulness, and nothing is more common with new beginners than errors in posting.

Commence with the first account in the Journal, and write it as a heading in the Ledger. See if the amount opposite, be in the debit or credit Journal column, and enter it on the corresponding side in the Ledger, using as an expression the opposite journal entry. Suppose, for example, the journal entry to be "Merchandise Dr. To Cash." This expression implies, of course, that Merchandise is to be debited, and Cash credited. Under Merchandise account in the Ledger, on the *debit* side, we say, "To Cash," and carry the amount to the money column. Also, under Cash account on the *credit* side, we say, "By Merchandise," and carry the amount into the credit column. In posting from the Journal, be careful to enter in the column at the left of the account, the *page* of the Ledger to which the amount is posted, *immediately* upon its being entered in the Ledger.

DAY BOOK,—SET I.

QUEBEC, JANUARY 2, 1871.

✓	A. J. Hall invests in business this day Cash amounting to.....		$4500	
	"			
✓	He also owes R. Green on %, which is to be paid from the business.....		350	
	5			
✓	Bought of A. Lynch, for Cash,			
	217 yds. English Black Cloth, @ $5	$1085		
	140 yds. Canada Gray Cloth 4.20	588	1673	
	8			
✓	Sold B. Foster, for Cash,			
	109 yds. English Black Cloth, @ $6.30		686	70
			$7209	70

QUEBEC, JANUARY 12, 1871.

	Amount brought forward,	$7209	70
✓	Sold Jos. Murray, on %, 15 yds. Black Cloth, @ $6.50	97	50
	17		
✓	Bought of N. S. Power, on our note due February 22, next, 21 yds. Black Satin, @ $4.10	86	10
	20		
✓	Accepted R. Green's draft on us at 15 days' sight, favor of M. Duval,	350	
	26		
✓	Sold J. N. Benson, on his note payable Feb. 27, next, 56 yds. Canada Gray Cloth, @ $5.40	302	40
	31		
✓	Paid Cash as follows: For Painting of Store, $15.75 For Family Expenses, 42 60 For Rent of Store, one month, 30.00	88	35
	February 1		
✓	Received Cash of Jos. Murray, on %,	50	
	2		
✓	Bought of Myler & Lee, at 3 months, 432 yds. Irish Linen, @ $.45	194	40
	4		
✓	Lent Cash to D. Murphy, until 15th inst.,	120	
	6		
✓	Sold W. S. Reid, 84 yds. Gray Cloth, @ $5.50 Received in Payment, 3 Shawls, @ $60, $180 His Note at 40 days, for 200 Balance on %, 82—462	462	
	7		
✓	Paid our acceptance, favor of M. Duval, in Cash,	350	
		$9310	45

QUEBEC, FEBRUARY 10, 1871.

	Amount brought forward,	$9310	45
✓	Received Cash of Jos. Murray, in full of %,	47	50
	15		
✓	Received Cash of D. Murphy, in full for loan of 4th inst.,	120	
	18		
	Bought of C. Phelan, 86 yds. Silk Velvet, @ $7.50	645	
✓	Paid him, Cash, $150		
	Gave my Note @ 60 days, for 250		
	Balance on %, 245—645		
	22		
✓	Paid Cash for my Note in favor of N. Power, now due,	86	10
	25		
✓	Exchanged Notes with L. White for our mutual accommodation, each drawn at 40 days,	300	
	27		
✓	Received Cash for J. N. Benson's Note now due,	302	40
	"		
	Bought of L. A. Taylor,		
	500 yds. French Merino, @ $.75	375	
✓	Gave in Payment,		
	W. S. Reid's Note at 40 days, for $200		
	Cash, 140		
	Order on W. S. Reid, in full of %, 35—375		
	28		
✓	Paid Cash for Store Rent to date $30		
	" Family Expenses, etc., 48.50	78	50
		$11264	95

QUEBEC, JANUARY 2, 1871.

		Dr.		Cr.	
1	CASH *Dr.*	$4500			
1	To STOCK.			$4500	
	STOCK is the title chosen to represent the person investing; in this case, A. J. Hall. It is *credited* with the investment according to *Principle* 1. Cash is here *received* by the concern, and is made *Dr.*, according to *Principle* 2.				
	"				
1	STOCK *Dr.*	350			
3	To R. GREEN.			350	
	Stock is *debited* for the liability assumed by the concern, *Prin.* 1. R. Green is *credited* according to *Prin.* 6.				
	5				
2	MERCHANDISE *Dr.*	1613			
1	To CASH.			1673	
	Merchandise *cost* $1673, and is *debited*, *Prin.* 3.—Cash was *paid* for merchandise, and is *credited*, *Prin.* 2.				
	8				
1	CASH *Dr.*	686	70		
2	To MERCHANDISE.			686	70
	Cash is *debited* for its receipts, *Prin.* 2.—Merchandise is *credited* for its proceeds, *Prin.* 3.				
	12				
3	JOS. MURRAY *Dr.*	97	50		
2	To MERCHANDISE.			97	50
	Jos. Murray Dr., *Prin.* 6.—Merchandise Cr., *Prin.* 3.				
		$7307	20	$7307	20

QUEBEC, JANUARY 17, 1871.

		Dr.		Cr.	
	Amounts brought forward,	$7307	20	$7307	20
2	MERCHANDISE *Dr.*	86	10		
3	To BILLS PAYABLE.			86	10
	Merchandise Dr., *Prin.* 3.—Bills Payable Cr., *Prin.* 5.				
	20				
3	R. GREEN *Dr.*	350			
3	To BILLS PAYABLE.			350	
	R. Green is here made Dr. because we have *canceled* our indebtedness to him by promising to pay the amount to another person whom he has authorized to receive it, *Prin.* 6.—Bills Payable is *credited* for our new liability thus assumed, *Prin.* 5. The only change wrought in our affairs by this transaction is the transfer of a liability from a personal account to a note. We must now meet this obligation at its maturity, or be disgraced by having our paper protested.				
	26				
2	BILLS RECEIVABLE *Dr.*	302	40		
2	To MERCHANDISE.			302	40
	Bills Receivable Dr., *Prin.* 4. Merchandise Cr., *Prin.* 3.				
	31				
4	EXPENSE *Dr.*	88	35		
1	To CASH.			88	35
	Expense Dr., *Prin.* 7.—Cash Cr., *Prin.* 2.				
	February 1				
1	CASH *Dr.*	50			
3	To JOS. MURRAY.			50	
	Cash Dr., *Prin.* 2.—Jos. Murray Cr., *Prin.* 6.				
		$8184	05	$8184	05

QUEBEC, FEBRUARY 2, 1871.

		Dr.		Cr.	
	Amounts brought forward,	$8184	05	$8184	05
2	MERCHANDISE *Dr.*	194	40		
4	TO MYLER & LEE.			194	40
	Merchandise Dr., *Prin.* 3.—Myler & Lee Cr., *Prin.* 6.				
	4				
4	D. MURPHY *Dr.*	120			
1	TO CASH.			120	
	D. Murphy Dr., *Prin.* 6.—Cash Cr., *Prin.* 2.				
	6				
2	SUNDRIES *Dr.* TO MERCHANDISE.			462	
2	MERCHANDISE	180			
2	BILLS RECEIVABLE	200			
5	W. S. REID	82			
	Merchandise Dr., *Prin.* 3. Bills Receivable Dr., *Prin.* 4. W. S. Reid Dr., *Prin.* 6. Merchandise Cr., *Prin.* 3. The term SUNDRIES is here used for the first time. It means, simply, *Sundry Accounts*, and is convenient as a Journal expression, and to avoid the necessity of enumerating the items which comprise the totals carried to the Ledger accounts.				
	7				
3	BILLS PAYABLE *Dr.*	350			
1	TO CASH.			350	
	Bills Payable Dr., *Prin.* 5.—Cash Cr., *Prin.* 2.				
	10				
1	CASH *Dr.*	47	50		
3	TO JOS. MURRAY.			47	50
	Cash Dr., *Prin.* 2.—Jos. Murray Cr., *Prin.* 6.				
		$9357	95	$9357	95

	QUEBEC, FEBRUARY 15, 1871.	Dr.		Cr.	
	Amounts brought forward,	$9357	95	$9357	95
1	CASH *Dr.*	120			
4	To D. MURPHY.			120	
	Same *Prin.* as for the preceding entry.				
	18				
2	MERCHANDISE *Dr.* To SUNDRIES.	645			
1	To CASH.			150	
3	" BILLS PAYABLE.			250	
4	" C. PHELAN.			245	
	Merchandise Dr., *Prin.* 3.—Cash Cr., *Prin.* 2; Bills Payable, Cr., *Prin.* 5; C. Phelan Cr., *Prin.* 6.				
	22				
3	BILLS PAYABLE *Dr.*	86	10		
1	To CASH.			86	10
	Bills Pay. Dr., *Prin.* 5.—Cash Cr., *Prin.* 2.				
	25				
2	BILLS RECEIVABLE *Dr.*	300			
3	To BILLS PAYABLE.			300	
	B. Rec. Dr., *Prin.* 4.—B. Pay. Cr., *Prin.* 5.				
	27				
1	CASH *Dr.*	302	40		
2	To BILLS RECEIVABLE.			302	40
	Cash Dr., *Prin.* 2.—B. Rec. Cr., *Prin.* 4.				
	"				
2	MERCHANDISE *Dr.* To SUNDRIES.	375			
2	To BILLS RECEIVABLE.			200	
1	" CASH			140	
5	" W. S. REID.			35	
	Mdse. Dr., *Prin.* 3.—B. Rec. Cr., *Prin.* 4; Cash Cr., *Prin.* 2; W. S. Reid Cr., *Prin.* 6.				
	28				
4	EXPENSE *Dr.*	78	50		
1	To CASH.			78	50
	Expense Dr., *Prin.* 7.—Cash Cr., *Prin.* 2.				
		$11264	95	$11264	95

Dr. (*Liabilities assumed for the proprietor and accounts drawn out.*) Stock. (*Capital invested by the Proprietor.*) Cr.

1871						1871					
Jan.	2	To R. Green,	1	$ 350	00	Jan.	2	By Cash,	1	$4500	00
Feb.	28	" *Balance,*		4323	75	Feb.	28	" Loss and Gain,		173	75
				4673	75					4673	75
						March	1	By Balance,		$4323	75

Dr. (*Received.*) Cash. (*Paid out.*) Cr.

1871						1871					
Jan.	2	To Stock,	1	$4500	00	Jan.	5	By Mdse.,	1	$1673	00
"	8	" Mdse., 5186·70	1	686	70	"	31	" Expense, 1761·35	2	88	35
Feb.	1	" Jos. Murray,	2	50	00	Feb.	4	" D. Murphy,	3	120	00
"	10	" " "	3	47	50	"	7	" Bills Payable,	3	350	00
"	15	" D. Murphy,	4	120	00	"	18	" Mdse.,	4	150	00
"	27	" Bills Receivable,	4	302	40	"	22	" Bills Payable,	4	86	10
						"	27	" Mdse.,	4	140	00
						"	28	" Expense,	4	78	50
						"	"	" *Balance,*		3020	65
				5706	60					5706	60
Mar.	1	To Balance,		$3020	65						

Dr. (*What Mdse. has cost.*) **Merchandise.** (*Proceeds from Sales.*) *Cr.*

1871						1871					
Jan.	5	To Cash,	1	$1673	00	Jan.	8	By Cash,	1	$ 686	70
"	17	" Bills Payable,	2	86	10	"	12	" Jos. Murray,	1	97	50
Feb.	2	" Myler & Lee,	3	194	40	"	26	" Bills Receivable, 1086.60	2	302	40
"	6	" Mdse.,	4	180	00	Feb.	6	" Sundries,	3	462	00
"	18	" Sundries,	4	645	00	"	28	" *Balance*,		1945	50
"	27	" "	4	375	00						
"	28	" Loss & Gain,		340	60						
				3494	10					3494	10
Mar.	1	To Balance,		$1945	50						

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Dr. (*Others' notes received.*) **Bills Receivable.** (*Others' notes disposed of.*) *Cr.*

1871						1871					
Jan.	26	To Mdse.,	2	$ 302	40	Feb.	27	By Cash,	4	$ 302	40
Feb.	6	" "	3	200	00	"	"	" Mdse.,	4	200	00
"	25	" Bills Payable,	3	300	00	"	28	" *Balance*,		300	00
				802	40					802	40
Mar.	1	To Balance,		$ 300	00						

Bills Payable.

Dr. (*Our notes redeemed.*) (*Our notes issued.*) *Cr.*

1871						1871					
Feb.	7	To Cash,	3	$ 350	00	Jan.	17	By Mdse.,	2	$ 86	10
"	22	" "	4	86	10	"	20	" R. Green.	2	350	00
"	28	" *Balance,*		550	00	Feb.	18	" Mdse. 436.10	4	250	00
						"	25	" Bills Receivable,	4	300	00
				986	10					986	10
						Mar.	1	By Balance,		$ 550	00

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R. Green.

Dr. (*Our % against him.*) (*His % against us.*) *Cr.*

1871						1871					
Jan.	20	To Bills Payable,	2	$ 350	00	Jan.	2	By Stock,	1	$ 350	00

Jos. Murray.

Dr. (*Our % against him.*) (*His % against us.*) *Cr.*

1871						1871					
Jan.	12	To Mdse.,	1	$ 97	50	Feb.	1	By Cash,	2	$ 50	00
						"	10	" "	3	47	50
				97	50					97	50

Dr. (*Outlay.*) **Expense.** *Cr.*

1871						1871					
Jan.	31	To Cash,	2	$ 88	35	Feb.	28	By *Loss and Gain*,		$ 166	85
Feb.	28	" "	4	78	50						
				166	85					166	85

Dr. (*Our % against him.*) **Myler & Lee.** (*His % against us.*) *Cr.*

1871						1871					
Feb.	28	To *Balance*,		$ 194	40	Feb.	2	By Mdse.,	3	$ 194	40
						Mar.	1	By Balance,		$ 194	40

Dr. (*Our % against him.*) **D. Murphy.** (*His % against us.*) *Cr.*

1871						1871					
Feb.	4	To Cash,	3	$ 120	00	Feb.	15	By Cash,	4	$ 120	00

Dr. (*Our % against him.*) **C. Phelan.** (*His % against us.*) *Cr.*

1871						1871					
Feb.	28	To *Balance*,		$ 245	00	Feb.	18	By Mdse.,	4	$ 245	00
						Mar.	1	By Balance,		$ 245	00

Dr. (*Our % against him.*) **W. S. Reid.** (*His % against us.*) **Cr.**

1871							1871						
Feb.	6	To Mdse.,	3	$ 82	00		Feb.	27	By Mdse.,	4	$ 35	00
							"	28	" *Balance*,		47	00
				82	00						82	00
Mar.	1	To Balance,		$ 47	00							

Dr. **Loss and Gain.** **Cr.**

1871						1871					
Feb.	28	To Expense,	4	$ 166	85	Feb.	28	By Mdse.,	4	$ 340	60
"	"	" *Stock*,		173	75						
				340	60					340	60

18

Dr. (*Resources.*) **Balance.** (*Liabilities.*) **Cr.**

1871					1871				
Feb.	28	To Mdse.,	$1945	50	Feb.	28	By Bills Payable,	$ 550	00
"	"	" Cash,	3020	65	"	"	" Myler & Lee,	194	40
"	"	" Bills Receivable,	300	00	"	"	" C. Phelan,	245	00
"	"	" W. S. Reid,	47	00	"	"	" Stock,	4323	75
			5313	15				5313	15

A. J. HALL'S BALANCE SHEET OF FEBRUARY 28, 1871,—SET I.

The object of the Balance Sheet is to obtain a correct view of the true state and result of the business of which the books give the records. Accounts, however, are closed or balanced under other circumstances—upon a cessation of business, dissolution of partnership, and when the books are full and there is occasion for new ones.—The Balance Sheet exhibits, under two heads, the entire result of our mercantile operations. Under the heading of "*Balances of our Resources and Liabilities,*" are brought together all that owe us, and that we possess, and all that we owe or are liable for; under the heading of "*Balances of our Losses and Gains,*" are shown all the balances of loss or gain that may be found in the Ledger; and consequently the balance of one will be the net capital, of the other, the net gain. —The Balance Sheet is chiefly composed from the Ledger, and an inventory of the merchandise on hand belonging to us, which should be taken at the cost price, or less, if the goods are depreciated in value, but never at more than cost.

Dr. **Balances of his Resources and Liabilities.** *Cr.*

1	Cash,	balance on hand,	$3020	65	3	Bills Payable,	balance outstanding,	$ 550	00
2	Merchandise,	bal. on hand per Inv.	1945	50	4	Myler & Lee,	bal. in their favor,	194	40
2	Bills Receivable,	balance on hand,	300	00	4	C. Phelan,	bal. in his favor,	245	00
5	W. S. Reid,	balance in his favor,	47	00	1	Stock,	balance of it,	4323	75
			$5313	15				$5313	15

Dr. **Balances of his Losses and Gains.** *Cr.*

4	Expense,	loss,	$ 166	85	2	Merchandise,	gain on sales,	$ 340	60
	Stock, or A. J. Hall's	Net gain,	173	75					
			$ 340	60					

ANOTHER FORM OF BALANCE SHEET.

A. J. HALL'S BALANCE SHEET,—SET I. *

Taken Feb. 28, 1871.	*L. Folio.*	Trial Balance. *Dr.*	Trial Balance. *Cr.*	Inventory.	Representative. *Loss.*	Representative. *Gain.*	Stock. *Dr.*	Stock. *Cr.*	Real. *Resources*	Real. *Liabilit.*
Stock,	1	350 .	4500 .	.. .	.. .	.. .	.. .	4150 .	.. .	.. .
Cash,	1	5706 60	2685 95	.. .	.. .	.. .	.. .	.. .	3020 65	.. .
Merchandise,	2	3153 50	1548 60	1945 50	.. .	340 60	.. .	.. .	1945 50	.. .
Bills Receivable,	2	802 40	502 40	.. .	.. .	.. .	.. .	.. .	300 .	.. .
Bills Payable,	3	436 10	986 10	.. .	.. .	.. .	.. .	.. .	.. .	550 .
Expense,	4	166 85	.. .	.. .	166 85	.. .	.. .	.. .	.. .	.. .
Myler & Lee,	4	.. .	194 40	.. .	.. .	.. .	.. .	.. .	.. .	194 40
C. Phelan,	4	.. .	245 .	.. .	.. .	.. .	.. .	.. .	.. .	245 .
W. S. Reid,	5	82 .	35 .	.. .	.. .	.. .	.. .	.. .	47 .	.. .
		10697 45	10697 45							
To Stock—net gain,					173 75		.. .	173 75		
					340 60	340 60				
To Balance—net capital,							4323 75	.. .	.. .	4323 75
							4323 75	4323 75	5313 15	5313 15

* NOTE.—The process of showing results in this form of statement is precisely similar in its order, and the results the same, as in "closing the Ledger," and will need no further explanation.

PROCESS OF CLOSING.

It is supposed that the student has gone through with the process of journalizing and posting the transactions, and that his Ledger accounts present an equilibrium of debits and credits. To test this fact, before proceeding farther, we will take a

Dr. TRIAL BALANCE. *Cr.*

Differences.	*Face of Ledger.*		*Face of Ledger.*	*Differences.*
	350	Stock,	4500	4150
3020 65	5706 60	Cash,	2685 95	
1604 90	3153 50	Merchandise,	1548 60	
300	802 40	Bills Receivable,	502 40	
	436 10	Bills Payable,	986 10	550
	350	R. Green,	350	
	97 50	Jos. Murray,	97 50	
166 85	166 85	Expense,		
		Myler & Lee,	194 40	194 40
	120	D. Murphy,	120	
		C. Phelan,	245	245
47	82	W. S. Reid,	35	
5139 40	11264 95	*Equilibrium*,	11264 95	5139 40

It will be seen, in the above form of the Trial Balance, that the footings of the "Face of Ledger" columns exactly agree with the footings of the Day Book and Journal; which affords conclusive evidence that all the transactions have found their way to the Ledger. The columns of "Differences," which must also balance, will afford a convenient test of the results of each account.

Satisfied that the transactions have been properly posted, we now proceed to close the Ledger accounts. But let it not be forgotten that the object of closing the Ledger is to present, in a proper manner, both the *present condition* of our business and its *progress*. Its present condition can be shown by a list of its resources and liabilities; and its progress, by a list of its gains and losses.

By an examination of the facts, it will be seen that *Resources* are shown by an excess of the *Debit* side of REAL accounts, and LIABILITIES by an excess of the *Credit* side of REAL accounts; and that *Losses* are shown by an excess of the *Debit* side of REPRESENTATIVE accounts, and *Gains* by an excess of the *Credit*

side of REPRESENTATIVE accounts. This will suggest the propriety of opening two accounts for these general results: one to contain the resources and liabilities, and the other the gains and losses. We will now open these accounts under the titles of "Loss and Gain," and "Balance," the former to contain the results of the REPRESENTATIVE, and the latter of the REAL accounts. Before proceeding to close the accounts, we must ascertain if they are all in a condition to show the results desired. The Merchandise account, as it now stands, shows an excess of the debit side, and would therefore represent a *loss*, if the merchandise were all sold. The account itself does not show whether the property is all sold; and the only means of ascertaining the facts in the case, is to take an actual inventory, or a valuation of that which remains unsold. This we now proceed to do, with the following result:

INVENTORY.

Merchandise remaining unsold, February 28, 1871.

93 yards	English Black Cloth.	@ $5	$ 465	
21 "	Black Satin,	" 4.10	86	10
432 "	Irish Linen,	" .45	194	40
86 "	Silk Velvet,	" 7.50	645	
500 "	French Merino,	" .75	375	
3	Shawls,	" 60.00	180	
			$1945	50

Hence, we see that the unsold merchandise is worth $1945 50, which amount we enter on the credit side of Merchandise account in *red ink*, * and transfer the same immediately to Balance account. The accounts are now in a condition to close; and we will take them in their order. The first account (after Stock, which is the proprietor's own account) is Cash account. This account represents a resource consisting of cash on hand; the debit side showing the money received, and the credit side that disposed of. We *close* the account by entering the difference, *in red ink*, on the *credit* side, and footing up the sides, drawing double lines underneath. The red ink entry, or *balance*, is transferred immediately to the *debit* side of Balance account. The next account, Merchandise, shows a *gain*, and the balance is trans-

* An entry in *red ink* on the Ledger, denotes that the amount thus written *is to be transferred*, either to some other account, or to another position under the same account. Red ink entries are always transferred to the *opposite side* from where they first appear, for the reason that they indicate an excess of that side.

ferred to the *credit* side of Loss and Gain account. The next, Bills Receivable account, is closed in the same manner as Cash account, the balance being transferred as a resource to Balance account. Bills Payable account shows a *liability*, and the balance is transferred to the *credit* side of Balance account. R. Green's account already balances, and we close it by simply ruling the double lines. Jos. Murray's account balances, and we close it in the same manner as the preceding account. Expense account shows a *loss*, and is transferred to the *debit* side of Loss and Gain. Myler & Lee's account shows a *liability*, and the balance is transferred to the *credit* side of Balance account. D. Murphy's account balances, and we close it by simply ruling the double lines. C. Phelan's account shows a *liability*, and the balance is transferred to the *credit* side of Balance account. W. S. Reid's account shows a *resource*, and is transferred to the *debit* of Balance account.

We have now the *results* of all the accounts exhibited under the heads of Loss and Gain and Balance, and if the balances have been properly transferred, these accounts, together with the (unclosed) Stock account, must be in equilibrium. To test this, we next take a Trial Balance of these three accounts, which we call the

SECOND TRIAL BALANCE.

	Dr.		Cr.	
Stock,	$ 350		$4500	
Loss and Gain,	166	85	340	60
Balance,	5313	15	989	40
	$5830		$5830	

The balances being properly transferred, we proceed to accomplish the grand object of closing the Ledger, by carrying the net gain from the Loss and Gain to the *credit* side of Stock account, increasing the investment; but, had it been a loss, to the debit side, decreasing the investment. The Stock account now contains the capital invested increased by the gain, which must, of course, equal the *present worth*, as shown by the Balance account. We now close Stock account into Balance, which must produce an equilibrium of the Balance account; and complete, in that account, the record of resources and liabilities.

The object of closing the Ledger accounts is to restore the proprietor's account to the same relative position towards the business which it occupied at the commencement; viz., as showing the net investment, or net interest of the proprietor.

ORDER OF CLOSING.

The student will do well to observe particularly, and to follow out in practice, the following order of closing the Ledger.

First.—Open an account with "Loss and Gain," (if not already opened,) and another with "Balance"; the former to exhibit the *losses* and *gains*, and the latter the *resources* and *liabilities*.

Second.—Ascertain from the inventory if any property remains unsold; and, if so, credit each account for which such property was originally debited with the value of that unsold, making the entry in *red ink*, "By Balance," and transferring the amount directly to the debit side of Balance account, making this entry in *black ink*, "To Merchandise," or "To Real Estate," or any other account from which the amount is transferred. The Ledger accounts will each show, now, one of the four following results; viz., a Resource, a Liability, a Gain, or a Loss.

Third.—Omitting Stock account, (or Partner's accounts.) commence with the first account in the Ledger. First ascertain which of the above results it shows, and make the closing entry accordingly. If the difference represent a resource or a liability, enter upon the smaller side, *in red ink*, "*To*" or "*By* Balance," as the case may be, and transfer the amount *in black ink* to the opposite side of Balance account. If the difference represent a gain or loss, enter on the smaller side in red ink, "To" or "By Loss and Gain," and transfer the amount, in the same manner, to Loss and Gain account. Close all the accounts (except Stock or Partners',) and transfer the balances as directed. The Loss and Gain account will now show, on the debit side, all the losses, and on the credit side, all the gains, the difference being the net gain or net loss. The Balance account will show on the debit side all the resources, and on the credit side all the liabilities, (excepting the result of Stock or Partners' accounts,) the difference being the real interest or present investment of the proprietor or proprietors.

Fourth.—Take a "Second Trial Balance," or a Trial Balance of the remaining open accounts, Stock or Partners', Loss and Gain, and Balance. If the balances have been properly transferred, the debits and credits of these accounts, taken together, must be equal.

Fifth.—Close the Loss and Gain account into Stock, or, if it be a partnership business, into the partners' accounts, dividing the gain or loss according to agreement. The Stock or Partners' accounts will now show the original investment, increased by the gain, or decreased by the loss; the difference being the *present*

net investment. As the Balance account shows the same thing, they must, of course, agree.

Sixth.—Close Stock account (or Partners' accounts) into Balance account, which must equalize that account, it showing now, on one side, the total resources, and on the other, the total liabilities, and presenting, in the most condensed form, the exact present condition of the business.

PRACTICAL EXERCISES.

The following memoranda will comprise each a month's business, and the student is required to write up therefrom all the books represented in the preceding series. The form and arrangement of the books he will of course gather from the examples given; and he will find no point of difficulty which has not been fully discussed in connection with Set I. These exercises will require him to study well the form of expression in the Day Book, and the principles which govern the Journal, and will afford an excellent test of his proficiency in what he has passed over.

MEMORANDUM I.

January 2, 1871. I, L. N. Holmes, commenced the Dry goods business with the following Resources: 350 yards Elbeuf Cloth, at £1 3; 600 yds. Belgian Linen, at 2s. 6d.; 310 yds. American Cloth, at 10s.; 212 yards Cassimere, at 6s. 3d.; and £500 in cash.—**3.** Bought of J. Brown, for cash, 105 yds. Indian cotton, at 9d.—**4.** Sold L. Harris, for cash, 18 yds. Elbeuf Cloth at £1,7 6; 16½ yds. Belgian Linen, at 3s. 6d.—**5.** Sold L Newton on his note at 30 days, 150 yds. Elbeuf Cloth, at £1 15 7.—**6.** Bo't of H. Simon & Co., on my note due Feb. 15, 50 yds. gray cloth, at 12s.—**7.** Bo't of C. Dery, to be paid on the 18th instant, 30 yds. Cotton Velvet, at 4s. 3d.—**9.** Sold E. G. Irvine, on %, 36 yds. Indian Cotton, at 1s. 2½d.—**10.** Bo't of Bedard & Jordan, 120 yds. Merino, at 5s. 7½d.; paid them cash, £25: and the balance, at 60 days.—**11.** Bo't of Morgan & Co., on my note at 40 days, 28 yds. Silk velvet, at £2 6 3½.—**12.** Sold D. St. Just, 30 yds. Cotton Velvet, at 6s. 9d.; 40 yds. Merino, at 7s. 6½d. Received in payment, cash, £15 4 2, and his note at 30 days for the balance.—**13.** Bo't of C. Harkin, on %, 4 doz. Silk Handkerchiefs, at 3s. 6d.—**14.** Sold W. Dixon, 200 yds. American Cloth, at 12s. 9d. Received in payment, cash, £27 10, and the balance at 15 days.—**16.** Paid cash for Fuel, £2 2 6.—**17.** Sold B. Morency, at 60 days, 150 yds. Cassimere, at 8s. 3½d.—**18.** Sold to Sundry persons, for cash, 5 yds. Cassimere, at 9s.; 6 Silk Handkerchiefs, at 4s. 6d.; 80 yds. Belgian Linen, at 3s. 7d. On the same day, paid C. Dery, cash, for bal. of account now due, £6 7 6.—**19.** Accepted C. Harkin's Draft on me at 8 days, in favor of A. Sykes, for £8 8.—**20.** Received of E. G. Irvine, cash, on %, £1 5.—**21.** Bo't of S. McGill, on my note at 2 months, 108 yds. Dutch Linen, at 2s. 8d.—**23.** Sold F. Audibert 140 yds. Elbeuf Cloth, at £1 18 5. Rec'd in paym't his note at 15 da., for £150, and cash for the bal.—**24.** Taken from the

Store for my own use, 4 Silk handkerchiefs, at 3s. 6d.—**25**. Sold D. N. Patton, 28 yds. Silk Velvet, at £2 15. Received in payment, 100 yds. Blue Cloth, at 15s., and cash for the bal.—**26**. Received of B. Morency, 50 yds. Yellow Cotton, at 1s. 4d., on %.—**27**. Bo't of D. St. Just 30 yds. Sedan Cloth, at £1 2 6½. Gave in payment his note of the 12th instant, for £10; the balance on %.—**28**. Lent F. Audibert, cash, £12 10, until 10th of February next.—**30**. Paid cash for acceptance favor of A. Sykes, 19th inst.; on the same day, received cash of W. Dixon, in full of %.—**31**.—Paid cash for sundry expenses, £6 10 4.

Take the detailed Inventory of the Merchandise unsold on January 31st, *and quoted at the cost price, the amount of which is* £372 6 8.

Net Gain realized on January 31st,	£ 266	11 2
My Net Capital " "	1465	6 2

MEMORANDUM II.

February 1, I continue the same business with the following resources and liabilities, shown in Balance account of last month's Ledger; viz., RESOURCES: Merchandise, as per Inventory, $1489.33; Cash, $2982.36; Bills Receivable, $1667.50; E. G. Irvine's account, $3.70; B. Morency's do., $235.41; F. Audibert's do., $50; LIABILITIES: Notes outstanding, for $436.83; Bedard and Jordan's acct., $35; D. St. Just's do., $95.25.—**2**. Rec'd cash of F. Audibert, in full payment of the loan of last January 28th.—**3**. Gave D. St. Just, an order on B. Morency for $60, to be paid in cash.—**4**. Paid cash for Insurance in the Royal Insurance Co., on Merchandise amounting to $1400, at 1¼ % premium.—**6**. Sold Kelly & Shea, at 3 months, 20½ yds. Elbeuf Cloth, at $5.60; 217 yds. Belgian Linen, at 62½ cts.; 57 yds. Cassimere, at $1.70; 69 yds. Indian Cotton, at 22½ cts.—**7**. Rec'd cash of L. Newton, in full for his note of $1067.50, due this day.—**9**. Sold H. T. Perry, 65 yds. American Cloth, at $2.70; 24¼ yds. Gray Cloth, at $2.75; 20¾ yds. Merino, at $1.45; 31 yds. Yellow Cotton, at 32½ cts. Received in payment his note at 30 days, for $150, his order on F. Audibert, for $30, and cash for the balance.—**10**. Rec'd of F. Audibert, in payment for his note of January 25, last, amount'g to $600, and due this day; viz., 32 yds Silk Velvet, at $9.30, and cash for the balance.—**11**. Bought of A. Gibb, 218 yds White Flannel, at 87½ cts.; 195 yds. Red Flannel, at 92 cts. Gave in payment my Draft, at sight, on B. Morency, for $150; the bal. at 1 month.—**13**. Sold G. S. Convey on %, 71½ yds. Silk Velvet, at $10.70.—On the same day, sold to sundry persons, for cash, 8¾ yds. White Flannel, at $1.12; 37½ yds. Blue Cloth, at $4.20.—**14**. Rec'd for my portion in my aunt's bequest, $560.75, in cash, which I have deposited in the Union Bank.—**15**. Paid in cash my note in favor of H. Simon & Co., for $120, due this day.—**17**. Sold C. R. McGrath, at 8 days, 8 Silk Handkerchiefs, at 85 cts.—**18**. Bo't of A. Lane & Co. 215¾ yds. Black Watered Silk, at $4.36. Gave in payment H. T. Perry's note in my favor, for $150; my note, at 40 days, for $500;

and cash for the balance.—**20**. Accepted A. Gibb's Draft on me at 30 days, in fav. of G. Lafont, for $220.15, in full of his %.—**21**. Paid cash to Bedard & Jordan, in full of %.—**23**. Paid cash to my shoemaker, in full of his account, $10.80.—**24**. Sold for cash 3½ yds. Black watered Silk, at $5.20; 18 yds. Red Flannel, at $1.10; 25 yds. Sedan Cloth, at $5.30.—On the same day, Discounted my acceptance of 20th inst., favor of G. Lafont, due in 30 days from date. Paid for face of acceptance, less discount for 29 days at 6 %. in cash, $219.09.—**27**. Received $108.40, in cash, as the net proceeds of the sale at auction of the Merchandise saved from the fire that destroyed my Store on the 24th inst.—**28**. Rec'd from the Royal Insurance Co., in cash, $1400, amt. for which my Merchandise was insured.

BALANCE ACCOUNT.

RESOURCES.			LIABILITIES.		
Cash,	$5657	75	Bills Payable,	$ 816	83
E. G. Irvine,	3	70	D. St. Just,	35	25
B. Morency,	25	41	Stock,	5875	43
F. Audibert,	30				
Kelly & Shea,	362	85			
G. S. Convey,	80	25			
Union Bank,	560	75			
C. R. McGrath,	6	80			
	$6727	51		$6727	51

MEMORANDUM III.

March 1, I, A. J. Hall, have this day joined Produce and Grocery, to my Dry Good business. My Resources and Liabilities are as follows: Cash on hand, £755 3 3; Notes on hand, £75; Merchandise, as per Inventory, £486 7 6; W. S. Reid owes me on %, £11 15. Notes outstanding amt'g to £137 10; I owe Myler & Lee, £48 12, and C. Phelan, £61 5.—On the same day, Sold P. Lewis, for cash, 40 yds. Irish Linen, at 3s.; 18 yds. French Merino, at 4s. 6d.; 5½ yds. Silk Velvet, at £2 2 6.—**2**. Bought of F. Morin, for cash. 8 bbls. Extra Superior Flour, at £1 10; 3 bbls. Extra Flour, at £1 4 6; 7 bbls. Superfine Flour (Canada wheat), at £1 3.—**3**. Bought of H. Lawlor & Son on my note due May 3rd next, 2 bbls. Oatmeal, at £1 10; 80 bush. Peas, at 4s. 6d.; 30 bush. Barley, at 5s.; 108 lbs. Butter, at 8½d.—On the same day, Sold J. B. Davis, on %, 12 yds. English Black Cloth, at £1 16 8; 3½ yds. Black Satin, at £1 3 6; 25 yds. Irish Linen, at 2s. 9d.—**5**. Sold S. Reeve on his note due April 18, 40 bush. Peas, at 5s. 6d.; 1 bbl. Extra Superior Flour, at £1 18; 2 bbls. Superfine Flour, at £1 6 2.—**6**. Bo't of L. Crawford & Co., at 15 days, 2 dozen Felt hats, at 4s. 2½d.; 1½ doz. Black Caps, at 7s. 3d.; 130½ yds. White Flannel, at 3s. 6d.—**7**. Sold C.

Maurice for Cash, 1 bbl. Extra Superior Flour, £1 19.—On the same day, Accepted Myler & Lee's Draft on me at 10 days, in favor of T. Lebel, for £37 10.—**8**. Rec'd of J. B. Davis, cash, on %, £12 10. —On the same day, Bo't for cash of Smith & O'Neil, 50 lbs. coffee, at 1s. 2d.; 20 lbs. Tea, at 2s. 6d.; 60 lbs. Brown Sugar, at 5½d.; 15 lbs. Chocolate, at 1s. 5d.; 24 lbs. Cheese, at 9d.—**9**. Sold W. Rolland, at one month, 15 yds. Irish Linen, at 3s.; 30 yds. Red Flannel, at 4s. 4d.—**10**. Bo't of A. Hamel & Co. 54½ yds. Alpaca, at 2s. 1d.; 113¼ yds. French Merino, at 5s. 3d.; 6 Carpet-bags, at 5s. 6d.; 3 doz. handkerchiefs, at 10s. 5d. Gave in payment my note at 90 days, for £20 10; the bal. on %.—**12**. Rec'd of W. S. Reid, in cash, £11 15, in full of %.—**13**. Paid £3 8 9 in cash for the purchase, cartage, etc., of 2½ cords of fire-wood.—**14**. Sold B. Jones on his note at 2 months, 70 yds. Red Flannel, at 4s. 7d.; 15 yds. English Black Cloth, at £1 16 9⅓: 28 yds. White Flannel, at 4s. 2½d.—**15**. Sold for cash, 16 lbs. Butter, at 1s.; 5 bu. Barley, at 6s. 7½d.—**16**. Rec'd of J. B. Davis a check on C. Howard, for £17 1, payable to the bearer, which was paid me this day, in cash, in full of %.—**17**. Sold G. Lemay, 2 bbls. Superfine Flour, at £1 7; 2 bbls. Extra Superior Flour, at £1 17 8; 1 barrel Oatmeal, at £1 11; 40 lbs. Butter, at 10½d. Rec'd in payment, cash, £2 10 6; the balance on acct.—**19**. Paid L. Crawford & Co. cash, in full of %.—**20**. Paid cash for acceptance favor of T. Lebel, 7th inst.—**21**. Paid cash for a horse and harness, £43 5.—**22**. Bought of F. R. Meredith, 61¼ yds. Cassimere, at 9s. 3d. Paid in cash, £18; the bal. at 20 days.—**23**. Bo't of Myler & Lee, 78½ yds. Woolen Carpet, at 2s. 7d.; 85 yds. Printed Calico, at 10½d.; 18 pair Cotton Gloves, at 1s. 1½d.; 15 yds. Welsh Flannel, at 2s. 6d.; 6 Silk Umbrellas, at 18s. 9d. Gave in payment, cash, £6 4 6; an order on G. Lemay for £5; the bal. on %.—**24**. Sold J. Bell, on %, 1 bbl. Extra Flour, £1 7; 5 bush. peas, at 6s. 2d.; 3 Felt Hats, at 5s. 6d.—On the same day, Sold to sundry persons for cash, 5 yds. French Merino, at 7s. 4½d.; 1 carpet-bag, 8s. 4d.; 2 Black Caps, at 9s. 3d.—**26**. Sold B. Nolan 40 yds. French Merino, at 6s. 8d. Rec'd in payment 5 bbls. Apples, at £1 2; and cash for the balance.—**27**. Sold S. A. Hunt, 15 yds. Alpaca, at 3s. 4d.; 1 doz. Handkerchiefs, 12s. 5d.; 25 lbs. Coffee, at 1s. 10d.; 10 lbs. Tea, at 3s. 5d. Rec'd in payment his note at 40 days, for £3 11 2½; and cash for the bal. —On the same day, Paid cash, 3s. 6d., for cartage of the above sale. —**28**. Rec'd of Gauthier & Barry, Montreal, as per their Bill of Invoice of the 26th inst.; viz., 5 bbls. Rye Meal, at £1 7; 50 bushels Indian Corn, at 4s.; 60 bu. Oats, at 3s. 9d., which I paid, pursuant to their order, to J. Rogers, their agent, as follows: S. Reeve's note due April 17, for £15 10 4; and cash for the balance.—On the same day, Paid cash for freight and other expenses of the above Invoice, £1 3 7.—**29**. Paid cash for 1 pair of pants and 1 overcoat for my own use, £5 10.—**30**. Taken from the Store for Family expenses during the month; viz., 8 lbs. Butter, at 8½d.; 5 lbs. Coffee, at 1s. 2d.; 3 lbs. Tea, at 2s. 6d.—On the same day, lent A Smith, in cash, £10 12 6, previous to balancing my accounts, and whose entry I had omitted.—**31**. Paid cash for sundry expenses during the month; viz., for Rent of Store, £6 10; for Family expenses, etc., £7 5 6.

Take the detailed Inventory of the Merchandise unsold on March 31st, *and quoted at the cost price, the amount of which is* £578 16 11.

Net Loss on March 31	£ 30 5 2½
My net capital " "	1061 6 0½

MEMORANDUM IV.

April 1, I continue my business with the following Resources and Liabilities taken from my Ledger of the preceding month, minus a few cents; viz., Cash on hand, $2525.17; Merchandise as per Inventory, $2315.38; Notes on hand, $512.30; W. Rolland owes me $35; G. Lemay do., $8.96; J. Bell do., $14.86; A. Smith do., $42.50. Notes outstanding amtg. to $761.30. I owe as follows: Myler & Lee $88.90; C. Phelan $245; A. Hamel & Co. $72.47; F. R. Meredith $41.31.—**2**. Bo't on my note, at 60 days, of J. Dawson, 5 bbls. Herrings, at $7; 3 bbls. Codfish, at $7.40; 3 bbls. Mackerel, at $7.20.—**3**. Bo't for cash 20 Shares Montreal Bank Stock, at $104.—**4**. Rec'd cash in full of % of G. Lemay, $8.96.—**5**. Sold to sundry persons for cash, 5 lbs. Brown Sugar, at 12 cts.; 10 lbs. Chocolate, at 44 cts.; 24 lbs. Cheese, at 19 cts.—**6**. Sold P. S. Roberts 30 yds. Irish Linen, at 58½ cts.; 12 yds. Black Satin, at $4.85; 20 yds. Cassimere, at $2.50. Rec'd in payment, cash, $73, and bal. at 15 days.—**7**. Sold B. Nolan, 39¼ yds. Woolen Carpet, at 76 cts.; 40 yds. French Merino, at 83½ cts.; 37⅞ yds. White Flannel, at 90 cts.; 2 doz. Handkerchiefs, at $3.15. Rec'd in payment, his note at 40 days, for $50; his Order on Myler & Lee, for $26.45; the balance on acct.—**9**. Lent G. S. Convey on his note payable on the 25th inst., and without interest, $60. —On the same day, Sold P. Maguire, for cash, 6 Shares Montreal Bank Stock, at $109.—On the same day, Rec'd cash from W. Rolland, in full of acct.—**10**. Sold J. M. Lindsay, Richmond. 2 bbls. Extra Superior Flour, at $12.45; 2 bbls. Extra Flour, at $11; 1 bbl. Superfine Flour, $6.40; 1 bbl. Oatmeal, $8.32; 8 bu. Barley, at $1.43¾; 40 lbs. Brown Sugar, at 11¼ cts.; 4 bbls. Apples, at $7.68. Rec'd in payment, 100 bu. potatoes, at 41¾ cts., and his note at 60 days for the balance.—On the same day, Paid the Grand Trunk for freight of the above Invoice, in cash, $2.55.—**11**. Paid cash to F. R. Meredith, in full for his sale of March 22.—**12**. Sold for cash, 6¼ yds. English Black Cloth, at $7.72; 3½ yds. Black Satin, at $6.48; 35 yds. Silk Velvet, at $8.65; 5 Felt Hats, at $1.20.—**13**. Deposited $800, cash, in the Union Bank.—**14**. Rec'd of P. S. Roberts his note at 90 days, dated the 6th inst., in full of %.—**16**. Sold H. Collins, at 3 months, 45 yds. Printed Calico, at 25 cts.; 9 pair Cotton Gloves, at 46⅔ cts.; 12 yds. Welsh Flannel, at 72½ cts.; 5 Silk Umbrellas, at $4.55,—**17**. Accepted Myler & Lee's note on me at 8 days in favor of C. Maynard in full of %.—**18**. Sold N. Graham, 4 bbls. Rye Meal, at $8.80; 45 bu. Indian Corn, at $1.10; 30 bu. Oats, at 92½ cts. Rec'd in payment, cash for half of the sale, less 2 % disc., $55.10; the other half, that is, $56.22, remains due.—**19** Gave my shoemaker a check for

$17.28 on the Union Bank, in payment of his account.—**20**. Rec'd $84 in cash for 6 % dividend on 14 Shares Montreal Bank Stock. —**21**. Rec'd of J. Beaudry, Sorel, 150 bu. Oats, at 60 cts.; 300 bu. Rye Meal, at 90 cts., which I sold immediately with $80 profit, to E. Stephens. Rec'd in payment of the latter, a note at 20 days, for $200; cash, $120; the balance on %.—**23**. Bo't for cash 10 bbls. Extra Flour, at $4.80; 15 bbls. Fancy Flour, at $4.70; 8 bbls. Superfine Flour, at $4.60; 8 bbls. Oatmeal, at $6.—**24**. Lent cash to P. Fremont on his note at 40 days, and without interest, endorsed by A. Sauran, $65.—**25**. Rec'd cash of G. S. Convey in payment for his note of the 9th inst., due this day.—**26**. Sold C. A. Simpson, 8 Shares Montreal Bank Stock, at $112. Rec'd in payment, 128 yds. Elbeuf Cloth, at $6; and cash which I deposited in the Union Bank.—**27**. Sold N. O. Day on his note at 2 months, 35 bu. Peas, at $1.13; 17 bu. Barley, at $1.29; 20 lbs. Coffee, at 30 cts.; 4 bbls. Herrings, at $7.85.—**28**. Paid cash for Myler & Lee's Draft, in favor of C. Maynard, $62.45.—On the same day, Gave the carpenter an order on N. Graham for $5.10, for repairs of Store Fixtures.—**30**. Sold J. S. O'Brien, 200 yds. Irish Linen, at 90 cts.; 40 yds. Silk Velvet, at $9.20; 12 Felt Hats, at $1.98¾; 12 Black caps, at $1.90. Rec'd in payment, Neil & Roche's note, at 40 days, for $240; cash, $203; discount allowed for the payment in cash, $3.80; the bal. on %.— On the same day, Paid cash for sundry expenses; viz., Taxes and Gas, $5.63, Family expenses, $24.35; Rent of Store, $26.

Take the detailed Inventory of the Merchandise unsold April 30th, *and quoted at the cost price.*

The Merchandise amounts to	$2232.95.
The Shares of the Montreal Bank Stock, to	624.00.

BALANCE ACCOUNT.

RESOURCES.			LIABILITIES.		
Cash,	$ 835	26	Bills Payable,	$ 840	10
Merchandise,	2232	95	C. Phelan,	245	00
Bills Receivable,	1285	45	A. Hamel & Co.,	72	47
J. Bell,	14	86	J. Beaudry,	360	00
A. Smith,	42	50	Stock,	4821	02
Montreal Bank Stock,	624	00			
B. Nolan,	26	98			
Union Bank,	910	72			
H. Collins,	46	90			
N. Graham,	51	12			
E. Stephens,	120	00			
J. S. O'Brien,	147	85			
	$6338	59		$6338	59

SET II.

DAY BOOK, JOURNAL, CASH BOOK, BILL BOOK, COMMISSION SALES BOOK, ACCOUNT SALES.

DAY BOOK,—SET II.

QUEBEC, APRIL 1st, 1871.

J. Byrne and F. O'Reilly have this day entered into copartnership, under the style and firm of BYRNE & O'REILLY, to conduct a Produce, Grocery, Domestic Shipping business, and for buying and selling Real Estate, Steamboat Stocks, etc.: — the parties to invest equal amounts of net capital, and to share alike in gains and losses.			
J. Byrne's Resources are as follows:			
Cash on deposit in the National Bank,	$16000		
One-half of the Tow-boat Nestor, valued at	7800		
One-fourth of the Tow-boat Levis, " "	5400	29200	00
His Liabilities are:			
A Note, favor of Barclay & Co., due April 14th, which the Firm assume,	$8750		
J. L. Fraser, amt. due him,	250	9000	00
Making his Net Capital $20200.			

QUEBEC, APRIL 1st, 1871.

F. O'Reilly's Resources are:

Cash on hand,	$19600		
A Note in his favor, drawn by L. Clint, due May 25th,	4230.60	23830	60
His Liabilities, which the Firm assume, are:			
Young & Talbot, amt. due them	$1068		
R. Fisher & Son, " " "	2562.60	3630	60
Making his Net Capital $20200.			

1

Deposited Cash in the National Bank,		18000	00

2

Bought of C. Ross & Bro., Store and Fixtures, at		6000	00
Paid them, Check on National B'k, $2000			
Bond and Mortgage for balance, 4000—$6000			

3

Sold ¼ of the Tow boat Nestor for cash deposited in the National Bank,		4200	00

4

Bought of L. R. O'Connor & Co.,				
150 bbls. Superfine Flour,	@	$ 5.40	$810	
120 " Extra Mess Pork,	"	10.00	1200	
720 " Mess Beef,	"	12.00	8640	
125 " Prime Beef,	"	9.00	1125	
60 " Beef Hams,	"	16.50	990	
50 " Pearl Ashes,	"	5.50	275	
8 hhds. Sugar, 8800 lbs.,	"	.07½	660	13700 00
Gave in payment,				
Our two Notes—1 @ 40 da., for		$2000		
1 @ 3 mo., for		4000		
Our Check on the National Bank, for		3700		
Balance on %,		4000	—$13700	

5

Drew Cash from National Bank, per Check,	400	00

QUEBEC, APRIL 5, 1871.

Paid cash for ¼ Repairs of Tow-boat Levis,	90	00
6		
Shipped per Steamboat St. Alban, and consigned to McLean & Co., Montreal, to be sold on our % and risk,		
200 bbls. Mess Beef, @ $12 2400		
30 " Prime " " 9 270		
4 hhds. Sugar, 4400 lbs., " $.07½ 330—3000		
Paid drayage on same in cash, 4	3004	00
8		
Bo't of E. S. Pierce, a House, 24 St. Louis street, for	7000	00
Gave in payment,		
⅛ of Tow-boat Nestor, for $2200		
Check on the National Bank, for 1500		
Bond, secured by Mortgage, payable in 6 months, for 3300—7000		
9		
Received per Grand Trunk R. R., from L. Shaw & Co., Toronto, to be sold on their % and risk,		
600 bush. Wheat, invoiced @ $1.40		
800 " Corn, " " .65		
4200 lbs. Butter, " " .14		
Paid transportation charges, in cash,	95	00
10		
Purchased by check on National Bank, of Lewis & Wright, of this city, their Bill of Exchange at sight on Watson & Co., Montreal, and remitted the same this day to Young & Talbot, in full of their %, $1068		
Paid in cash, ¼ % Prem. for the Bill, 2.67	1070	67
11		
Sold H. W. Cooper, on his note @ 60 days,		
600 bush. Wheat, (L. S. & Co.'s Consignment) @ $1.70,	1020	00

QUEBEC, APRIL 12, 1871.

Received intelligence that the Tow-boat Levis sunk yesterday in the St. Lawrence river, near Green Island, and has been delivered over to the Underwriters.		
The boat being insured for $21500, we have received in Cash, (which we have deposited in the National Bank,) from the Quebec Ins. Co., our ¼ of same, $5375, less Expenses $110, = $5265		
Lost the Bal. of our Share of the cost of said Boat, ($5400 + $90 − $5265) = 225	5490	00
13		
Rec'd per Steamer Anna, from F. J. Ray, Halifax, N. S., to be sold on his % and risk,		
400 bbls. Codfish, invoiced @ $4.50		
600 " Mackerel, " " 6.50		
500 " Herrings, " " 5.00		
Paid Freight and Insurance, in cash,	150	00
15		
Bought of A. Stars & Co., per Check on National Bank,		
30 Shares National Bank Stock, @ $48 per S.	1440	00
16		
Sold B.W. Hardy, for cash, from L. S. & Co.'s Consignment,		
4200 lbs. Butter, @ $.16 $672		
800 bush. Corn, " .80 640	1312	00
"		
Closed L. Shaw & Co.'s Consignment, and rendered them an Account Sales of the same.		
Our charges for Storage and Adver., $ 20.00		
Our Commission on Sales, 64.13		
L. Shaw & Co.'s net proceeds, 2152.87	2237	00
17		
Bought, at Auction, ⅕ of Steamboat Sorel, for	6400	00

Gave in payment,		
The Note of L. Clint, favor of F. O'Reilly and invested by him on commencing Business, due 25th proximo, $4230.60		
Our check on National B'k for 1500.00		
Cash for bal. including disct. on Note, 696.19—6426.79		
The disct. on Note, $4230.60 for 38 days, is	26	79
"		
Gave our Note, @ 40 days, to the Quebec Ins. Co., for Ins. on our Share of the Steamboat Sorel, for $6500, @ 2 % = $130, and Policy $1.	131	00
18		
Shipped per Steamboat Alfred, and consigned to Price & Co., Kingston, Ontario, to be sold on our % and risk,		
50 bbls. Pearl Ashes, from Store, valued @ $7 $ 350		
4 hhds. 4400 lbs. Sugar, " " @ $.09 396		
600 bbls. Mackerel, (F. I. R's Consignment) @ $7.50 4500		
Paid cash for Ins.—Premium and Policy, 20	5266	00
19		
The Steamboat St. Alban, on which we shipped goods to McLean & Co., Montreal, got on fire at her arrival in port, and our goods, which were rescued in a damaged condition, and upon which there was no insurance, were sold at auction for cash,	2500	00
"		
Shipped, per Steamboat Glory, to W. S. Kelly, Three Rivers, pursuant to his order, and for his %,		
50 bbls. Superfine Flour, @ $ 6.12 366		
50 " Extra Mess Pork, " 11.00 550—916		
Paid drayage on same in cash, 4	920	00

QUEBEC, APRIL 20, 1871.

Sold E. G. Henry, for cash, 400 bbls. Codfish, (F. I. R's Con.) @ $5 2000 500 " Herrings " " " 6 3000	5000	00
"		
Closed F. I. Ray's Consignment, and rendered him an Account Sales of the same. Our charges for Storage and Advertising, 50.00 Our Commission on sales, 237.50 F. I. R. net proceeds, remitted in cash, 9062.50	9350	00
22		
Sold to J. L. Fraser, 25 Shares National Bank Stock, at @ $52	1300	00
Received payment as follows: Canceling for our indebtedness to him, $250.00 Interest on same allowed by us, 1.50 Cash for the balance, 1048.50—1300		
23		
Received from G. Doyle & Son, Ottawa, to be sold on their % and risk, 1000 bush. Wheat, 800 " Oats, 200 bbls. Tallow,		
Paid Freight in cash,	100	00
24		
Sold our House, No. 24 St. Louis street, to R. Fisher & Son, for	8000	00
Offset, as part payment, the am't which we owe them on %, $2562.60 Rec'd their Note at 18 months, secured by Mortgage on Property, for 5000.00 And Cash, for the balance, 437.40—8000		
25		
Sold E. F. Andrews, at 40 days, on %, 200 bbls. Tallow, (G. D. & Son's Cons.) @ $8,	1600	00

QUEBEC, APRIL 26, 1871.

Paid our Note, favor of Barclay & Co., due April 14, by Check on National Bank, $8750.00 Interest due on same to date, 8.75	8758	75
27		
Paid cash for our Note of the 4th inst., at 40 days, favor of L. R. O'Connor & Co., Face of Note, Cash paid, 1993.34 Discount off to May 17, 6.66—2000	2000	00
29		
Received advice from Price & Co., Kingston, of the sale of 50 bbls. Pearl Ashes; 4400 lbs. Sugar; and 600 bbls. Mackerel, shipped them on the 18th inst. Net proceeds remitted in cash,	5100	00
30		
Sold O. S. Dion, for cash, 1000 bush. Wheat, (G. D. & Son's Consignment) @ $1.40	1400	00
"		
Closed G. Doyle & Son's Consignment, and rendered them an Account Sales—800 bush. Oats remaining unsold, Our charges for Storage and Adver., $ 30 Our Commission on Sales, .75 G. Doyle & Son's net proceeds 2795	2900	00
"		
F. O'Reilly has drawn cash for private use,	200	00
"		
Paid sundry expenses this month, in cash,	150	00

QUEBEC, APRIL 1st, 1871.	Dr.		Cr.	
SUNDRIES *Dr.* To J. BYRNE.			$29200	00
NATIONAL BANK	$16000	00		
TOW-BOAT NESTOR STOCK	7800	00		
TOW-BOAT LEVIS STOCK	5400	00		
"				
J. BYRNE *Dr.* To SUNDRIES.	9000	00		
" B. PAYABLE.			8750	00
" J. L. FRASER.			250	00
"				
SUNDRIES *Dr.* To F. O'REILLY.			23830	60
CASH	19600	00		
BILLS RECEIVABLE	4230	60		
"				
F. O'REILLY *Dr.* To SUNDRIES.	3630	60		
" YOUNG & TALBOT.			1068	00
" R. FISHER & SON.			2562	60
"				
NATIONAL BANK *Dr.*	18000	00		
To CASH.			18000	00
2				
REAL ESTATE *Dr.* To SUNDRIES.	6000	00		
" NATIONAL BANK.			2000	00
" MORTGAGE PAY.*			4000	00

* The term "Mortgage Payable" is but another name for Bills Payable: the accounts may be kept separate or together. There is a distinction between a promissory note and a mortgage on real estate; and the majority of business men would prefer to have that distinction preserved in their accounts.

QUEBEC, APRIL 3, 1871.	*Dr.*	*Cr.*
NATIONAL BANK *Dr.*	$4200 00	
To TOW-BOAT NESTOR STOCK.		$4200 00
4		
MERCHANDISE *Dr.* To SUNDRIES.	13700 00	
To BILLS PAYABLE.		6000 00
" NATIONAL BANK.		3700 00
" L. R. O'CONNOR & Co.		4000 00
5		
CASH *Dr.*	400 00	
To NATIONAL BANK.		400 00
"		
TOW-BOAT LEVIS STOCK *Dr.*	90 00	
To CASH.		90 00
6		
SHIPMENT TO MONTREAL * *Dr.*	3004 00	
To SUNDRIES.		
" MERCHANDISE.		3000 00
" CASH.		4 00
8		
REAL ESTATE *Dr.* To SUNDRIES.	7000 00	
To TOW-BOAT NESTOR STOCK.		2200 00
" NATIONAL BANK.		1500 00
" MORTGAGE PAYABLE.		3300 00

* "Shipment to Montreal" is a new account, opened to represent a particular enterprise, and although it relates to merchandise, it is distinct from the merchandise *in Store*, and is given this new name to mark that distinction. It is as though we had sold our merchandise for $3000, and immediately invested the same in this adventure. The shipment is debited with its cost, and merchandise and cash credited.

QUEBEC, APRIL 9, 1871.	*Dr.*	*Cr.*
L. SHAW & CO.'S CONS. * *Dr.*	$ 95 00	
To CASH.		$ 95 00
10		
SUNDRIES *Dr.* To SUNDRIES.		
YOUNG & TALBOT	1068 00	
PREMIUM	2 67	
To NATIONAL BANK.		1068 00
" CASH.		2 67
11		
BILLS RECEIVABLE *Dr.*	1020 00	
To L. SHAW & CO.'S CONSIGNMENT.		1020 00
12		
SUNDRIES *Dr.*		
To TOW-BOAT LEVIS STOCK.		5490 00
NATIONAL BANK	5265 00	
LOSS AND GAIN	225 00	
13		
F. I. RAY'S CONSIGNMENT *Dr.*	150 00	
To CASH.		150 00
15		
NATIONAL BANK STOCK *Dr.*	1440 00	
To NATIONAL BANK.		1440 00

* The account here opened—S. Shaw & Co.'s Consignment—is precisely the same, in effect, as would be an account with L. Shaw & Co., although it really represents the *property* of that firm, which we receive, as commission merchants, to sell. Instead, therefore, of debiting the Consignment account with the value of the property, we debit it only with what it has cost us.

QUEBEC, APRIL 16, 1871.	*Dr.*	*Cr.*
CASH *Dr.*	$1312 00	
To L. SHAW & Co.'s CONS,		$1312 00
"		
L. SHAW & Co.'s CONSIGN. *Dr.*	2237 00	
To SUNDRIES.		
" STORAGE & ADVERTISING.		20 00
" COMMISSION.		64 13
" L. SHAW & Co. *		2152 87
17		
SUNDRIES *Dr.* To SUNDRIES.		
STEAMBOAT SOREL STOCK	6400 00	
DISCOUNT	26 79	
To BILLS RECEIVABLE.		4230 60
" NATIONAL BANK.		1500 00
" CASH.		696 19
"		
STEAMBOAT SOREL STOCK *Dr.*	131 00	
To BILLS PAYABLE.		131 00
18		
SHIPMENT TO KINGSTON *Dr.*	5266 00	
To SUNDRIES.		
" MDSE.		746 00
" F. I. RAY'S CONSIGN.		4500 00
" CASH.		20 00

* This entry is made for the purpose of exhibiting on our books the net amt. owing to L. Shaw & Co., as the result of our business with them so far; and as their Consignment account was used to show the facts connected with the sale of their property, we can ascertain from this account how much they are entitled to, as net proceeds, which must be the difference between the sides of that account, when its entire cost and proceeds are properly shown; in this case, $21 2.87. The effect of this entry will be to close the Consignment account, and carry its results to the account of L. Shaw & Co.

QUEBEC, APRIL 19, 1871.	*Dr.*	*Cr.*
CASH *Dr.*	$2500 00	
To SHIPMENT TO MONTREAL (1)		$2500 00
"		
W. S. KELLY (2) *Dr.* To SUNDRIES.	920 00	
" MDSE.		916 00
" CASH.		4 00
20		
CASH *Dr.*	5000 00	
To F I. RAY'S CONSIGNMENT.		5000 00
"		
F. I. RAY'S CONSIGNMENT *Dr.*	9350 00	
To SUNDRIES.		
" STORAGE & ADVERTISING		50 00
" COMMISSION.		237 50
" CASH.		9062 50
22		
SUNDRIES *Dr.*		
To NATIONAL BANK STOCK.		1300 00
J. L. FRASER	250 00	
INTEREST	1 50	
CASH	1048 50	

(1) Shipment to Montreal is treated precisely as any property or representative account; having been debited with its costs, we now credit it with its proceeds. The difference will be, in this case, our loss.

(2) Here the goods are not shipped for our account, but for the account of another party who ordered them. This is therefore a regular sale.

QUEBEC, APRIL 23, 1871.	*Dr.*		*Cr.*	
G. DOYLE & SON'S CONSIGN. *Dr.*	$ 100	00		
To CASH.			$ 100	00
24				
SUNDRIES *Dr.* To REAL ESTATE.			8000	00
R. FISHER & SON	2562	60		
MORTGAGE RECEIVABLE	5000	00		
CASH	437	40		
25				
E. F. ANDREWS *Dr.*	1600	00		
To G. DOYLE & SON'S CONS.			1600	00
26				
SUNDRIES *Dr.* To NATIONAL BANK.			8758	75
BILLS PAYABLE (1)	8750	00		
INTEREST	8	75		
27				
BILLS PAYABLE (2) *Dr.* To SUNDRIES.	2000	00		
" CASH.			1993	34
" INTEREST.			6	66

(1) Notes, like cash, ought always to be debited and credited with the value written upon them. If they are really worth more or less than this expressed value, that difference must be shown in some other account. In the present case, the face of the note is $8750; but the *worth* of the note, with the interest due upon it, is $8758.75; and this is the amount we are obliged to pay, in order to cancel it. We therefore debit Bills Payable with the face of the note, and Interest with the amount we pay for Interest.

(2) In this case, the note is really worth *less* than its expressed value, as we are obliged to pay for it only $1993.33, which is $6.67 less than its face. We here debit Bills Payable with the face of the note, according to principles laid down, and credit Cash for the amount of cash paid, and Interest for the difference, that being the amount produced by Interest, or by paying our note before it is due.

Quebec, April 29, 1871.		Dr.		Cr.	
Cash	*Dr.*	$5100	00		
To Shipment to Kingston.				$5100	00
30					
Cash	*Dr.*	1400	00		
To G. Doyle & Son's Cons.				1400	00
"					
G. Doyle & Son's Cons.	*Dr.*	2900	00		
To Sundries.					
" Storage & Advertising.				30	00
" Commission.				75	00
" G. Doyle & Son.				2795	00
"					
F O'Reilly	*Dr.*	200	00		
To Cash.				200	00
"					
Expense	*Dr.*	150	00		
To Cash.				150	00

We have omitted the Ledger in this Set, believing the student to be fully capable to post the accounts without assistance of this kind. We shall adhere to this plan hereafter, except in cases where some new principle or application may be otherwise more clearly shown.

The student will make his Ledger conform to the following Trial Balance, and close it in accordance with the Statement which follows

TRIAL BALANCE.

Balances.	Total Footings.	LEDGER ACCOUNTS.	Total Footings.	Balances.
	9000	J. Byrne	29200	20200
	3830 60	F. O'Reilly	23830 60	20000
23098 25	43465	National Bank	20366 75	
1400	7800	Tow-boat Nestor Stock	6400	
	5490	Tow-boat Levis Stock	5490	
	10750	Bills Payable	14881	4131
	250	J. L. Fraser	250	
1020	5250 60	Bills Receivable	4230 60	
6230 20	36797 90	Cash	30567 70	
	1068	Young & Talbot	1068	
	2562 60	R. Fisher & Son	2562 60	
5000	13000	Real Estate	8000	
		Mortgage Payable	7300	7300
9038	13700	Merchandise	4662	
		L. R. O'Connor & Co.	4000	4000
504	3004	Shipment to Montreal	2500	
	2332	L. Shaw & Co.'s Consign.	2332	
33 05	39 71	Premium, Disc't, & Int.	6 66	
225	225	Loss and Gain		
	9500	F. I. Ray's Consignment	9500	
140	1440	National Bank Stock	1300	
		Storage and Advertising	100	100
		Commission	376 63	376 63
		L. Shaw & Co.	2152 87	2152 87
6531	6531	Steamboat Sorel Stock		
166	5266	Shipment to Kingston	5100	
920	920	W. S. Kelly		
	3000	G. Doyle & Son's Cons.	3000	
5000	5000	Mortgage Receivable		
1600	1600	E. F. Andrews		
		G. Doyle & Son	2795	2795
150	150	Expense		
61055 50	191972 41		191972 41	61055 50

INVENTORY OF UNSOLD PROPERTY.

Merchandise,	$9325 00
$\frac{1}{3}$ of Tow-boat Nestor Stock,	1950 00
Real Estate,	6300 00
$\frac{1}{5}$ Steamboat Sorel Stock,	6531 00
5 Shares National Bank Stock, @ $50,	250 00
	$24356 00

STATEMENT,—SET II.

LOSSES AND GAINS.

			Losses.	Gains.
TOW-BOAT NESTOR STOCK,...	Proceeds from sales	6400		
	Value of unsold.....	1950		
		8350		
	Cost.....	7800		
	Gain.....	550		550 00
TOW-BOAT LEVIS STOCK,.....	Cost.....	5490		
	Proceeds from Ins.	5265		
	Loss.....	225	225 00	
REAL ESTATE,.....	Proceeds from sales.	8000		
	Value of Unsold.....	6300		
		14300		
	Cost.....	13000		
	Gain.....	1300		1300 00
MERCHANDISE,.....	Proceeds from sales.	4662		
	Mdse uns. (per Inv.).	9325		
		13987		
	Cost.....	13700		
	Gain.....	287		287 00
SHIPMENT TO MONTREAL,.....	Cost.....	3004		
	Proceeds.....	2500		
	Loss.....	504	504 00	
PREMIUM, DISC'T, INTEREST,..	Cost.....	39.71		
	Proceeds.....	6.66		
	Loss.....	33.05	33 05	
NATIONAL BANK STOCK,.....	Proceeds from sales.	1300		
	Value of unsold.....	250		
		1550		
	Cost.....	1440		
	Gain.....	110		110 00
STORAGE AND ADVERTISING,..	Proceeds.....			100 00
COMMISSION,.....	"			376 63
SHIPMENT TO KINGSTON,.....	Cost.....	5266		
	Proceeds.....	5100		
	Loss.....	166	166 00	
EXPENSE,.....	Cost.....		150 00	
	Net gain.....		1645 58	
			2723 63	2723 63

From Inventories of Unsold Property.			*Res.*	*Liabil.*
TOW BOAT NESTOR STOCK,			$1950	
REAL ESTATE,			6300	
MERCHANDISE,	Per Inventory		9325	
NATIONAL BANK STOCK,			250	
STEAMBOAT SOREL STOCK,			6531	
From Ledger Accounts.				
NATIONAL BANK,	Amount on Deposit		23098 25	
BILLS PAYABLE,	Outstanding Notes			4131
BILLS RECEIVABLE,	Amount on hand		1020	
CASH,	" "		6230 20	
MORTGAGE PAYABLE,	Outstanding Notes			7300
L. R. O'CONNOR & Co.,	We owe them			4000
L. SHAW & Co.,	We owe them			2152 87
W. S. KELLY,	Owes us		920	
MORTGAGE RECEIVABLE,	Amount on hand		5000	
E. F. ANDREWS,	Owes us		1600	
G. DOYLE & SON,	We owe them			2795
J. BYRNE,	Capital invested (net)	20200.00		
	His half of net gain	822.79		
	His present interest in the concern	21022.79		21022 79
F. O'REILLY,	Capital invested (net)	20200.00		
	Drawn out	200.00		
	His half of net gain	822.79		
		622.79		
	His present interest in the concern	20822.79		20822 79
			62224 45	62224 45

From the foregoing statement the student will be enabled to close up this Ledger with certainty and to produce the results in his Balance account which are exhibited under the head of Resources and Liabilities there shown.

CASH BOOK,

Cash Received.

April	1	To F. O'Reilly,.....Rec'd of him as capital.	$19600	
"	5	" National Bank,....Rec'd of it.	400	
"	16	" L. Shaw & Co's Consignment,.....Rec'd for sale of this date.	1312	
"	19	" Ship't to Montreal,.Rec'd for sale at auction.	2500	
"	20	" F. I. Ray's Consignment,.....Rec'd for sale of this date.	5000	
"	22	" Nat. Bank Stock,..Rec'd the balance of sale of this date.	1048	50
"	24	" Real Estate,.....Rec'd for bal. of sale.	437	40
"	29	" Ship't to Kingston,.Rec'd for sale of goods.	5100	
"	30	" G. Doyle & Son's Consignment, ..Rec'd for sale of this date.	1400	
			$36797	90
		To Balance,.....From old %.	6230	20

BILL BOOK,

Bills

No.	When rec'd.	Drawers.	In whose favor.	For what received.	Where pay.
	1871				
1	April 1	L. Clint,	F. O'Reilly.	Investment.	Our Office.
2	" 11	H. W. Cooper.	Ourselves.	Merchandise.	" "

Bills

No.	When issued.	Drawers.	In whose favor.	For what given.	Where payable.
	1871				
1	Mar. 18	J. Byrne.	Barclay & Co.	To Bal. %.	Quebec.
2	April 4	Ourselves.	L. R. O'Connor & Co.	Mdse.	"
3	" 4	"	"	"	"
4	" 17	"	Quebec Ins. Co.	Insurance.	"

—SET II.

Cash Disbursed.

April	1	By National Bank,.....	Deposited.	$18000	
"	5	" Tow-boat Levis Stock,.	Paid for repairs.	90	
"	6	" Shipm't to Montreal,.	Paid for Drayage.	4	
"	9	" L. Shaw & Co.'s Consignment,.....	Paid for Freight.	95	
"	10	" Premium,.....	Paid for it.	2	67
"	13	" F. I. Ray's Consign..	Paid Freight & Ins.	150	
"	17	" Steamb't Sorel Stock,.	Paid for balance of purchase of her.	696	19
"	18	" Shipm't to Kingston,.	Paid for Insurance.	20	
"	19	" W. S. Kelly,.....	Paid for Drayage.	4	
"	20	" F. I. Ray's Consign .	Paid for net proceeds.	9062	50
"	23	" G. Doyle & Son's Consignment,.....	Paid for Freight.	100	
"	27	" Bills Payable,.....	Paid our Note No. 2.	1993	34
"	30	" F. O'Reilly,.....	Paid him on %.	200	
"	30	" Expense,.....	Paid sundry expenses.	150	
"	30	" *Balance,*.....	*Amount on hand.*	6230	20
				$36797	90

—SET II.

Receivable.

Date of Note.		Time to run.	When due.	Amount.		Remarks.
1871			1871			
March	23	60 days.	May $\frac{22}{25}$	$4230	60	Paid April 17.
April	11	60 days.	June $\frac{10}{13}$	1020	00	

Payable.

Date of Note.		Time to run.	When due.	Amount.		Remarks.
1871			1871			
March	18	30 days.	April $\frac{17}{20}$	$ 8750	00	Paid April 26.
April	4	40 days.	May $\frac{14}{17}$	2000	00	Paid April 27
"	4	3 mo's.	July $\frac{4}{7}$	4000	00	
"	17	40 days.	May $\frac{27}{30}$	131	00	

COMMISSION SALES

Dr. L. SHAW & Co.'s

1871				
April	9	Invoice, Per Grand Trunk R. R., of 600 bu. Wheat, @ $1.40		
		800 bu. Corn, @ .65		
		4200 lbs. Butter, @ .14		
"	"	To CASH, Paid Freight	$ 95	00
"	16	" STORAGE & ADVERTISING,	20	00
"	"	" COMMISSION, 2¾ % on $2332	64	13
"	"	" L. SHAW & Co., Net proceeds	2152	87
		Due by Equation May 13, 1871 *	$2332	00

Dr. F. I. RAY'S

1871				
April	13	Invoice, Per Steamer Anna, of 400 bbls. Codfish, @ $4.50		
		600 " Mackerel, @ 6.50		
		500 " Herrings, @ 5.00		
"	"	To CASH, Paid Freight & Insurance	$ 150	00
"	20	" STORAGE & ADVERTISING,	50	00
"	"	" COMMISSION, 2½ % on $9500	237	50
"	"	" CASH, Net proceeds remitted	9062	50
			$9500	00

Dr. G. DOYLE & SON'S

1871				
April	23	Invoice, Per Richelieu Co.'s Line, of 1000 bush. Wheat,		
		800 " Oats,		
		200 bbls. Tallow,		
"	"	To CASH, Paid Freight	$ 100	00
"	30	" STORAGE & ADVERTISING	30	00
"	"	" COMMISSION, On $3000 @ 2½ %	75	00
"	"	" G. DOYLE & SON, Net pro. due May 20, 1871	2795	00
			$3000	00

* The calculation for averaging this account, to ascertain when the net proceeds

CONSIGNMENT. *Cr.*

1871				
April	11	By Bills Receivable, Sold H. W. Cooper on his Note at 60 days. 600 bu. Wheat, @ $1.70.	$1020	00
"	16	" Cash, Sold B. W. Hardy. 4200 lbs. Butter, @ $.16 $672		
		800 bu. Corn, @ .80 640	1312	00
			$2332	00

CONSIGNMENT. *Cr.*

1871				
April	18	By Ship't to K., Assumed and ship'd Price & Co. 600 bbls. Mackerel, @ $7.50,	$4500	00
"	20	" Cash, 400 " Codfish, @ 5.00 $2000		
		500 " Herrings, @ 6.00 3000	5000	00
			$9500	00

CONSIGNMENT. *Cr.*

1871				
April	25	By E. F. Andrews, Sold him, @ 40 days, 200 bbls. Tallow, @ $8	$1600	00
"	30	" Cash, 1000 bu. Wheat, @ 1.40	1400	00
			$3000	00

are due, will be found in the Commercial Arithmetic from p. 266 to 279.

ACCOUNT SALES,—SET II.

Account Sales of { 600 bu. Wheat, / 800 " Corn, / 4200 lbs. Butter, } **on % and risk of**

L. Shaw & Co.

1871					
April	11	Sold H. W. Cooper, on his Note @ 60 days,			
		600 bu. Wheat, @ $1.70	$1020.00		
"	16	Sold B. W. Hardy, for cash,			
		4200 lbs. Butter, @ $.16	672.00		
		800 bu. Corn, @ .80	640.00	2332	00
		Charges			
"	9	Paid Freight, in cash,	95.00		
"	16	Storage & Advertising,	20.00		
"	"	Commission, 2¾ % on $2332,	64.13	179	13
		L. Shaw & Co.'s net proceeds,		$2152	87
		Due by Equation, May 13, 1871,			
		E. E. BYRNE & O'REILLY,			
		Quebec, April 16, 1871. *Per J. Maguire.*			

Sales of Goods by order and for % of F. I. Ray.

1871					
April	18	Taken to our account,			
		600 bbls. Mackerel, @ $7.50	$4500.00		
"	20	Sold for Cash.			
		400 bbls. Codfish, @ $5	2000.00		
		500 " Herrings, @ $6	3000.00	9500	00
		Charges			
"	13	Paid Freight & Insurance, in cash,	$150.00		
"	20	Storage & Advertising,	50.00		
"	"	Commission, 2½ % on $9500,	237.50	437	50
		F. I. Ray's net proceeds remitted		$9062	50
		E. E. BYRNE & O'REILLY,			
		Quebec, April 20, 1871. *Per J. Maguire.*			

Sales of { 200 bbls. Tallow, / 1000 bu. Wheat, } **for % of G. Doyle & Son.**

1871					
April	25	Sold E. F. Andrews, @ 40 days, 200 bbls. Tallow, @ $8	$1600.00		
"	30	Sold for cash, 1000 bu. Wheat, @ $1.40	1400.00	3000	00
		Charges			
"	23	Paid Freight, in cash,	$100.00		
"	30	Storage & Advertising,	30.00		
"	"	Commission, 2½ % on $3000,	75.00	205	00
		G. Doyle & Son's net pro. due May 20, 1871,		$2795	00

E. & O. E. BYRNE & O'REILLY,
Quebec, April 20, 1871. *Per J. Maguire.*

MEMORANDUM.

May 1, 1871, A. J. Hall and R. S. Griffin have this day entered into copartnership, under the style and firm of HALL & GRIFFIN, in the prosecution of a *produce, grocery, domestic shipping* business, and for buying and selling *Bank Stocks*, etc. They are to furnish the Capital as agreed. A. J. Hall is to share one-third, and R. S. Griffin, two-thirds of the gains or losses. A. J. Hall's Resources and Liabilities are taken from the Balance Account of his Ledger of April, Memorandum IV, p. 30. R. S. Griffin's Resources and Liabilities are as follows: Cash, $8755.40; Merchandise, per Inventory, $3125; Bills Payable, per B. B., $2238.36.—**2.** Sold L. Parkins, 68¼ yds. Merino, at $1.32; 5½ yds. Silk Velvet, at $8.50; 59¾ yds. Black Cloth, at $5.80. Received in payment, cash, $200; his note at 40 days, for $150; the balance at 3 months.—On the same day, Sold J. Morgan & Co., 150 bbls. Mess Pork, at $9.50; 100 bbls. Lard, 20000 lbs., at 6½ cts. Rec'd in payment, 14 hhds. Sugar, 15400 lbs., at 6 cts.; R. S. Griffin's note, their favor, assumed by the Firm at commencing business, due 29th inst., for $1500; and cash for bal. of sale and for discount on note, $257.75. Disc't on R. S. Griffin's note for 27 days is $6.75.—**3.** Gave H. Lawlor & Son, in payment of A. J. Hall's note their favor, due this day, 30 bush. Oats, at 80 cts.; and cash for the balance, $105.30.—On the same day, paid in Cash R. S. Griffin's note, favor of Talbot & Smith, assumed by the Firm at commencing business, dated February 1st, at 90 days, with int. from date. Amt. of Note, $738.36. The int. for 93 days (includ. grace) am'ts to $11.44.—On the same day, Bo't of O. C. Lawrence, 125 bbls. Extra Superfine Flour, at $4.25; 100 bbls. Extra Mess Pork, at $9; 200 bbls. Mess Beef, at $11; 100 bbls, Prime Beef, at $8.50; 50 bbls. Beef

Hams, at $16; 50 bbls. Pearl Ashes, at $4.30. Gave in payment, 3 bbls. Codfish, at $8.60; 100 bushels Potatoes, at 48 cts.; 14 hhds. Sugar, 15400 lbs., at 7½ cts.; and cash for the balance.—**4.** Shipped per Steamer Prince Arthur, and consigned to Blanchard & Kelly, Halifax, to be sold on our % and risk, 125 bbls. Extra Superfine Flour, at $4.25; 200 bbls. Mess Beef, at $11; 15 bbls. Fancy Flour at $4.70. Paid drayage in cash, $11.25. Passed our note, at 30 days to the Quebec Insurance Co., for $2800, at 1¼ %.—**5.** Rec'd of S. A. Hunt, in payment of his note of March 27, amounting to $14.24, due on the 9th inst.; viz., 2 bbls. Herrings, at $6; and cash for the balance.—**7.** Paid cash for repairs of Store, $25.—On the same day, Bought of J. S. O'Dowd & Co. on %, 200 bbls. Mess Beef, at $11.12½.—On the same day, Shipped per Steamer Champlain, Capt. Belleau, and consigned to R. J. Wilson, St. John, N. B., to be sold on our % and risk, 50 bbls. Pearl Ashes, at $4.30; 200 bbls. Mess Beef, at $11.12½. Paid cash for drayage, etc., $7; also to the Montreal Insurance Co., for Ins. on $2480, at 1¼ % and Policy $1.—**8.** Rec'd of E. Stephens, in full of %, his note at 30 days, for $100; and cash for the balance, $20.—**9.** Discounted, at the Union Bank, Neil & Roche's note, favor of J. S. O'Brien, for $240. Disct. for 35 days, $1.40; cash received, $238.60.—**10.** Paid C. Phelan cash in full of %.—**11.** Rec'd per Grand Trunk R. R. from Fisher & Lee, Toronto, Mdse., previously ordered by us, viz., 30 hhds. Cuba Molasses, 3000 gals. at 25 cts.; 30 hhds. Cuba Sugar, 30750 lbs. at 4¼ cts. Paid in cash for freight, drayage, etc., $96.—**12.** Bought for cash of R. Lyons & Co., their Bill of Exchange on Hamel & Norris, Toronto, and remitted the same this day to Fisher & Lee, in payment of am't due them, $2133.75. Paid ¼ % premium for the bill.—**14.** Bo't of C. L. Murray, on our note at 4 mos., 2500 bu. Red Wheat, delivered on board the Steamer Victoria, Capt. Barry, at $1.03 per bush., and shipped the same to N. C. Moreau, Pictou, to be sold for our % and risk. Issued our note, at 15 days, to the Quebec Insurance Co., for Ins. on $2608.61 at 1¼ % and for Policy $1.—**15.** E. Stephens' note for $200 is due and not paid.—**16.** Renewed B. Jones' note for $198.06, now due. Rec'd his new note at 141 days, for $150, and cash for the bal. and 144 days' int. at 7 %.—**17.** Sold to Carroll & Samson, 20 hhds. Cuba Molasses, 2000 gals., at 27 cts.; 100 bbls. Prime Beef, at $9; 100 bbls. Mess Beef, at $11.50. Rec'd cash in part, $1295; their note at 60 days, for balance, including discount, $1308.74. The disc't on the note is for 63 days.—**18.** Accepted J. S. O'Dowd & Co.'s draft on us, at 30 days' sight, favor of E. L. Tessier, for $1000.—**19.** B. Nolan has this day renewed his note, favor of A. J. Hall, for $50, assumed by the Firm, by another note for the same amt. and time, endorsed by J. Kerwin, and paid cash for 43 days' int. on the new note, 36 cts.—**21.** Bo't of H. Collins, 120 bbls. Middlings, at $4.20; 60 bbls. Rye Flour, at $3.60. Gave in payment, 35 bbls. Mess Pork, at $10; our check on the Union Bank, for $250; bal. on %.—**23.** Shipped per Steamer Laval, to S. Larue & Co., Gaspe, for their %, and pursuant to their order, 150 bbls. Mess Beef, at $11.50. Paid cash for drayage, $4.75.—**24.** Sold to sundry persons, for cash, 3 barrels Herrings, at $7.50: 3 bbls. Mackerel, at $8.40; 10 bbls.

Middlings, at $5; 1 piece, 32 yds., Elbeuf Cloth, at $7.—**25**. Rec'd intelligence that the Steamer Victoria, on which we made a Shipm't on the 14th inst., was wrecked in the Gulf of St. Lawrence on the 20th inst.,—Steamer and Cargo, total loss.—**26**. Discounted our acceptance of the 18th inst., at 30 days' sight, E. L. Tessier's favor, amt'g to $1000. Discount for 25 days at 6 %, on $1000, is $4.17. Bal. paid in cash.—**28**. Paid R. S. Griffin cash for private use, $200. —On the same day, sold as follows: to C. E. Panet, on %, 342 yds. Red Flannel, at 80 cts., and 2¼ yds. Black Satin, at $5; to N. Blake, on an order of J. B. Davis, 39¼ yds. Woolen Carpet, at 60 cts.; 8 bbls. Superfine Flour, at $4.80.—On the same day, Sold T. A. Bardy, 4 Shares of the Montreal Bank Stock, at $106. Rec'd in payment, 80 bbls. Gray Apples, at $5; and cash for the bal.—**29**. The Quebec Insurance Co. has failed, and is able to pay but 10 cts. on the dollar of its indebtedness. Accordingly, we have received from the Assignees 10 % on the am't of our Ins. on shipm't to Pictou of 14th inst., per Steamer Victoria, which was wrecked in the Gulf of St. Lawrence on the 20th inst. Amount insured $2608.61 at 10 % = $260.86, for which we received our note of 14th inst., at 15 days, $33.61; and cash for balance.—On the same day, received an Account Sales from R. J. Wilson, St. John N. B., of Pearl Ashes and Beef sent him on the 7th inst. Net proceeds, $2976.15. Rec'd also a draft, remitted by them, on Viger & Roy, at sight (for part proceeds) which has been paid in cash $1500.—**30**. Rec'd an Account Sales from Blanchard & Kelly, Halifax, of 200 bbls. Mess Beef, belonging to ship't to them of 4th inst. Net proceeds, $2380.86.—On the same day, sold to sundry persons for cash, 15 lbs. Brown Sugar, at 10 cts.; 5 lbs. Chocolate, at 30 cts.; 44 lbs. Butter, at 18 cts.; 5 bush. Indian Corn, at 90 cts.; 3 bbls. Beef Hams, at $18; 2 bbls. Extra Superior Flour, at $6.50. —**31**. Paid cash for Rent of Store $75; for Clerk hire $100; for Laborers, $38.

INVENTORY OF UNSOLD PROPERTY.

Merchandise,	$2753	81
Shipment to Halifax, balance of Mdse.,	601	75
Shares of the Montreal Bank Stock,	208	00
	$3563	56

The net losses, May 31, am't to	$1477.22
Of which A. J. Hall's third is	492.41
And R. S. Griffin's two-thirds,	984.81
A. J. Hall's capital is	4328.61
R. S. Griffin's capital is	8457.23

SET III.

JOURNAL DAY BOOK,

INVOICE BOOK, SALES BOOK, COMMISSION SALES BOOK, ACCOUNT SALES,

FORMS OF NOTES, DRAFTS, LETTERS, ETC.

PARTNERSHIP BUSINESS.

REMARK.—The Sets of books thus far shown in this work, have all been conducted upon the Italian method of historical Day Book, with separate Journal. We did so on account of its greater simplicity, and not to distract the mind from more important considerations which it was necessary to inforce. The student being now more thoroughly grounded in the science, we shall henceforth give a little attention to the more practical forms in use, and to a greater variety of entries than heretofore. We wish him particularly to note the peculiar form of the Journal Day Book introduced in this Set, that he may be able to express, in this manner, any conceivable transaction, combining all the essential points of the separate Day Book and Journal. Where more severely practical forms—for the purposes of condensation—are not in use, the Journal Day Book meets with great favor, as being both plain and practical.

In the transactions of this Set, we have introduced a new feature; viz., *Mdse. Co. transactions.* It will, of course, be understood that by "Merchandise Companies" is meant the temporary copartnerships existing between the consignor and consignee, having reference to the sale of particular consignments of merchandise. The nature of this species of copartnership differs from that of a general copartnership only in its duration, and the manner of conducting its sales. In Mdse. Co. business, one of the partners—the consignee—is the commission merchant, and, in that capacity, receives and disposes of the property

as he would of a simple consignment; the only difference being that he is interested in the gains and losses. The two methods given in this Set, and which are fully illustrated, differ only as regards the opening and closing entries. In the *first* method—exemplified by Mdse. Co. account, "A,"—the principle recognized is, that the *holder* of the property is responsible for it. Thus, when we receive from C. Lortie & Bro. an invoice to be sold on joint %, we debit "Mdse. Co. A" with the invoice and expenses, and credit the consignors with the cost of the invoice, thus making ourselves responsible for the property as though it were all our own. The consignor's entry, if recognizing the same principle, will be to debit us for the entire cost of the merchandise. In the *second* method, the principle recognized is that the *owner* of the property is responsible. Thus, when we receive from C. Lortie & Bro., mdse. to be sold on joint %, we debit "Mdse. Co. A" with *our own share* only, and credit the consignor. The consignor's entry, in this case, if made to correspond with ours, would be to debit us for our share, and "Shipment in Co. to Quebec" for his share.

Where there are more than two parties interested, if the accounts are kept by the *first* method, the consignee should, as before, debit the Mdse. Co. account with its entire cost—invoice and expenses—and credit the consignor with their (the consignee's and consignor's) joint share, and any other party or parties with his or their share. The consignor would, in such a case, debit the consignee with their joint share, and each of the other parties with his or their share. The other parties would, if making an entry to correspond, debit the consignee and credit the consignor each for his own share.

Where there are more than two parties interested, and the accounts are kept by the *second* method, the consignee should debit "Mdse. Co." account *for his own share* and all the charges, and credit the consignor for his (the consignee's share). The consignor, on the other hand, should debit each of the parties for their respective shares, and "Ship't in Co." for his own share. Each of the other parties should debit "Shipment in Co.," and credit the consignor each for his own share.

JOURNAL DAY BOOK,—SET III.

QUEBEC, MAY 3rd, 1871.

C. S. Mitchell and R. A. Hudon have, per articles of agreement dated 1st inst., entered into partnership, under the firm and style of "Mitchell & Hudon," for the prosecution of a Wholesale Grocery & Wine Business, and General Commission; each to invest $12000, and share alike in gains and losses.

QUEBEC, MAY 3rd, 1871.

SUNDRIES	*Dr.*	To C. S. MITCHELL (1)		$12000 00
		For Effects invested:		
CASH		Deposited.	$9000	
BILLS RECEIVABLE		Notes in his favor; viz., one drawn by P. Racine, due May 28,	1500	
		Another drawn by S. Lewis, due June 6,	1000	
P. ALLARD		Balance of %.	500	12000 00
		"		
SUNDRIES	*Dr.*	To R. A. HUDON.		12000 00
		For effects invested:		
CASH		Deposited.	$8000	
BILLS RECEIVABLE		Note in his favor, drawn by D. Aylwin, due June 1	1000	
MERCHANDISE		As per Inventory, Inv. B.	2700	
A. RINFRET		Balance of %.	300	12000 00
		4		
SHIP'T TO MONTREAL	*Dr.*	To SUNDRIES.		575 00
		Shipped per Steamboat Quebec, and consigned to G. S. Walls, Montreal, to be sold for our %.		
To MERCHANDISE		Inv. of produce, as per S. B.	$573	
" CASH		Paid Freight and Drayage	2	575 00
		6		
S. WHITE & CO.'S CONSIGNMENT		*Dr.*		
To CASH		Paid Freight and Drayage of an Invoice of Flour, as per I. B., amt'g to $1018.25, rec'd from S. White & Co, to be sold on their %.		10 00

(1) There is supposed to be two columns for dollars and cents—ruled lines for the inner column are not given, in the present instance, for want of room.

QUEBEC, MAY 7, 1871.

MERCHANDISE	*Dr.*	TO SUNDRIES.	1561	10
	Rec'd per Steamb. Champlain, from D. C. Peachy & Son, Montreal, and consigned to us for our acct., an Inv. of Wines, as per I. B.			
To D. C. PEACHY & SON	For amount of	$1549.60		
" CASH	Paid for Freight & Dray.	11.50	1561	10
	8			
NATIONAL BANK		*Dr.*		
To CASH	Deposited.		10000	00
	"			
SUNDRIES *Dr.*		To BILLS RECEIVABLE.	1000	00
	Discounted D. Aylwin's note, fav. R. A. Hudon, due June 1st.			
CASH	Proceeds of note	$996		
DISCOUNT	Amount of 24 ds., at 6 %	4	1000	00
	10			
L. DOUGLAS & CO.	*Dr.*	TO SUNDRIES.	1272	68
	Forwarded per Grand Trunk R. R., pursuant to their order, an Invoice of Mdse., as follows:			
To MDSE.	Sundry produce, per S. B.	$1240.00		
" CASH	Paid Freight and Ins.	30.20		
" COMMISSION	For Expedition and Ins.	2.48	1272	68
	11			
BILLS RECEIVABLE		*Dr.*		
	To S. WHITE & CO.'S CONSIGNMENT.		624	00
	For sale of Flour to S. Doran, as per C. S. B.			

QUEBEC, MAY 12, 1871.

CASH		*Dr.*		
	To S. WHITE & Co.'s CONSIGNMENT.			584 00
	For sale of Flour, as per C. S. B.			
	"			
S. WHITE & Co.'s CON.	*Dr.*	To SUNDRIES.		1198 00
	Closed S. White & Co.'s Consignment, and rendered them an Account Sales.			
To STORAGE & ADVER.	Our charges		$ 18.50	
" COMMISSION	2½ % on $1208		30.20	
" S. WHITE & Co.	Their net proceeds		1149.30	1198 00
	14			
MERCHANDISE	*Dr.*	To SUNDRIES.		2581 90
	Bo't of L. McCord, produce, as per I. B., amt. $2581 90, and paid as follows :			
To P. ALLARD	Our order on him, for		$300.00	
" NATIONAL BANK	Our check, for		750.00	
" BILLS RECEIVABLE	S. Doran's note of the 11th inst., for		624.00	
" " PAYABLE	Gave our note, at 30 d's, for		500.00	
" DISCOUNT	Allowed on the bal.		12.30	
" CASH	For balance.		395.60	2581 90
	15			
C. S. MITCHELL, Private		*Dr.*		
To CASH.	Drew on private %.			30 50
	16			
MDSE. Co. A.	*Dr.*	To SUNDRIES.		3590 00
	Rec'd from C. Lortie & Bro., Halifax, to be sold on our joint acct. and risk, each ½, Mdse. as per I. B., amount $3550.			
To C. LORTIE & BRO.	Their invoice as above		$3550	
" CASH	Paid Freight		40	3590 00

QUEBEC, MAY 17, 1871.

G. S. WALLS	*Dr.*		
	TO SHIPMENT TO MONTREAL.		650 40
	Received an Account Sales of the Mdse. sent him on the 4th inst.		
	18		
S. WHITE & CO.	*Dr.*		
	TO BILLS PAYABLE.		1149 30
	Accepted their Draft, at 8 days' sight, in favor of L. Raymond, for $1149.30.		
	19		
SUNDRIES	*Dr.* TO SUNDRIES.		
	Ship'd M. Blanchet & Co., Pictou, to be sold on joint acct., each ½, Mdse. as per S. B., amt.	$1280.00	
	Freight on same	2.75	
	Ins. at ½ % on $1280	6.45	
		$1289.20	
M. BLANCHET & CO.	For their ½ above invoice	$644.60	
SHIPMENT IN CO.	" our ½	644.60	1289 20
TO MERCHANDISE	As above	1280.00	
" CASH	Paid Drayage and Ins.	9.20	1289 20
	21		
MDSE CO. B.	*Dr.* TO SUNDRIES.		414 86
	Rec'd of G. Quinn & Co., Montreal, to be sold on joint acct. of themselves, E. Cary & Son, and ourselves, each ⅓, an Invoice of produce, as per I. B., amount'g to $1160.28.		
To G. QUINN & CO.	For our ⅓ invoice	$386.76	
" CASH	Paid Freight	28.10	414 86

QUEBEC, MAY 22, 1871.

SHIP'T TO THREE RIVERS	*Dr.*	TO SUNDRIES.	310	50
	Shipped per Brig St. Maurice, and consigned to J. N. Carbray, Three Rivers, to be sold on our acct. and risk, Mdse. as per S. B.			
TO MERCHANDISE	Invoice of produce	$306.00		
" CASH	Paid expenses	4.50	310	50

23

SUNDRIES	*Dr.*	TO MDSE. CO. A.	4140	00
	Sold O. Martel, Mdse. Co. A., as per C. S. B., amounting to $4140.			
BILLS RECEIVABLE	His note at 15 days, for	$2000		
CASH	For Balance	2140	4140	00

"

MDSE. CO. A.	*Dr.*	TO SUNDRIES.	550	
	Closed sales in company with C. Lortie & Bro., and rendered them an Account Sales.			
TO STORAGE & ADVER.	Our charges	$ 12.00		
" COMMISSION	2½ % on $4140	103.50		
" C. LORTIE & BRO.	Their ½ net gain	217.25		
" LOSS AND GAIN	Our " "	217.25	550	00

25

M. BLANCHET & CO.		*Dr.*		
		TO SHIPMENT IN CO.	610	60
	Rec'd an Account Sales of Mdse. shipped them on the 19th inst. Our net proceeds as above.			

QUEBEC, MAY 26, 1871.

CASH	*Dr.*		
	TO SHIPMENT TO THREE RIVERS.	115	00
	Rec'd of J. N. Carbray, Three Rivers, a check at sight on the National Bank, for $115, as net proceeds of the Mdse. shipped him on the 22nd inst. per Brig St. Maurice. The ship having sunk, the Mdse., which was not insured, was saved, but much damaged, and sold at auction for cash.		
	28		
D. C. PEACHY & SON	*Dr.*		
	TO MDSE. CO. B.	1273	50
	Sold them, at 6 mos., Mdse., as per C. S. B.		
	"		
CASH	*Dr.*		
	TO BILLS RECEIVABLE.	1500	00
	P. Racine has paid his note, due this day.		
	"		
MDSE CO. B.	*Dr.* TO SUNDRIES.	858	64
	Closed Mdse. Co. B., and rendered Account Sales of the same to G. Quinn & Co., Montreal.		
TO STORAGE & ADVER.	Our charges	$ 3.55	
" COMMISSION	2% on $1273.50	25.47	
" G. QUINN & CO.	Their net proceeds	405.46	
" E. CARY & SON	" " "	405.46	
" LOSS AND GAIN	Our ½ net gain	18.70	858 64

QUEBEC, MAY 29, 1871.

INTEREST	*Dr.*		
	To CASH.	6	32
Renewed S. White & Co.'s draft, for $1149.30, which we accepted on the 18th inst., by our note for same amount, to the 1st of July. We paid cash for int. on our note $6.32.			
	30		
BILLS RECEIVABLE	*Dr.*		
	To L. DOUGLAS & Co.	1272	68
Rec'd of L. Douglas & Co., their Bill of Exchange on A. Simms & Devaux, London, at 60 days' sight, for $1272.68, in full of acct.			
	"		
SUNDRIES *Dr.*	To BILLS RECEIVABLE.	1272	68
CASH	Sold L. Douglas & Co.'s Bill of Exch., our favor, for $1259.95		
DISCOUNT	Lost 1% 12.73	1272	68
	31		
SUNDRIES	*Dr.* To CASH.	262	25
BILLS RECEIVABLE	Purchase of N. Caron's draft, on Molson, at 8 days' sight, $260.00		
DISCOUNT	Paid 2.25	262	25
	"		
D. C. PEACHY & SON	*Dr.*		
	To BILLS RECEIVABLE.	260	00
For N. Caron's draft, our favor, sent them in payment acct.			
	"		
EXPENSE	*Dr.*		
	To CASH.	45	30
For sundry expenses.			

TRIAL BALANCE.

Balances.		Total Footings.			Total Footings.		Balances.	
				C. S. Mitchell	12000		12000	
				R. A. Hudon	12000		12000	
12719	48	23594	95	Cash	10875	47		
3000		7656	68	Bills Receivable	4656	68		
				Bills Payable	1649	30	1649	30
3444		6843		Merchandise	3399			
200		500		P. Allard	300			
300		300		A. Rinfret				
		575		Shipment to Montreal	650	40	75	40
		1208		S. White & Co.'s Consign't	1208			
				Storage and Advertising	34	05	34	05
		1533	50	D. C. Peachy & Son	1549	60	16	10
9250		10000		National Bank	750			
13		25	30	Discount and Interest	12	30		
		1272	68	L. Douglas & Co.	1272	68		
				Commission	161	65	161	65
		1149	30	S. White & Co.	1149	30		
30	50	30	50	C. S. Mitchell's private %				
		4140		Mdse. Co. A.	4140			
				C. Lortie & Bro.	3767	25	3767	25
650	40	650	40	G. S. Walls				
1255	20	1255	20	M. Blanchet & Co.				
34		644	60	Shipment in Co.	610	60		
		1273	50	Mdse. Co. B.	1273	50		
				G. Quinn & Co.	792	22	792	22
195	50	310	50	Shipment to Three Rivers	115			
				Loss and Gain	235	95	235	95
				E. Cary & Son	405	46	405	46
45	30	45	30	Expense				
31137	38	63008	41		63008	41	31137	38

INVENTORY.

The Mdse. remaining unsold, May 31, 1871, amounts to $3530.25.

C. S. MITCHELL & R. A. HUDON'S BALANCE SHEET,—SET III.

Balances of their Resources and Liabilities.

Dr.							*Cr.*			
Cash,	balance on hand.	$12719	48		Bills Payable,	balance outstanding.	$ 1649	30
Bills Receivable,	" " "	3000	00		D. C. Peachy & Son,	bal. in their favor.	16	10
Merchandise,	bal. on hand per Inv.	3530	25		C. Lortie & Bro.,	" " "	3767	25
P. Allard,	balance in our favor.	200	00		G. Quinn & Co.,	" " "	792	22
A. Rinfret,	" " " "	300	00		E. Cary & Son,	" " "	405	46
National Bank,	balance of deposit.	9250	00		C. S. MITCHELL's share of capital,		12122.25	
G. S. Walls,	balance in our favor.	650	40		R. A. HUDON'S " "		12152.75	
M. Blanchet & Co.,	" " " "	1255	20		NET CAPITAL,		24275	00
		$30905	33				$30905	33

Balances of their Losses and Gains.

Dr.							*Cr.*			
Discount and Interest,	loss.	$ 13	00		Merchandise,	gain on sales.	$ 86	25
Shipment in Co.,	loss.	34	00		Shipment to Montreal,	gain.	75	40
Shipment to Three Rivers,	loss.	195	50		Storage and Advertising,	gain.	34	05
Expense,	loss.	45	30		Commission,	gain.	161	65
C. S. MITCHELL's, half net gain,	$152.75				Mdse. Co. A.,	gain.	217	25
R. A. HUDON's, " " "	152.75				Mdse. Co. B.,	gain.	18	70
NET GAIN,		305	50					
		$ 593	30				$ 593	30

QUEBEC, MAY 3, 1871.

INVENTORY of Merchandise advanced by R. A. Hudon, as Capital:—

740 lbs. Lard,	@	$ .10	$ 74.00	
1900 " Ham,	"	.13	247.00	
50 bbls. Apples,	"	3.72	186.00	
60 " Extra Superior Flour	"	5.50	330.00	
46 " Fancy Flour,	"	4.50	207.00	
50 bags, 7000 lbs. Coffee,	"	.15	1050.00	
8 casks Bordeaux Wine,	"	50.00	400.00	
120 bottles Champagne Wine,	"	.80	96.00	
22 gals. Cyprus Wine,	"	5.00	110.00	$2700 00

Quebec, May 1, 1871.

Signed R. A. HUDON.

6

M. H. INVOICE of Flour sent per Grand Trunk R. R., and consigned to Mitchell & Hudon, Quebec, to be sold on our % and risk:

80 bbls. Superfine Flour,	@	$5.00	$ 400.00
70 " Oatmeal,	"	6.10	427.00
45 " Rye Flour,	"	4.25	191.25
			$1018.25

Ottawa, May 4, 1871.

S. WHITE & Co.

7

M. H. INVOICE of Merchandise shipped per Steamboat Champlain, Capt. Ricard, consigned to Mitchell & Hudon, Quebec, pursuant to their order and for their %, viz.:—

50 bbls., 1500 gals. Coal Oil,	@	$ .60	$900.00	
20 " 560 " Linseed Oil,	"	1.00	560.00	
15 " Herrings,	"	5.25	78.75	
			$1538.75	

Charges.

Insurance @ ½ % on $1550,	$7.75		
Drayage,	3.10	10.85	1549 60

Montreal, May 5, 1871.

D. C. PEACHY & SON.

QUEBEC, MAY 14, 1871.

Paid by note, order, and cash.

Quebec, May 14, 1871.

MESSRS. MITCHELL & HUDON,

Bo't of L. McCORD.

1500 bush.	Red Wheat,	@ $.90	$1350.00	
1200 "	Oats,	" .56¼	675.00	
350 "	Peas,	" .80	280.00	
42 tubs, 1846 lbs.	Butter,	" .15	276.00	$2581 90

Received payment,

L. McCORD.

16

SHIPPED per Brig Victoria, consigned to Messrs. Mitchell & Hudon, Quebec, to be sold on joint %, each ½, viz.:—

250 boxes, 5000 lbs.	Soap,	@ $ .07	$ 350	
160 " 4000 "	Chocolate,	" .20	800	
M. H. 309 "	Sperm Candles,	" 8.00	2400	
			$3550	

Halifax, May 7, 1871.

C. LORTIE & BRO.

21

SHIPPED per Brig Vaudreuil, consigned to Messrs. Mitchell & Hudon, Quebec, to be sold on joint % of E. Cary & Son, and themselves, each ⅓, viz.:—

36 bbls., 1441 lbs.	Plums,	@ $ .08	$ 115.28	
90 "	Green Apples,	" 3.60	324.00	
M. H. 175 "	Gray "	" 4.12	721.00	
			$1160.28	

Montreal, May 14, 1871.

G. QUINN & Co.

QUEBEC, MAY 4, 1871.

INVOICE of Merchandise shipped per Steamboat Quebec, Capt. Labelle, consigned to G. S. Walls, Montreal, to be sold on our %, viz. :—

G.S.W.	3 bbls., 620 lbs., Lard,	@ $ .10	$ 62.00	
	26 " Apples,	" 3.50	91.00	
M.	20 bags, 2800 lbs., Coffee,	" .15	420.00	
			$573.00	
	Charges.			
M.&H.	Drayage,		2.00	$ 575 00

MITCHELL & HUDON.

Quebec, May 4, 1871.

10

INVOICE of Merchandise sent per Grand Trunk R. R., and consigned to L. Douglas & Co., pursuant to their order of the 4th inst. and for their %, viz. :—

L. D. & Co.	9 bbls., 1640 lbs., Ham,	@ $ .16	$ 262.40	
	60 " Extra Superior Flour,	" 6.00	360.00	
	22 bags, 3088 lbs., Coffee,	" .20	617.60	
			$1240.00	
	Charges.			
M.&H.	Paid for Drayage,	$ 5.30		
	" " Insurance, @ 2% premium on $1245,	24.90		
	Our Commis. for Ins. and Exped.	2.48	32.68	$1272 68

MITCHELL & HUDON.

Quebec, May 10, 1871.

SALES BOOK,—SET III.

QUEBEC, MAY 19, 1871.

INVOICE of Merchandise shipped per Steamer Cartier, Capt. Roy, and consigned to M. Blanchet & Co., Pictou, to be sold on joint %, each ½.

B. & C.	40 bbls., 1200 gals. Coal Oil,	@ $ .60	$720.00	
	20 " 560 " Linseed Oil, "	1.00	560.00	$1280 00
	Charges.			
	Paid for Drayage,		$2.75	
	" " Insurance @ ½ % on $1290		6.45	9 20
				$1289 20

MITCHELL & HUDON.

Quebec, May 19, 1871.

E. E.

INVOICE of Merchandise per Brig St. Maurice, consigned to J. N. Carbray, Three Rivers, to be sold on our % and risk.

J.N.C.	40 sacks, 160 bu. Red Wheat,	@ $.90	$144.00	
	90 " 360 " Oats,	" .45	162.00	$ 306 00
	Charges.			
	Paid for Drayage, etc.,			4 50
				$ 310 50

MITCHELL & HUDON.

Quebec, May 22, 1871.

E. E.

COMMISSION SALES BOOK,—SET III.

Dr. **S. White & Co.'s Consignment.** *Cr.*

1871					1871				
May	6	To Cash, Paid Freight and Drayage.	$ 10	00	May	11	By Bills Receivable, Sold S. Doran on his note at 40 days, viz. :—		
"	12	" Storage & Advertising, Charges.	18	50			80 bbls. Superfine Flour, at $6, $480		
"	"	" Commission, On $1208, @ 2½ %.	30	20			20 bbls. Oatmeal, at $7.20, 144	$ 624	00
"	"	" S. White & Co., Their net proceeds, due June 4, 1871.	1149	30	"	12	" Cash, Sold C. D. Gibb, 50 bbls. Oatmeal, at $7, $350		
							45 " Rye Flour, at $5.20, 234	584	00
			$1208	00				$ 1208	00

Account Sales rendered May 12.

E. E.

Dr. **Merchandise Co. A.** *Cr.*

1871					1871				
May	16	To C. LORTIE & BRO., Rec'd from C. L. & Bro., Halifax, to be sold on our joint % and risk, each ½, as per S. B., amounting to	$3550	00	May	23	BY BILLS RECEIVABLE, Sold O. Martel, viz.: 250 boxes, 5000 lbs., Soap, at $.08½ $ 425.00		
"	"	" CASH, Paid Freight.	40	00			160 boxes, 4000 lbs., Chocolate, at $.22 880.00		
"	23	" STORAGE & ADVER., Our charges.	12	00			300 boxes, Sperm Candles, at $9.45 2835.00		
"	"	" COMMISSION, 2½ % on $4140.	103	50			$4140.00		
"	"	" C. LORTIE & BRO., Their ½ net gain.	217	25			Rec'd his note, at 10 days, for	$2000	00
"	"	" Loss & GAIN, Our ½ net gain.	217	25	"	"	" CASH, for the Balance	2140	00
			$4140	00				$4140	00

Account Sales rendered May 23.
C. Lortie & Bro.'s net proceeds, $3767.25. Due May 29, 1871.

E. E.

72

Dr. **Merchandise Co. B.** *Cr.*

1871					1871				
May	21	To G. Quinn & Co., Rec'd from G. Q. & Co., Montreal, to be sold on joint ⅔ of themselves, E. Cary & Son, and ourselves, each ⅓, as per S. B., amount, $1160.28,			May	28	By D. C. Peachy & Son, Sold them, at 6 months,		
		Our ⅓ above invoice,	386	76			36 bbls., 1440 lbs. Plums, at $.10,	$144.00	
"	"	" Cash, Paid Freight,	28	10			90 bbls. Green Apples, at $3.80,	342.00	
"	28	" Storage & Adver., Our charges.	3	55			175 bbls. Gray Apples, at $4.50,	787.50	
"	"	" Commission, 2 % on $1273.50	25	47					
"	"	" G. Quinn & Co., Their ⅓ invoice $386.76, and net gain, $18.70.	405	46					
"	"	" E. Cary & Son, Their ⅓ invoice $386.76, and net gain, $18.70.	405	46					
"	"	" Loss & Gain, Our ⅓ net gain.	18	70					
			$1273	50				$1273	50

Account Sales rendered May 28.
G. Quinn & Co.'s net proc., $405.46
E. Cary & Son's " " 405.46

E. E. Due Dec. 5. 1871.

Account Sales of { 80 bbls. Superfine Flour / 45 " Rye " / 70 " Oatmeal } **for account of**

S. White & Co.

1871				
May	11	Sold S. Doran, on his note at 40 days,		
		80 bbls. Superfine Flour, @ $6.00	$480.00	
		20 " Oatmeal, " 7.20	144.00	624 00
"	12	Sold C. Lee, for cash,		
		50 bbls. Oatmeal, @ $7.00	$350.00	
		45 " Rye Flour, " 5.20	234.00	584 00
		Charges.		1208 00
"	6	Paid Freight and Drayage, in cash,	$10.00	
"	12	Storage and Advertising,	18.50	
"	"	Commission, 2½ % on $1208,	30.20	58 70
		S. White & Co.'s net proceeds, due June 4, 1871		$1149 30
		E. E. MITCHELL & HUDON.		
		Quebec, May 12, 1871. *Per J. Madison.*		

Account Sales of Merchandise, on joint % of C. Lortie & Bro., and ourselves, each ½.

1871				
May	23	Sold O. Martel,		
		250 boxes, 5000 lbs. Soap, @ $ .08½	$425	
		160 " 4000 " Chocolate, " .22	880	
		300 " Sperm Candles, " 9.45	2835	4140 00
		Cash, $2140—Note at 15 days, $2000		
		Charges.		
"	16	Paid cash for Freight,	$ 40.00	
"	23	Storage and Advertising,	12.00	
"	"	Commission, 2½ % on $4140,	103.50	
"	"	Our ½ net gain on Sales,	217.25	372 75
		C. Lortie & Bro.'s net proceeds,		$3767 25
		INVOICE,		
		250 boxes, 5000 lbs. Soap, @ $.07	$350.00	
		160 " 4000 " Chocolate, " .20	800.00	
		300 " Sperm Candles, " 8.00	2400.00	
		½ net gain,	217.25	
		Net proceeds as above,	$3767.25	
		E. E. Due by Equation, June 1st.		
		MITCHELL & HUDON,		
		Quebec, May 23, 1871. *Per J. Madison.*		

Sales of Merchandise on joint account of G. Quinn & Co., E. Cary & Son, and ourselves, each ⅓.

1871						
May	28	Sold D. C. Peachy & Son, at 6 months,				
		36 bbls., 1440 lbs. Plums, @ $ .10		$144.00		
		90 " Green Apples, " 3.80		342.00		
		175 " Gray " " 4.50		787.50	1273	50
		Charges.				
"	21	Paid Freight and Drayage,	$ 28.10			
"	28	Storage and Advertising,	3.55			
"	"	Commission, 2 % on $1273.50,	25.47			
"	"	E. Cary & Son's net proceeds,	405.46			
"	"	Our ⅓ net gain,	18.70		481	28
		G. Quinn & Co.'s net proceeds,			$ 792	22
		INVOICE,				
		36 bbls., 1441 lbs., @ $.08		$115.28		
		90 " Green Apples, " 3.60		324.00		
		175 " Gray " " 4.12		721.00	1160.28	
		Your and our ⅔ invoice,		773.52		
		Your ⅓ net gain,		18.70		
		Net proceeds as above,		$792.22		
		E. E. Due by Equation, Dec. 5.				
		Quebec, May 28, 1871. MITCHELL & HUDON.				

CHECK-BOOK,—SET III.

The **Check-Book** is a book of blank-checks, with a margin for memoranda, containing, in brief, the Bank Account. The checks are filled and torn off, leaving the memoranda. By adding deposits and deducting checks, the balance in bank is always apparent. Many houses keep no other bank account than this, in which case, the balance in bank is always included in the Cash Account.

NATIONAL BANK.

1871								
May	8	Deposited	10000	00	No. 1.			
"	14	Check No. 1	750	00	L. McCord.			
			9250	00	For Mdse.			
					May 14, 1871.	750	00	

No. 1. Quebec, May 14, 1871.

NATIONAL BANK.

Pay to L. McCord, or Bearer,

Seven Hundred and Fifty ——— 100 Dollars.

$750. MITCHELL & HUDON.

No. ___ Quebec, _____ 187__.

NATIONAL BANK.

Pay to _____ or Bearer,

_____ 100 Dollars.

$_____ _____

RECEIPTS, NOTES, DRAFTS, &c.

RECEIPTS.

(From transaction of Jan. 31, p. 8.)

Quebec, January 31, 1871.

Received of Mr. A. J. Hall, Thirty Dollars, in full for one month's rent of store, up to date.

E. R. TRUDEL.

(From transaction of Feb. 18, p. 9.)

$250. $\frac{00}{100}$ Quebec, February 18, 1871.

Received of Mr. A. J Hall, his note at sixty days, dated this day, for Two hundred & fifty Dollars.

C. PHELAN.

NOTES.

(From transaction of May 3, p. 58.)

$1500. $\frac{00}{100}$ Quebec, April 25, 1871.

Thirty days after date, I promise to pay to A. J. Hall, or order, Fifteen hundred Dollars; value received.

Due May 28, 1871. P. RACINE.

(From transaction of Feb. 18, p. 9.)

£62 10 0 Quebec, February 18, 1871.

Sixty days from date, I promise to pay to the order of C. Phelan, at my office, Sixty-two pounds ten shillings, old Canadian Currency; value received.

Due April 22, 1871. A. J. HALL.

DRAFTS.

(From transaction of May 18, p. 61.)

$1149. $\frac{30}{100}$ Ottawa, May 15, 1871.

At eight days sight, pay to the order of J. Raymond, Eleven hundred forty-nine $\frac{30}{100}$ Dollars, value received, and charge the same to account of

S. WHITE & Co.

To Mitchell & Hudon, Quebec.

NOTE.—For the *acceptation*, Mitchell & Hudon have written across the following words: "Accepted May 18, 1871," under which they have signed.

(From transaction of May 26, p. 63.)

$115. Three Rivers, May 26, 1871.

At sight, without grace, pay to the order of Mitchell & Hudon, One hundred and fifteen Dollars, value received, and charge to account of

To F. T. Perron, Cashier,
National Bank, Quebec.

J. N. CARBRAY.

BILL OF EXCHANGE.

£36 5 6 Quebec, June 2, 1871.

Fifteen days after sight of this our first of exchange (second and third of same tenor and date, unpaid), pay to the order of Mr. D. Saucier, Thirty-six pounds Five shillings Six pence, value received, and charge to account of

Your obedient servant,

To Simms & Devaux, Bankers,
Wellington street,
London.

C. S. MITCHELL.

LETTER BOOK,—SET III.

REMARKS.—This book is used for taking copies of all business letters of importance, written to or received from others. But letters received are usually filed away.

We give herewith letters in connection with the transactions of Set III. But we do not submit them as absolute models in their way. It would be as difficult to afford a model of a business letter, that is, one which it would be proper for every one to *copy*, as it would for an artist to produce a cast of features that *everybody* would consider perfect.

To be able to write a good business letter is no small accomplishment, nor can it be acquired by studying models, although much aid may be secured in this way, pertaining to form, arrangement, and even style, if undertaken with no undue surrender of individuality; for a good business letter should be neither more nor less than the transcript of a man's thoughts, or what he would say were he to speak with care and deliberation. As no two men ever think or talk exactly alike, so no two men could be expected to write alike.

There are, in business letters, certain qualifications which are equally essential to all, and with reference to which, general instructions may be given. We will enumerate a few of these points:—1st Like all other documents in manuscript, a business letter should be, chirographically, *well written*, so as to commend itself at once to the reader. Neatness and legibility are the chief requisites in a handwriting. 2nd The grammatical construction should be faultless; and, above all, no document should be *disfigured* with misspelled words. 3rd The subject matter should be immediately apparent, stated without circumlocution, and in terms not to be misconstrued. A business document should be written in brief terms, and yet explicitly.

There is no qualification which will more surely commend young men to the favor of an employer than proficiency in Business Correspondence.

(CIRCULAR.)

G. S. Walls, Esq., QUEBEC, May 1, 1871.
Montreal.

SIR:—

We, the subscribers, respectfully announce to you that we have formed a copartnership under the firm of MITCHELL & HUDON, for the prosecution of a wholesale Grocery, Wine Business, and General Commission. We take the liberty of assuring you that all business intrusted to our care, shall receive from us, personally, prompt and faithful attention; in a word, that we will correspond to the confidence placed in us.

Very respectfully,

Your obedient servants,

MITCHELL & HUDON.

Messrs. Mitchell & Hudon, MONTREAL, May 2, 1871.
Quebec.

GENTLEMEN:—In reply to your circular of the 1st inst., I beg leave to solicit the favor of your patronage for a general commission business, and pledge myself for the strict observance of your commands, and faithful performance of my duty.

Respectfully yours,

G. S. WALLS.

G. S. Walls. Esq.,
Montreal.

QUEBEC, May 4, 1871.

SIR:—Enclosed we remit to you Bill of Lading and Invoice of Merchandise, amounting to $575, which we consign to you per Steamboat Quebec, to be sold for our %- You will do us the favor to use all possible despatch in making sales and rendering account.

Yours,

MITCHELL & HUDON.

Messrs. S. White & Co.,
Ottawa.

QUEBEC, May 6, 1871.

GENTLEMEN:—We have the honor of informing you of the arrival, in good order, of your Consignment of Flour, pursuant to our order, and of which, your favor of 4th inst, gave us advice.

We find it conformable to the Invoice, amounting to $1018.25, which we have placed to the credit of your %.

We beg leave to assure you that we will pay all possible attention to your orders. Offering you our sincerest thanks, we remain,

Your obedient servants,

MITCHELL & HUDON.

Messrs. D. C. Peachy & Son,
Montreal.

QUEBEC, May 7, 1871.

GENTS:—We are in receipt of the goods you consigned to us, pursuant to our order of 3rd inst., and of which you gave advice by your favor of 5th inst. Save a few barrels of Herrings whose quality appears to us inferior, the rest is satisfactory.

Your account is credited for the amount of Invoice, $1549.60.

Very respectfully yours,

MITCHELL & HUDON.

Messrs. L. Douglas & Co.,
Toronto.

QUEBEC, May 10, 1871.

GENTS:—Enclosed, please find Invoice of Merchandise amounting to $1272.68, which we forward to you per Grand Trunk R. R., pursuant to your order of 4th inst.

Be so kind as to credit us for the same.

Truly yours,

MITCHELL & HUDON.

Messrs. S. White & Co., QUEBEC, May 12, 1871.
Ottawa.

GENTS :—We send you, enclosed, Account Sales of Merchandise forwarded to us on 4th inst. The net proceeds, due on June 4, is $1149.30.

Hoping to be favored with new orders, and being grateful for those already received, we remain, gentlemen,

Respectfully yours,

MITCHELL & HUDON.

Messrs. Mitchell & Hudon, HALIFAX, May 7, 1871.
Quebec.

GENTLEMEN :—In accordance with the agreement made between us on the 3rd inst., we ship you to day, per Brig Victoria, sundry Merchandise amounting, as per enclosed Invoice, to $3550, to be sold on joint account.

As these goods are said to be in great demand in your city, perhaps you might find it to our advantage to dispose of them at first opportunity. However, knowing your abilities in business, we leave the whole to your sound judgment.

Very respectfully,

C. LORTIE & BRO.

Messrs. Mitchell & Hudon, MONTREAL, May 15, 1871.
Quebec.

GENTS :—Please find enclosed, Account Sales of the Merchandise you shipped me on the 4th inst. Your net proceeds, including Invoice and Gains on Sales, is $650, which I have entered to your credit.

Your obedient servant,

G. S. WALLS.

Messrs. M. Blanchet & Co., QUEBEC, May 19, 1871.
Pictou.

GENTS :—Herein enclosed, you will find Invoice of Merchandise which we ship you per Steamer Cartier, to be sold on joint %. We have debited you for half of the Invoice, $644.60.

Hoping that you may have fair opportunities to sell advantageously, we remain, Gentlemen,

Your humble servants,

MITCHELL & HUDON.

Messrs. Mitchell & Hudon,
Quebec.

MONTREAL, May 14, 1871.

GENTS:—We accept with pleasure your proposition to join in a Company Speculation. We, accordingly, ship you, per Schooner Vaudreuil, which is to sail to morrow, Merchandise, as per enclosed Invoice, to be sold in joint account with yourselves, E. Cary & Son, and ourselves, each one third.

We have debited you for $\frac{1}{3}$ of Invoice.

Wishing you complete success in the sales of them, we beg to subscribe ourselves,

Very truly yours,

G. QUINN & Co.

J. N. Carbray, Esq.,
Three Rivers.

QUEBEC, May 22. 1871.

SIR:—Yours of the 16th inst. is at hand. Your propositions are gratefully accepted. In accordance therewith, we ship you per Schooner St. Maurice, 40 bags Red Wheat. and 90 bags Oats, as per enclosed Invoice, amounting to $310.50, which we consign to you to be sold on our % and risk.

Hoping you will study our best interests, we remain, sir,

Yours respectfully,

MITCHELL & HUDON.

Messrs. C. Lortie & Bro.,
Halifax.

QUEBEC, May 23, 1871.

GENTLEMEN:—We send you enclosed, Account Sales of the Merchandise forwarded on 7th inst. We have been quite successful in the sales of them, and we are of opinion, from actual appearances, that the good market shall continue for sometime. If you think advisable to risk a new consignment, we shall be happy to join you in it, or to sell for you on Commission.

Very respectfully,

MITCHELL & HUDON.

Messrs. Mitchell & Hudon,
Quebec.

PICTOU, May 23, 1871.

GENTS:—Enclosed, please find Account Sales of the Merchandise you shipped us on the 19th inst. Your net proceeds is

$610.60. The season seems favorable for the sale of the like goods. We shall be happy to join you again for any reasonable amount. Please advise us thereupon, and believe us,

Truly yours,

M. BLANCHET & Co.

Messrs. Mitchell & Hudon,
Quebec.

THREE RIVERS, May 24, 1871.

GENTS: — You shall have heard through the newspapers before the receipt of this letter, of the stranding of the Schooner St. Maurice, near Three Rivers, on the 23rd inst. The cargo was saved, but your Consignment on me being rather damaged, I thought prudent to dispose of it at auction without delay. Enclosed you will find Account Sales, and Check on National Bank for $115, as net proceeds.

Waiting for a new Consignment which may prove more successful, I have the honor, Gentlemen, to be,

Yours respectfully

J. N. CARBRAY.

J. N. Carbray, Esq.,
Three Rivers.

QUEBEC, May 26, 1871.

SIR:—Your favor of the 24th inst. was received, and the enclosed Check was honored by the National Bank.

We approve of your proceedings concerning this sale, and beg of you to accept our sincere thanks.

Believe us ever disposed to honor you with our confidence,

Truly yours,

MITCHELL & HUDON.

Messrs. G. Quinn & Co.,
Montreal.

QUEBEC, May 28, 1871.

GENTS:—We send you enclosed, Account Sales of your Invoice of Fruits of 21st inst.

Hoping you will find the result satisfactory, we beg to subscribe ourselves, Gentlemen,

Very truly yours,

MITCHELL & HUDON.

Messrs. L. Douglas & Co.,
Toronto.

QUEBEC, May 30, 1871.

GENTS:—We are in receipt of your favor of the 27th inst., containing a draft at sixty days on A. Simms & Devaux, London, for $1272.68, which is placed to your credit.

Please accept, Gentlemen, the sincere thanks of

Your obedient servants,

MITCHELL & HUDON.

Messrs. D. C. Peachy & Son,
Montreal.

QUEBEC, May 31, 1871.

GENTS:—Enclosed, you will find a Draft at eight days' sight on N. Caron for $260, for which you will please to credit us.

We have the honor, Gentlemen, to remain,

Yours gratefully,

MITCHELL & HUDON..

MEMORANDUM I.

June 1, We, Mitchell & Hudon, continue our business with the Resources and Liabilities taken from our Balance Sheet p. 66.—**2.** Received advice from Douglas & Co., Toronto, that they have purchased, as per agreement, 90 bbls. Extra Flour, to be sold on our joint %, each ½, and that they have debited us for ½ the cost price which, as per bill, amounts to $585.—**3.** Shipped per Brig St. Hubert, and consigned to S. McManus, St. Johns, Newfoundland, to be sold on our % and risk, produce, (S. B.) amtg. to $1864. Passed our note No. 3, at 6 mos., to the North Insurance Co., for ins. on $2040, at 1¼ %, and paid in cash for Policy, $1.25.—Rec'd per Grand Trunk R. R., from L. Dion, Montreal, Bordeaux Wines (I. B.) amounting to $120; and accepted his draft on us, favor Jones & Co., at 20 days' sight, for the amt. of invoice.—**4.** Gave Merchandise (S. B.) in payment of an order from P. Allard, for $369.20.—Exchanged our note No. 5 with E. Cary & Son's, for our mutual accommodation, each drawn at 30 days, for $320; discounted theirs at the National Bank, and rec'd in cash, $318.24. Discount was taken for 33 days, at 6 %. —**5.** Bo't on joint acct. with G. S. Walls, each ½, 5000 lbs. Chocolate, at 25 cts. We are to receive 5 % commission on the sale. Paid in cash for our half, $625.—**6.** Rec'd of S. Lewis, in payment of his note No. 1 for $1000, due this day, Merchandise (I. B.) amtg. to $500, and cash for the bal.—**7.** Rec'd per Brig Columbia, Capt. Russell, from C. A. Molson, Limerick, pursuant to our order and for our %, Mdse. (I. B.) due in Limerick on Aug. 1st next, amounting to $4800. Gave our bonds to the Custom-house for duties, at 3 and 6 mos., for

$1440; paid Freight in cash, $74. Rec'd at the same time by the same Brig Columbia from C. A. Molson, 50 casks Sicily Wine (I. B.), amt. $4400, to be sold on his % and risk. Gave our bonds to the Custom-house for duties at 3 months, for $1606.40; paid Freight in cash, $97.70.—**9**. Sold E. Dunn, on his note at 2 mos., endorsed by F. Laroche, 5 casks Sicily Wine, of C. A. Molson's Consignment (C. S. B.), amtg. to $760.—**10**. O. Martel's note, No. 2, for $2000, is due and not paid.—Shipped per Brig Victoria, consigned to C. Lortie & Bro., Halifax, Merchandise to be sold on % of C. A. Molson's Consignment (C. S. B.), amtg. to $2481. Paid cash for Freight, etc., $21.75.—Shipped at the same time by the same Brig, and consigned to C. Lortie & Bro. to be sold on our %, Merchandise from our Store (S. B.) amtg. to $258.75, and an Invoice of Oats (C. S. B.), bo't of G. Morin, at 2 months, amtg. to $180. Paid cash for Freight, etc. $17.65.—**11**. Paid cash to the Quebec Insurance Co., for ins. on $9600 of our stock, at 1½ %, policy included, $144.—Gave in payment of C. A. Molson's draft on us, at sight, for $3200, favor of Jos. S. Aubert; viz., Merchandise (S. B.) for $600; 5 casks wine of C. A. M's Consignment (C. S. B.), amt. $720; and cash for the bal.—**12**. O. Martel has paid in cash, his note No. 2, in our favor, which was protested, $2000.—Rec'd cash for C. Lortie & Bro., in payment of their draft on Lepage & Garneau, for $1600. Our commission at ½ % is $8. Net proceeds in cash, $1592.—**13**. Bo't of J. B. Walls, for % of N. S. Robertson, and pursuant to his order, shipped per Steamer Prussia, consigned to R. Murray, Portland, Mdse. (C. S. B.), amtg. to $5000 for which we have given our acceptance, at 40 days, due July 26, for $5000. Expenses for loading, etc., amt. to $156. Our commission is $77.33.—**14**. Bo't at auction, on joint account with P. Brady, each ½, Mdse. (I. B.), amtg. to $9700. Our ½ purchase is $4850, for which we gave our acceptance at 30 days, favor of King & Co.—**16**. Paid cash for our note No. 1, favor of L. McCord, for $500, due this day.—**17**. Bo't of Healy & Cameron, the Brig Maria, for $10000, which we paid as follows: our draft at 30 days' sight, on M. Blanchet & Co., for $1200; E. Dunn's note, for $760; and our check on National Bank for the bal.—**18**. Sent per Grand Trunk R. R., to S. White & Co., Ottawa, pursuant to their order, Mdse. (C. S. B.), of C. A. M's Consignment, amtg. to $1360. Rec'd in payment, our note No. 2, in their favor, for $1149.30, and their note at 40 days, for the bal.—**19**. Paid cash for repairing Brig Maria, $33.10.—**20**. Shipped per Brig Maria, and consigned to C. A. Molson, Limerick, to be sold on joint % of himself, H. Brook, and ourselves, each ⅓, 10 T. White Sugar from Mdse. Co. E., amtg. to $4850. Paid cash for loading, etc., $20.60; gave our note at 2 mos., for ins. on $5000, at ½ %. Our commission on $4870.60, at 2½ %, is $121.76½; our commission for ins., at ¼ %, is $12.50. We charged for the Freight by our Brig Maria, $80. H. Brook's ⅓ is $1703.28⅝; C. A. Molson's ⅓, $1703.28⅝; and ours, $1703.28⅜.—**21**. Insured our Brig Maria in the Gulf Insurance Co., for $10000, at ½ %, together with $1 policy, which we have paid in cash, $51.—**23**. Rec'd an Account Sales from L. Douglas & Co., of the Flour bought by them on the 2nd inst., on joint %; the proceeds amounting to $897.70. Our ½ is $448,85, and our net gain, $156.35.

—**24**. Rec'd of S. McManus, St. Johns, Newfoundland, Account Sales of the Mdse. consigned to him by Brig St. Hubert. Net proceeds amtg. to $2120. Rec'd in payment an Invoice of Fish (I. B.), amtg. to $2120. Paid for Freight, and other expenses, in cash, $61.34. Closed our Invoice to St. Johns with a gain of $229.25.—**25**. Paid cash for our acceptance of L. Dion's draft, favor of Jones & Co., for $120.—**26**. Sold J. Merault 4000 lbs. Chocolate from Mdse. Co. D, at 35 cts. Rec'd cash, $800; the bal. at 2 mos.—Sold R. Woods, on his note at 30 days, 1000 lbs. Chocolate from Mdse. Co. D., at 40 cts. —**27**. Closed Mdse. Co. D., and rendered G. S. Walls an Account Sales of the same. Our charges for Storage, Advertising, etc., $23.60; our Commission, 5 % on $1800. G. S. Walls' net gain, $218.20. Our ½ net gain, $.....—**28**. Rec'd from C. Lortie & Bro., Halifax, Account Sales of the Mdse. shipped them from C. A. Molson's Consignment. Net proceeds, $2962 for which we rec'd their draft, at 60 days, on Hamel & Bros., which was accepted.—Taken to our %, at 2 mos., the remaining 10 casks, of C. A. Molson's Consignment, at $128. Closed C. A. Molson's Consignment, and rendered him an Account Sales of the same. The expenses for Duty, etc., to this day, amt. to $1725.85. Our Commission on Sales, at 5 %, is $354.10; Storage and Advertising, $12. Net proceeds due C. A. Molson, on ....., $4990.05.—**30**, Rec'd from C. Lortie & Bro. an Account Sales of our shipment of the 10th inst. by Brig Victoria. Net proceeds, $843. Rec'd also a draft from them, at 10 days' sight on Garneau & Co. Paid cash for clerk hire and other expenses, $104.75.

BALANCE ACCOUNT, JUNE 30.

RESOURCES.			LIABILITIES.		
Bills Receivable.	$ 4415	70	Bills Payable.	$13266	90
Cash.	14185	58	G. S. Walls.	192	80
National Bank.	1210	00	D. C. Peachy & Son.	16	10
Merchandise.	12000	00	C. Lortie & Bro.	5359	25
P. Allard.	569	20	E. Cary & Son.	405	46
A. Rinfret.	300	00	G. Quinn & Co.	792	22
M. Blanchet & Co.	55	20	C. A. Molson.	4886	76½
L. Douglas & Co.	156	35	G. Morin.	180	00
N. S. Robertson.	5233	33	*C. S. Mitchell.*	13500	97½
H. Brook.	1703	28⅝	*R. A. Hudon.*	13531	47½
Brig Maria.	10000	00			
Sh'pt to Limerick.	1703	28⅝			
J. Merault.	600	00			
	$52131	93¾		$52131	93¾

MEMORANDUM II.

June 1st, F. Belmont & L. Moore, this day entered into co-partnership. each investing $15000. F. Belmont's investment is on hand. L. Moore's investment is deposited in the Quebec Bank.—**3.** Bo't of F. Belmont his store for $18750. In payment, assumed mortgage on the property for $13500, and Interest due on mortgage to date $126; paid cash for the bal. $5124.—Bo't of Fremont & Co. on %, 10 hhds. Sugar, 11250 lbs. at 8 cts.; 200 bbls. Thin Mess, at $13.50; 15000 lbs. Cheese, at 15 cts.—Paid cash for carpenter's work and painting, $112.50.—**4.** Received from Benning & Son, Toronto, to be sold on our joint %, and risk, each ½, 600 bbls. Extra Flour, at $6; 150 bbls. Prime Pork, at $14; paid freight on same, in cash, $150.—Rec'd from J. Arnold & Bro., Kingston, to be sold on our joint % and risk, each ½, 5000 bush. Western Wheat, at $1.20.—**5.** Sold for cash to H. Merrill, 300 bbls. Extra Flour, (Mdse. Co., A,) at $7.—**6.** Sold. N. Harris, on his note at 30 days, 300 bbls. Extra Flour, (Mdse. Co. A.,) at $7.50; 150 bbls. Prime Pork, (Mdse. Co. A.,) at $18.—Closed company sales with Benning & Son, and rendered them an account of the same. Our charges for Storage & Advertising, $15; Commission 2½ % on sales $....; our ½ net gain, $504.37; Benning & Son's do. $504.38.—Sent to J. Arnold & Bro. to be sold on our joint %, each ½, the following merchandise, bo't on our note, at 90 days, of Jordan & Sewell, 22500 lbs. Maple Sugar, at 8 cts.; 200 tubs Butter, 14000 lbs., at 15 cts.; 150 bbls. Green Apples, at $3; paid insurance ½ % on $4500, in cash, $22.50.—**8.** Sold H. Gregory, at 30 days, 5000 bush. Western Wheat, (Mdse. Co. B.,) at $1.35.—Closed sales in company with J. Arnold & Bro., Kingston, and rendered them an Account Sales. Our charges for Storage and Advertising, $30; Commission on Sales, 2½ %. J. Arnold & Bro's. ½ net gain, $275.62; ours do., $275.63.—**9.** Sold B. A. Chalmers, 15000 lbs. Cheese, at 18 cts. Rec'd in payment his note at 30 days, for $1500; and Cash, for the balance.—Accepted Benning & Son's draft on us at 30 days sight, favor G. Ross & Co., in full of their %.—Rec'd per Steamboat Frontenac, Montreal, from J. D. Roe, to be sold on joint % of himself, P. E. Onslow, Sorel, and ourselves, each ⅓, as per contract, 850 bbls. Mess Pork, at $15; paid freight, in cash, $300. Our and J. D. Roe's ⅓ invoice, $8500; P. E. Onslow's ⅓ do. $4250.—**12.** Bo't of Nelson & Co., ¼ Steamboat Europa, for $15000. Gave in payment, cash, $7500, and our note at 90 days for the bal.—**13.** Sold L. J. Nolan, Quebec, 850 bbls. Mess Pork, (Mdse. Co. C.,) at $17.65. Rec'd in payment his note at 40 days, for $10500, and cash for the bal.—Closed Mdse. Co. C., and rendered J. D. Roe and P. E. Onslow each an account of the Sales. Our charges for Storage and Advertising, $47.44; our Commission 2½ % on sales, $375.06. J. D. Roe's ⅓ net gain, $510: P. E. Onslow's ⅓ net gain, $510; our ⅓ net gain $510.—**15.** Sold W. J. Lyons, for cash, 10 hhds. Sugar, 11250 lbs., at 12 cts.—**16.** Bo't of Jordan & Sewell, 30 hhds. Brown Sugar, 30000 lbs., at 9 cts.; paid in cash, $1200; bal. on %.—**18.** Paid

cash as follows: for clerk hire to 15th inst., $75 ; to L. Moore, on %., $225.—**20**. B. A. Chalmers has, this day, discounted his note in our favor, du. July 12th ; proceeds of the note $1493.60 ; discount off, for 22 days, $6.40.—Rec'd of J. Arnold & Bro., an Account Sales of the Mdse. sent them on the 7th inst, to be sold on our joint %. Our ½ net gain, $450.—**22**. Shipped P. E. Onslow, Sorel, to be sold on joint % of himself, J. D. Roe, Montreal, and ourselves, each ⅓, 30 hhds. Brown Sugar, 30000 lbs., at 9¾ cts. ; paid freight in cash, on same $75.—**25**. F. Belmont has drawn on the Quebec Bank, for personal expense, $300.—Paid J. Arnold & Bro's draft on us, favor of C. Russell, per check on Quebec Bank, for $1453.12.—**28**. Rec'd cash for rent of part of our Store $2250. Per statement rendered this day, our share of earnings of last trip of Steamboat Europa, amts. to $375.—**29**. Paid cash for sundry expenses to date, $238.50.—**30**. Rec'd from P. E. Onslow, Accouut Sales of the Sugar shipped him on the 22nd inst. Our ⅓ net loss, $172.50. F. Belmont has this day invested in the firm, in cash, $4033.35.—**July 1st**. Rec'd from C. R. Kerney, Halifax to be sold on his and our joint %, each ½, 150 bbls. Mackerel, invoiced at $7 ; 40 bbls. Herrings, invoiced at $4.50 ; 75 bbls. Linseed Oil, invoiced at $40 ; paid Freight per check on Quebec Bank, $75. Deposited cash in the Quebec Bank, $12750.—**2**. Shipped P. Gilmour & Co., St. John, N. B., to be sold on our joint %, each ½, 200 bbls. Thin Mess, at $13.50 ; paid Drayage, in cash, $27.—**3**. Sold R. S. Venner, for cash, 150 bbls. Mackerel, (Mdse. Co. D.,) at $7.50. Effected insurance for $7500, at ¾ %, on any property that may be in our warehouse, $56.25.—**4**. Shipped J. O'Regan & Co., Montreal, as per their order, at 60 days, the following Merchandise ; 75 bbls. Linseed Oil, (Mdse. Co. D.,) at $45 ; 40 bbls. Herrings, (Mdse. Co. D.,) at $4.50.—Closed Mdse. Co. D., and rendered C. R. Kerney an Account Sales of the same. Our charges for Storage, Advertising & Insurance, $75 ; our Commission, 2½ % on Sales $.....C. R. Kerney for his ½ Invoice, $2115, and net gain, $91.50. Our ½ net gain, $....—**5**. Paid by check on Quebec Bank, advertising bills of Morning Chronicle, $225.—**6**. Rec'd from Kane & Joly, Hamilton, to be sold on joint % of themselves, A. C. Miller, and ourselves, each ⅓, 150 hhds. Brown Sugar, invoiced at $60 ; paid Freight per check on Quebec Bank, $750.—**7**. Rec'd from B. Masson & Co., Sandwich, to be sold on theirs and our joint acct., each ½, 500 bbls. Prime Pork, at $13.50 ; 250 bbls. Lard, 50000 lbs., at 7½ cts. ; paid Freight per check on Quebec Bank, $750.—**8**. Sold J. N. Miles, Quebec, 150 hhds. Brown Sugar, (Mdse. Co. E.,) at $75. Rec'd in payment, J. Mountain & Co's. note, dated March 3, 1871, due one day after date, for $7500 ; due to date on said note $185.20 ; and cash for balance.—Closed Mdse. Co. E., and rendered Kane & Joly and A. C. Miller, each an account of the sales. Our charges for Storage, Advertising, etc., $75 ; our Commission 2½ % on sales, $....Kane & Joly's net proceeds $3381.25 ; A. C. Miller's, $3381.25 ; our net gain, $381.25.—**9**. Rec'd cash for N. Harris' note, due this day, $4950.—**10**. Deposited cash in the Quebec Bank, $6000.—**12**.

Gave our check on Quebec Bank, in payment on mortgage, favor of T. S. Poston, for $6986.25. The amt. applied on mortgage, is $6750; the interest in full to date, $236.25.—**13**. Paid our acceptance, favor of Benning & Son, due this day, in cash, $6204.38. Accepted Fremont & Co.'s draft on us payable at 10 days sight, for $5850.—**14**. Sold H. Carroll, for cash, 500 bbls. Prime Pork, (Mdse. Co. F..,) at $14.25.—**15**. Rec'd advice from J. O'Regan & Co., Montreal, of an error in an Account Sales of last year's business, in which we were credited too little by $412.50. Sold our draft on J. O'Regan & Co., at 30 days sight for which we received cash, $3900.07; disct. off $67.43.—**17**. Sold J. B. Lewis, for cash, 250 bbls. Lard, (Mdse. Co. F.,) at 6 cts.—Closed Mdse. Co. F., and rendered B. Masson & Co., Sandwich, an Account Sales of the same. Our charges for Storage, Cooperage, etc., $75; our Commission 2½ % on sales, $.... B. Masson & Co.'s ½ invoice, $5250, less ½ net loss, $726.57. Remitted B. Masson & Co. cash in full of acct., $9773.43. Rec'd cash for L. I. Nolan's note of the 13th June, $10500.—**23**. Deposited cash in Quebec Bank, $15000.—**25**. Rec'd from P. Gilmour & Co. an Account Sales of 200 bbls. Thin Mess, shipped them on the 2nd inst. Our net proceeds, $1200.—**26**. Accepted C. R. Kerney's note on us at 30 days sight, favor of P. McGinn, for amt. due him, $4321.50.—**27**. Paid our acceptance favor of Fremont & Co., due this day, in cash, $5850.—**28**. Rec'd cash of H. Gregory, in full of acct.—**30**. Paid Cash to Jordan & Sewell, to bal. acct.—**31**. Passed to F. Belmont's Stock acct., the amt. charged to his Private acct. Passed to Stock acct. of L. Moore, the amt. charged to his Private acct. Allowed 7 % on F. Belmont's additional investment of June 30.

INVENTORY, JULY 31.

Store, valued @		$22500.00
Steamboat Europa Stock		15000.00
Interest due us on Notes	$213.95	192.58
Less, interest due from us	21.37	

BALANCE ACCOUNT, JULY 31.

Resources.		Liabilities.	
Real Estate.	$22500 00	Mortgage Payable.	$ 6750 00
Cash.	660 76	Bills Payable.	16171 50
Quebec Bank.	38210 63	Interest Payable.	21 37
Bills Receivable.	7500 00	J. D. Roe.	8010 00
Interest Receivable.	213 95	P. E. Onslow.	2932 50
Steamb Europa Stock.	15000 00	Kane & Joly.	6381 25
Steamboat Europa.	375 00	A. C. Miller.	3381 25
P. Gilmour & Co.	2563 50	F. Belmont.	23678 92
		L. Moore.	19697 05
	$87023 84		$87023 84

SET IV.

JOBBING AND IMPORTING BUSINESS,

EMBRACING AS PRINCIPAL BOOKS,

CASH BOOK, DOMESTIC AND FOREIGN INVOICE BOOKS, SALES BOOK AND JOURNAL;

AND AS AUXILIARIES,

INVENTORY BOOK AND BILL BOOK.

WITH A ROUTINE TAKEN FROM AN EXTENSIVE BUSINESS HOUSE.

REMARKS.—The particular feature of this set consists in the manner and form of original entries, which are made in separate books,—elsewhere used as auxiliaries,—from which they are either journalized, or passed directly to the Ledger at stated periods. This method has many advantages over consecutive entries in the Day Book, and, in one form or other, is adopted generally in all large establishments. The labors of the Book-keeper are thus divided up, and the separate departments of the business receive such special record as to present all the facts in their clearest light. Thus, if any particular information is desired respecting purchases, all the facts can be found at once in the Invoice Book; in the same manner, the facts and condition of the sales can be found in the Sales Book; the receipts and disbursements of cash, in the Cash Book, etc.

In the previous sets, these books are represented; but they are used only as auxiliaries, the entries of the business being made in the other books without reference to them. This plan, it will be evident, although possessing some merits, involves a large amount of unnecessary labor, which would prove a great objection in extensive houses. The special books themselves, however, are so essential in every well-regulated business, that they would receive favor, even at the expense of this additional labor. If, therefore,

they can be used without additional labor, and even at a reduction of labor, it would seem that no farther argument would be needed to secure their adoption.

The Invoice Books, and the Sales Book which follow, contain all purchases and sales of merchandise, with the conditions thereof. All sales and purchases *not made for cash* pass from these books to the Ledger; all others, from the Cash Book.

ROUTINE FOR AUGUST 1871.

NOTE.—To get the full advantage of this Set, the student must pay strict attention to the routine of transactions as indicated below. Let him make the entries in the different books precisely in the order given. Let the invoices be copied with care, and all the calculations and extensions made by himself. The exercises connected with the Foreign Invoice Book, involving compound numbers, and reduction of currencies, are highly essential. In reducing sterling to Canadian decimal currency, we have taken $4.84 to the £ sterling. We have also added to each foreign invoice, the duties, which being paid in cash, are not extended with the invoice, but posted from the Cash Book. However, in many importing houses, the duties are not extended in the Invoice Book, but only in the Cash Book.

1. The books are opened, as per first Journal entry. (Inventory Book copied—Cash Book commenced with balance on hand—Bill Book written up to show the notes, receivable and payable, as indicated in Journal entry.)....Bo't of S. B. Madden for cash, Invoice of Prints, etc., $1303.76 (Dom. I. B.—C. B.)....Sold L. Beaudry, St. Thomas, on his note at 6 mos., Inv. of Goods, $699.06, (S. B.—B.B.) ....Sold Mdse. this day, per Petty Cash Book, $97.50 (C. B.).... Rec'd cash on acct. of S. R. Boyce, $600, (C. B.)—**3**. Paid A. Miller for Blank Books $15.25, (C. B.)....Lent J. E. Lawrence, for one day, $800, (C. B.).—**5**. Received of J. E. Lawrence, return loan, $800, (C. B.)....Paid cash for Postage Stamps, etc., $7.50, (C. B.).... Paid J. A. Hall, on private acct., $80, (C. B.)—**6**. Rec'd cash of S. R. Boyce, in full of acct., $1440, (C. B.)—**7**. Paid R. P. Davis cash, in full of acct., $175, (C. B.)—**8**. Sold Mdse. per Petty Cash Book, $120, (C. B.)—**10**. Bo't of P. McHugh & Co., on our note at 6 mos., Invoice of Cotton, $1006.64, (Dom. I. B.—B. B.) ...Received per steamer St. David, from J. Duffy & Co., Liverpool, Invoice of Cotton and Silk Goods, $221.11; Paid duties in cash, $32.09, (For. I. B.—C. B.)—**11**. Bo't of N. Casey & Bro., on our note at 8 months, Invoice of Bleached Shirtings, $660.73, (Dom. I. B.—B. B.)—**12**. Sold F. Peters & Co., Three Rivers, on note at 6 mos., Invoice of Hose and Gloves, $273.52, (S. B.—B. B.)....Paid cash for Drayage, Porterage, etc., $25, (C. B.)—**14**. Sold Hazel & Foy, l'Islet, on their note at 8 months, Invoice of Shirting, $869.38, (S. B.—B. B.).... Rec'd cash in full of J. N. Galt's note, $1264, (C. B.—B. B.)—**15**. Received per Steamer Nestorian, from C. Shields & Co., Manchester, Invoice of Goods, $246.36; Paid duties in cash, $36.95, (For. I. B.—C. B.)....Sold Mdse. for cash, as per Petty Cash Book, $110.50, (C. B.)—**18**. S. I. Perron's note discounted, $800; Discount off,

$15.60, (C. B.—B. B.)—**20**. Sold Stein & Co., St. Mary, P. Q., on their note, at 6 months, Invoice of Prints, $1425.48, (S. B.—B. B.) ....Paid T. J. Colston on private acct., $100, (C. B.)—**21**. Sold Mdse. for cash, per Petty Cash Book, $102.50, (C. B.)—**23**. Paid cash in full of note, favor of G. H. Shills, $3800, (C. B.—B. B.)—**25**. Sold Byrne & Son, Kamouraska, for cash, Invoice of Goods, $400, (S. B.—C. B.)....Stein & Co.'s note discounted; Face of note, $1425.48. Discount off, $50.44, (C. B.—B. B. ...Rec'd per steamer Asia, from J. A. Knight, Dublin, Invoice of Goods, $440.14; Paid duties in cash, $105.63, (For. I. B.—C. B.)....Bo't of L. Power & Co., for cash, Invoice of Prints, $893.63, (D. I. B.—C. B.) ...Paid clerk hire in cash, $50, (C. B.)—**27**. Sold Mdse. for cash, as per Petty Cash Book, $160, (C. B.)—**28**. Sold C. E. Lawson, Sorel, on his note at 8 months, Invoice of Goods, $171.04, (S. B.—B. B.)—**29**. Paid C. S. Mitchell, on private acct., $130, (C. B.)—**30**. Sold Mdse. as per Petty Cash Book, $72, (C. B.)—**31**. Received cash of W. E. Gray, in full of acct., $1480.20.

ROUTINE FOR SEPTEMBER 1871.

1. Sold A. M. Rooney & Co., on their note at 6 months, Invoice of Goods, $1432.89, (S. B.—B. B.)....Paid cash for Drayage and Porterage, $17.50, (C. B.)—**2**. Lent L. Morgan, $600, (C. B.)—**3**. Sold Mdse. as per Petty Cash Book, $70.20, (C. B.)—**5**. Discounted our note, favor of A. G. Cook; face of note $1500. Discount off, $29.75, (C. B.—B. B.)....Sold S. D. Higgins, Quebec, on his note at 8 mos., Invoice of Goods, $527, (S. B.—B. B.)—**6**. A. M. Rooney & Co.'s note discounted; face of note $1432.89. Discount off, $49.60, (C. B.)—**7**. Sold Mdse. as per Petty Cash Book, $150, (C. B.)—**8**. Sold J. F. Nestor, St. Thomas, on his note at 8 months, Invoice of Mdse. $752.57, (S. B.—B. B.)—**10**. Rec'd per steamer Africa, Glasgow, Invoice of Goods, $1453.19. Duties paid in cash, $276.10, (For. I. B.—C. B.)—**12**. Sold Mdse. as per Petty Cash Book, $218.50, (C. B.)....Paid cash for Drayage, $75, (C. B.)—**15**. Sold S. R. Woods, Ottawa, on his note at 8 months, Invoice of Goods, $908.29, (S. B.—B. B.)....Paid R. A. Hudon cash on private acct., $140, (C. B.)—**17**. Sold Mdse. as per Petty Cash Book, $362.40, (C. B.) —**20**. Rec'd per steamer St. Patrick, from J. Bailey & Son, Liverpool, Invoice of Goods, $188.62. Paid duties in cash, $28.29, (For. I. B.—C. B.) ...Bought of Bell & Archer, on our note at 6 months, Invoice of Cloths, $1926.14, (Dom. I. B.—B. B.)....Sold N. B. Roy, Levis, for cash, Mdse., $923.40, (S. B.—C. B.)—**23**. Sold Mdse. as per Petty Cash Book, $180, (C. B.)—**25**. Sold E. Curran, Richmond, for cash, Invoice of Gloves, $460.75, (S. B.—C. B.)... Paid Postage, Porterage, etc., in cash, $12, (C. B.)—**27**. Sold Lee & Strang, Toronto, on their note at 8 mos., Invoice of Mixtures, $3303.71, (S. B. —B. B.)—**28**. Sold T. Ross & Co., Kingston, on 8 months note, Invoice of Goods, $578.52, (S. B.—B. B.)—**30**. Sold A. R. Jacob, Batiscan, on note at 3 months, Invoice of Goods, $100, (S. B.—B. B.) ....Sold Mdse. per Petty Cash Book, $125, (C. B.)....Paid cash in full of Drayage acct., $20.75, (C. B.)

DOMESTIC INVOICE BOOK,—SET IV.

This book contains copies of all invoices of merchandise purchased from importers and others in this country, with the conditions of all such purchases. Each lot and package is distinguished by some peculiar mark, which is transferred to the invoice, thus serving an important purpose in checking the articles, adjusting disputes, etc.

The purchases on time, for which notes are given, are passed to the Ledger from this book; those for cash, from the Cash Book.

QUEBEC, AUGUST 1, 1871.

		S. B. MADDEN.				
		5 Cases Toronto Prints,				
A. T.	596	2137[2]				
A. S.	587	1913[3]				
X.	351	1935				
	341	1886[3]				
A. C.	411	1742–9615 yds. at 8 cts. less ½ ct.	721.13			
		1 Bale 2¼ lbs. Black Wadding,				
		30 yds. at 40 cts.	12.00			
	1141	2 Bales Brown Globe Drills,				
	1147	1032[3]				
		1029 2061[3] at 7⅝ cts.	157.21			
M.	481	4 Cases Harop Prints,				
C.	491	246 1332				
M.	509	1262 1581[2]				
M.	97	1108 1222				
		1276 1715[2] 5851 at 8½ cts. less 2½ %	482.91			
			1373.25			
		Less 5 %	68.67			
			1304.58			
		M. 97—1276, short 34 yds.	2.82			
			1301.76			
		Add error in extend'g Harop Prints	2.00			
		C. B.	1303.76	1303	76	
		Amount forward,		1303	76	

QUEBEC, AUGUST 10, 1871.

			Amount forward,			1303	76	
	C.	P. McHUGH & Co., (6 months.)						
	794	40 ps. Duck Drilling,	1411²	@ 17 c.	239.96			
	800	40 " " "	1389²	" 18	250.11			
	834	36 " Brown "	1415²	" 25	353.88			
	704	42 " W. B. Diaper,	2169¹	" 7½	162.69			1006 64
		Note at 6 mos. from Aug. 10.						
		11						
		N. CASEY & BRO. (8 months.)						
		4 cases 4.4 Bleached Shirting,						
	750	40	1732²					
	751	40	1736					
	753	40	1755					
	754	40	1731²	6955 yds. at 9½ cts.				660 73
		Note at 8 months from Aug. 11.						
		25						
		L. POWER & Co.,						
C. M.	3892	52 Prints,	973	1858				
E. A.	5788	52 "	965	1834				
E. A.	6202	53 "	967	1895¹	5587¹ yd. @ 8 c. 446.98			
D. C.	4187	51 "	972	1924²				
E. N.	5630	49 "	968	1954²				
E. O.	5685	49 "	971	1929	5808 yd. @ 8½ c. 493.68			
					940.66			
		Discount off 5 %			47.03			
		C. B.			893.63	893	63	
		Purchases on time (taken to Ledger),						1667 37
		Cash purchases (entered from C. B.),						2197 39
		Total for the month,						3864 76

QUEBEC, SEPTEMBER 20, 1871.

BELL & ARCHER, (6 months.)

1877	30	Belgian Cassimere,	1091			
1881	30	"	1178[2]			
1896	30	"	1137			
1915	28	"	1094[2]			
1903	30	"	1151[2]			
1737	33	World's Fair,	1268			
1775	30	"	1168			
1823	33	"	1279[3]			
1834	33	"	1261[2]			
1845	30	"	1147			
1906	30	Sebastopol Checks,	1168[3]			
1913	27	"	1245[2]=14191 yds. @ 13½c.	$1915.78		
		Cooperage		3.00		
				1918.78		
		Add 77 yds. at 13½ cts.	$10.39			
		Less Freight,	3.03	7.36	1926	14
		Note at 6 months.				
		Total purchase for the mo. (taken to Ledger.)			1926	14

FOREIGN INVOICE BOOK,—SET IV.

QUEBEC, AUGUST 10, 1871.

			Sterling Currency.						Canadian Currency.			
			£	s.	d.	£	s.	d.	$	c.	$	c.
B.&Co.		J. DUFFY & Co., Liverpool, per Steamer "St. David," July 4, 1871.										
	71	25 doz. Ladies' White Cotton Hose, @ 2s. 9d.	3	8	9							
		20 " " " 3s.	3									
		29 " " " 2s. 10d.	4	2	2							
	46	1 " Ladies' Pearl Spun Silk Hose, 8½, @ 20s.	1									
	46	1 " " " 9 20s.	1									
	1	4 " black " 9 19s.	3	16								
	6	2 " pearl " 9 53s.	5	6								
	6	½ " Men's Nov. Silk Shirts, @ 39s.		19	6							
	62	1 " " " 50s.	2	10								
	120	1 " " " 55s.	2	15								
	66	1 " " " 62s.	3	2								
	68	8 " Ladies' Lisle Gauntlets, 11s. 6d.	4	12								
	100	5 " " " 12s. 6d.	3	2	6							
	101	5 " " " 22s.	5	10								
		103½ " Drayage,		2	10							
		Cases, Oil Cloth, etc.		13	6							
			45	0	3							
		Commission 1½ %		13	5	45	13	8			221	11
		Duties, 15 %							32	09		
		C. B. Paid in Cash,										
		Amounts forward,							32	09	221	11

			Sterling Currency. £	s.	d.	£	s.	d.	Canadian Currency. $	c.	$	c.
		Amounts forward,							32	09	221	11
C. S. & Co.		C. SHIELDS & Co., Manchester, Per Steamer "Nestorian," July 15, 1871.										
	54	60¾ Lavellas assorted, 3028² @ 3⅞ d.	48	17	11							
		Making up, Casing, etc.	2	6	10							
			51	4	9							
		Discount 1¼ %		12	9							
			50	12								
		Drayage		6		50	18				246	36
		C. B. Duties 15 %—Paid in Cash							36	95		
		25										
C. S. & Co.		J. A. KNIGHT, Dublin, per Steamer "Asia," July 20, 1871.										
520	70	400 9-8 14⁰⁰ Robes a Les, 10 yds. each, 4s. 6d.	90	0	0							
		Off 36 ps. No. 1, 9s., and 1½ % £1 7s.	1	16	0							
			88	4	0							
		Card 4s., Box 17s., Com. etc., £1 12s.	2	13	0	90	17	0				
		Cartage					1	9				
						90	18	9			440	14
		Amounts forward,							69	04	907	61

FOREIGN INVOICE BOOK,—SET IV.

QUEBEC, AUGUST 25. 1871.

			Sterling Currency. £	s.	d.	£	s.	d.	Canadian Currency. $	c.	$	c.
		Amounts forward,							69	04	907	61
		C. B. Duties 24 %—Paid in cash							105	63		
		Total imports for the month							174	67	907	61
		Duties on the same									174	67
		September 10									1082	28
H. M. & Co.		W. MEADE & Co., Glasgow, per Steamer "Africa," Aug. 1, 1871.										
	29	60¾ Mohair Mixtures, 2996 @ 3⅛d.	39	0	2							
		Case, Pkg., etc., 14s.; Com. 19s. 6d.	1	13	6	40	13	8				
	30	60¾ Bye Cloths assorted, 2969, @ 3d.	37	2	3							
		Case, Pkg., etc., 13s.; Com. 18s. 6d.	1	11	6	38	13	9				
	31	60¾ Bye Cloths assorted, 2973, @ 3d.	37	3	3							
		Case, Pkg., etc., 13s.; Com. 18s. 7d.	1	11	7	38	14	10				
	32	60¾ Bye Cloths assorted, 2965, @ 3d.	37	1	3							
		Case, Pkg., etc., 13s.; Com. 18s. 6d.	1	11	6	38	12	9				
	33	60¾ Bye Cloths assorted, 2966, @ 3d.	37	1	6							
		Case, Pkg., etc., 13s.; Com. 18s. 6d.	1	11	6	38	13	0				
		Amount forward,				195	8	0				

			£	s.	d.	£	s.	d.	$	c.	$	c.
		Amount forward,				195	8	0				
	34	60¾ Bye Cloths assorted, 2965², @ 3d.	37	1	4							
		Case, Pkg., etc., 13s.; Com. 18s. 5d.	1	11	6	38	12	10				
	35	60¾ Fine Mohair Mixtures, 2877², @ 5½d.	65	18	10							
		Case, Pkg., etc., 12s. 6d.; Com. 32s. 11d.	2	5	5	68	4	3				
						302	5	1				
		1¼ % discount				3	15	7				
						298	9	6				
		Carriage, etc.,				1	15	5				
						300	4	11			1453	19
		C. B. Duties 19 %—Paid in Cash							276	10		
		20										
H. M. & Co.		J. Bailey & Son, Liverpool, per Steamer "St. Patrick," Aug. 10.										
	674	25 ps. White Piques, 324 @ 2s. 4d.	37	16	0							
		Discount off		9	2							
			37	6	10							
		Charges, Commission, etc.,	1	12	7	38	19	5			188	62
		C. B. Duties 15 %—Paid in cash,							28	29		
		Total imports for the month,									1641	81
		Duties on same,									304	39
		Total costs of imports.									1946	20

SALES BOOK.

This book contains all the regular sales, either for cash or on time; the cash sales being extended in the inner column, are, of course, not included in the amount for which merchandise is credited from the Sales Book. These sales, together with the petty sales not entered on the Sales Book, are posted from the Cash Book. The total credit of the merchandise account for the month will agree, in amount, with the monthly recapitulation in the Sales Book.

QUEBEC, AUGUST 1, 1871.

		L. BEAUDRY, St. Thomas, P. Q.		
R.	192	1 case Black Velvet, 796 yds. at 36 cts.	$286.56	
		10 pcs. Fancy Cassimeres, 275 yds. at 70 cts.	192.50	
	1289	110 Robes, at $2	220.00	699 06
		Note at 3 months from Aug. 1.		
		12		
F. P. & Co.		F. Peters & Co. Three Rivers.		
	71	25 doz. Ladies' White Cotton Hose, @$1	$25.00	
		20 " " " 1.25	25.00	
		29 " " " 1.13	32.77	
	46	1 " Pearl Spun Silk Hose, 8¼	8.00	
	46	1 " " " 9	8.00	
	1	4 " Black " 9	$7.50 30.00	
	6	2 " Pearl "	20.00 40.00	
	68	8 " Ladies' Lisle Gauntlets,	4.50 36.00	
	100	5 " " "	4.75 23.75	
	101	5 " " "	9.00 45.00	273 52
		Note at 6 mos. from Aug. 12.		
		Amount forward,		972 58

QUEBEC, AUGUST 14, 1871.

			Amount forward,				972	58
H. & F.		HAZEL & FOY, L'Islet.						
		4 Cases Bleached Shirting.						
		750 40 1732[1]						
		751 40 1736						
		753 1745[1]						
		754 40 1741[2] 6955 yds. @ 12½ cts.					869	38
		Note at 8 mos. from Aug. 14.						
		20						
		STEIN & Co., St. Mary, P. Q.						
C. M.	3892	52 Prints, 973 1858						
E. A.	5788	52 " 965 1834						
E. A.	6202	53 " 967 1895 5587yd. @12 c. 670.44						
D. C.	4187	51 " 972 1924[2]						
E. N.	5630	49 " 968 1954						
E. O.	5685	49 " 971 1929[2] 5808yd. @13 c. 755.04					1425	48
		Note at 6 months.						
		25						
		BYRNE & SON, Kamouraska.						
520	70	200 9-8 14[00] Robes a Les (10 yds.) @ $2			400	00		
		Received Cash.						
		26						
		C. E. LAWSON, Sorel.						
674	14	6 White Piques, 119[1] @ 75 cts. $89.44						
	16	2 " 37[3] " 80 " 30.20						
	17	4 " 64[1] " 80 " 51.40					171	04
		Note at 8 mos. from date.						
		Sales on time,					3438	48
		Sales for Cash, entered herein but posted from C. B.,			400	00		
		Petty sales, entered alone on C. B.,			662	50	1062	50
		Total sales for the month,					4500	98

QUEBEC, SEPTEMBER 1, 1871.

		A. M. ROONEY & Co., Montreal.			
H.M.		1 bale Brown Sheeting, 563² yards at 14 cts.	$ 78.89		
	62	50 doz. Gent's Linen Hdkfs., at $5	250.00		
R.X.		1 case Cotton Damask, 540 yds., at 20 cts.	108.00		
	231	16 pcs. Black Bombasin, 558 yds. at $1.25	710.00		
		1 case Silecia, 2200 yds. at 13 c.	286.00	1432	89
		Note at 6 mos.			
		5			
		S. D. HIGGINS, Quebec.			
	231	10 pcs. Black Bombasin, 350 yards, at $1.10	$385.00		
	19	20 pcs. Duck, 710 yds. at 20 cts.	142.00	527	00
		Note at 8 mos.			
		8			
		J. F. NESTOR, St. Thomas.			
		1 bale Stark Brown Sheetings, 829 yds. at 10 cts.	$ 82.90		
	130	1 bale 4-4 Shaker Flannel, 337² yds. at 50 cts.	168.75		
		12 pcs. Green Veil Barege, 200 yards, at 35 cts.	70.00		
	1066	1 case Solid Check Ginghams, 2394 yds. at 18 cts.	430.92	752	57
		Note at 8 mos.			
		15			
		S. R. WOODS, Ottawa.			
		4 cases Harcp Prints,			
M.	481	246 1332			
C.	491	1262 1581²			
M.	509	1108 1222			
M.	97	1276 1715² 5851 yds. @ 12 cts.	$702.12		
		2 bales Brown Globe Drills,			
		1141 1032³			
		1147 1029 2061³ yds. @ 10 cts.	206.17	908	29
		Note at 8 mos.			
		20			
		N. B. ROY, Levis.			
		9 cases Cotton Damask, 4860 yards at 20 cts.	$972.00		
		5 % off,	48.60	923 40	
		Received cash.			
		Amounts forward,		923 40	3620 75

SALES BOOK,—SET IV.

QUEBEC, SEPTEMBER 25, 1871.

			Amounts forward,			923	40	3620	75
		ED. CURRAN,			Richmond.				
		90 doz. Ladies' Lisle Gauntlets,		@ $5	$450.00				
		5 " " Kid Gloves,		$7	35.00				
					485.00				
		5 % off,			24.25	460	75		
		Received cash.							
		27							
L. & S.		LEE & STRANG,			Toronto.				
	29	60¾ ps. Mohair Mixtures,	2996	@ 15 c.	$449.40				
	30	60¾ Bye Cloths assorted,	2969	14 c.	415.66				
	31	60¾ " "	2973	14 c.	416.22				
	32	60¾ " "	2965	14 c.	415.10				
	33	60¾ " "	2966	14 c.	415.24				
	34	60¾ " "	2965²	14 c.	415.17				
	35	60¾ ps. Fine M. Mixtures,	2877²	27 c.	776.92			3303	71
		Note at 8 months.							
		28							
		T. Ross & Co.,			Kingston.				
674	18	4 ps. White Piques,	75¹	@ $1.00	$75.25				
	19	1 "	15	1.25	18.75				
	54	60¾ ps. Lavellas,	3028¹	.16	484.52			578	52
		Note at 8 months.							
		30							
		A. R. JACOB,			Batiscan.				
	6	½ doz. Men's Novi Silk Shirts,		@ $20	$10				
	62	1 " " "			25 25				
	120	1 " " "			30 30				
	66	1 " " "			35 35			100	00
		Note at 3 months.							
		Sales on time,						7602	98
		Sales for cash, entered here and posted from C. B.				1384	15		
		Petty sales entered alone on C. B.				1106	10	2490	25
		Total sales for the month.						10093	23

This is the most convenient form for a Cash Book to be kept in connection with a general merchandise business; the feature of *special columns* may be extended, if desirable. It will be seen that *all* cash entries, debit and credit, are taken to the Ledger, either through the Journal or directly, from this book, together with all accounts producing or costing cash. The amounts distinguished as "per petty Cash Book," are entered here from a

Dr. **Cash.**

					Mdse.		*Sundr.*		*Bal.*	
1871										
Aug	1			*Amount on hand,*					7380	50
"	1		MDSE.,	Sales, per Petty C.-Book.	97	50				
"	1	√	S. R. BOYCE,	Rec'd on acct.			600	00		
"	5	√	LOAN,	Return from J. E. Lawrence.			800	00		
"	5	√	S. R. BOYCE,	Rec'd in full of acct.			1440	00		
"	8		MDSE.,	Sales, per Petty C.-Book.	120	00				
"	14	√	BILLS REC'BLE,	J. N. Galt's note due.			1264	00		
"	15		MDSE.,	Sales, per Petty C.-Book.	110	50				
"	18	√	BILLS REC'BLE,	S. I. Perron's note disc'td.			800	00		
"	21		MDSE.,	Sales, per Petty C.-Book.	102	50				
"	25		MDSE.,	Sold Byrne & Son, (S. B.)	400	00				
"	25	√	BILLS REC'BLE,	Stein & Co.'s note disc'td.			1425	48		
"	27		MDSE.,	Sales, per Petty C.-Book.	160	00				
"	30		MDSE.,	Sales, per Petty C.-Book.	72	00				
"	31	√	W. E. GRAY,	Rec'd in full of acct.			1480	20		
					1062	50	7809	68		
				Mdse. Sales for Cash			1062	50		
				Total Cash rec'd during the month			8872	18	8872	18
									16252	68

book containing sales too insignificant to be entered on the regular Sales Book. The column headed "Balances," will be found very convenient for the purposes for which it is used. The Check-marks, in the column following dates, are made to indicate that the amounts opposite in the "Sundries" column have been journalized. Were these amounts posted directly to the Ledger, the Ledger-page would be written instead of the Check-marks.

Cash. *Cr.*

					Mdse.		*Sundries.*	
1871								
Aug	1		MDSE.,	Madden's Invoice, per Dom. I. B.	1303	76		
"	3	√	EXPENSE,	Paid A. Miller's bill			15	25
"	3	√	LOAN,	Lent J. E. Lawrence for one day			800	00
"	5	√	EXPENSE,	Postage Stamps, $2; Drayage, $5.50			7	50
"	5	√	J. A. HALL,	Paid him on Private acct.			80	00
"	7	√	R. P. DAVIS,	Paid him in full of acct.			175	00
"	10		MDSE.,	Duties, as per Foreign I. B.	32	09		
"	12	√	EXPENSE,	Drayage and Porterage			25	00
"	15		MDSE.,	Duties, as per Foreign I. B.	36	95		
"	18	√	INTEREST,	Discount on Perron's note			15	60
"	20	√	T. J. COLSTON,	On Private acct.			100	00
"	23	√	BILLS PAYABLE,	Note favor G. H. Shills due			3800	00
"	25		MDSE.,	Duties, per Foreign I. B.	105	63		
"	25		MDSE.,	L. Power & Co.'s Invoice, per Dom. I. B.	893	63		
"	25	√	EXPENSE,	Clerk hire, $30; $20			50	00
"	25	√	INTEREST,	Discount on Stein & Co.'s note			50	44
"	29	√	C. S. MITCHELL,	On private acct.			139	00
					2372	06	5248	79
				Mdse. purchased for Cash			2372	06
				Total Cash paid out during the month			7620	85
				Balance on hand			8631	83
							16252	68

Dr. **Cash**

					Mdse.		*Sundr.*		*Bal.*	
1871										
Sept	1			*Balance on hand,*					8631	83
"	3		Mdse.,	Sales, per Petty C.-Book	70	20				
"	5	√	Interest,	Disc't on note fav. A. G. C.			29	75		
"	6	√	Bills Rec'ble,	Disc't A. M. Rooney & Co.'s note			1432	89		
"	7		Mdse.,	Sales, per Petty C.-Book	150	00				
"	12		Mdse.,	Sales, per Petty C.-Book	218	50				
"	17		Mdse.,	Sales, per Petty C.-Book	362	40				
"	20		Mdse.,	Sold N. B. Roy, per S. B.	923	40				
"	23		Mdse.,	Sales, per Petty C.-Book	180	00				
"	25		Mdse.,	Sold E. Curran, per S. B.	460	75				
"	30		Mdse.,	Sales, per Petty C.-Book	125	00				
					2490	25	1462	64		
				Mdse. Sales for Cash			2490	25	3952	89
				Total Cash rec'd during the month			3952	89	12584	72

					Mdse.		Sundries.	
1871								
Sept	1	√	EXPENSE,	Paid Drayage, $2; Porterage $8.50			17	50
"	2	√	LOAN,	Lent L. Morgan			600	00
"	5	√	BILLS PAYABLE,	Disc'td Note favor A. G. Cook			1500	00
"	6	√	INTEREST,	Discount on A. M. R. & Co.'s note			49	60
"	10		MDSE.,	Duties, as per For. I. B.	276	10		
"	12	√	EXPENSE,	Paid Drayage, on acct.			75	00
"	15	√	R. A. HUDON,	Paid on private acct.			140	00
"	20		MDSE.,	Duties, as per For. I. B.	28	29		
"	25	√	EXPENSE,	Postage, $4; Porterage, $5; alms, $3			12	00
"	30	√	EXPENSE,	Paid Drayage in full			20	75
					304	39	2414	85
				Mdse. purchased for Cash			304	39
				Total Cash paid for the month			2719	24
				Balance on hand			9865	48
							12584	72

The Bill Book can never, with advantage, be made a *principal* book, from which to post. The form presented below is the best

Bills

No.	When Rec'd.	Drawer or Endorser.	Drawee or Maker.
1	Aug. 1	W. H. Ellison.	J. N. Galt.
2	" 1	D. Atkinson.	S. I. Perron.
3	" 1	H. M. & Co.	L. Beaudry.
4	" 12	"	F. Peters & Co.
5	" 14	"	Hazel & Foy.
6	" 20	J. R. East.	Stein & Co.
7	" 28	H. M. & Co.	C. E. Lawson.
8	Sept. 1	"	A. M. Rooney & Co.
9	" 5	"	S. D. Higgins.
10	" 8	"	J. F. Nestor.
11	" 15	J. O. Moss.	S. R. Woods.
12	" 27	H. M. & Co.	Lee & Strang.
13	" 28	"	T. Ross & Co.
14	" 30	"	A. R. Jacob.

Bills

No.	When Issued.	Drawer or Endorser.	Drawee or Maker.
1	Feb. 20	G. H. Shills.	H. M. & Co.
2	April 1	S. A. Pugh.	"
3	May 12	A. G. Cook.	"
4	Aug. 10	P. McHugh & Co.	"
5	" 11	N. Casey & Bro.	"
6	Sept.	Bell & Archer.	"

for general purposes, although the arrangement in the former example is more comprehensive.

Receivable.

Date.	Time.	When Due.	Amount.	When and How disposed of.
1871		1871		
Feb. 11	6 mo.	Aug. 14	1264 00	Aug. 14 Paid.
April 12	8 mo.	Dec. 15	800 00	" 18 Discounted.
Aug. 1	3 mo.	Nov. 4	699 06	
" 12	6 mo.	Feb. 15	273 52	
" 14	8 mo.	April 17	869 38	
" 20	6 mo.	Feb. 23	1425 48	" 25 Discounted.
" 28	8 mo.	May 1	171 04	
Sept. 1	6 mo.	March 4	1432 89	" 6 Discounted.
" 5	8 mo.	May 8	527 00	
" 8	8 mo.	" 11	752 57	
" 15	8 mo.	" 18	908 29	
" 27	8 mo.	" 30	3303 71	
" 28	8 mo.	" 31	578 52	
" 30	3 mo.	Jan. 2	100 00	

Payable.

Date.	Time.	When Due.	Amount.	When and How redeemed.
		1871		
Feb. 20	6 mo.	Aug. 23	3800 00	Aug. 23 Paid.
Mar. 26	8 mo.	Nov. 29	3200 00	
May 12	6 mo.	" 15	1500 00	Sept. 5 Discounted.
Aug. 10	6 mo.	Feb. 13	1006 64	
" 11	8 mo.	April 14	660 73	
Sept. 20	6 mo.	Mar. 23	1926 14	

INVENTORY BOOK.

This book is used to enumerate the different articles of unsold merchandise, at such times as may be deemed desirable. It is, in this instance, purely an auxiliary, the amount of merchandise on hand being included in the opening journal entry. Inventories are frequently copied into one of the Invoice Books; but a separate book is preferable.

Mdse. on hand, August 1, 1871.

Marks.	Nos.		Yds.	Price.	Amount.
H. M.		1 bale Brown Sheetings	563^2	.11	61 98
R	192	1 case Black Velvet	796	$.26^3$	212 93
L. B.	1	1 case Paper Cambrics	2000	$.06^1$	125 00
	8	21 pairs White Blankets		3.43	72 03
		41 pcs. Black and White Tweeds	1369^2	.25	342 37
		21 " Fancy Cassimeres	576^1	.60	345 75
	197	17 " Black Satinet	469	$.52^2$	246 22
V. F.		1 case Woolen Shawls	60	4.89	293 40
	231	26 pcs. Black Bombasin	900	$.87^2$	787 50
	19	37 " Duck Canvas	1392	.15	208 80
		2 bales Black Wadding....doz.	80	.22	17 60
	1289	110 Robes		1.50	165 00
A. B.		1 case Cottonades	687^1	$.22^2$	154 63
R. & X.		10 cases Cotton Damask	5400	.16	864 00
	62	150 doz. Gent's Linen Hdkfs		4.50	675 00
	M.	150 pcs. Diaper		.90	135 00
		$50\frac{7}{8}$ Play Linens	1931^2	.17	328 35
	190	1 case Black Alpacas	910	$.27^2$	250 25
N. A.	4	1 " Opera Flannel	750	$.37^2$	281 25
		100 doz. Men's Gloves		2.50	250 00
		140 " Ladies' Lisle Gauntlets		4.46	624 40
		5 " " Kid Gloves		6.25	31 25
		1 bale Stark Brown Sheetings	829	$.08^2$	70 46
	130	1 " 4–4 Shaker Flannel	337^2	.45	151 88
		12 pcs. Green Veil Barege	200	.29	58 00
	1066	1 case Solid Check Ginghams	2394	.14	335 16
		25 pcs. Coburgs	525	.50	262 50
		1 case Silecia	2200	$.09^1$	203 50
	4595	1 " Linseys	1266^2	$.17^2$	221 64
		1 " Corset Jeans	1725^1	.06	103 51
B. S.		1 " Delaines	1300	.25	325 00
	3024	1 " D. Bege	864	.11	95 04
					8299 40

QUEBEC, AUGUST 1, 1871.

SUNDRIES		TO SUNDRIES.		
	Resources and Liabilities of A. J. Hall, C. S. Mitchell, and R. A. Hudon, partners in the firm of "Hall, Mitchell & Co.," doing a general Jobbing and Importing business in the City of Quebec; as taken from the Balance Sheet of their last Ledger:			
CASH	Amt. on hand, per C. Book	$7380.50		
MERCHANDISE	" " Inv. "	8299.40		
BILLS REC'BLE	Notes on hand, per Bill "	2064.00		
STORE RENT	Advance payment for rent	1600.00		
S. R. BOYCE	Balance of account	2040.00		
W. E. GRAY	" "	1480.20		
		22864.10		
TO BILLS PAYABLE	Notes outstanding, per B. B.	8500.00		
" R. P. DAVIS	Balance of account	175.00		
" A. J. HALL	Net Investment	4729.70		
" C. S. MITCHELL	" "	4729.70		
" R. A. HUDON	" "	4729.70	22864	10
	31			
MERCHANDISE		TO SUNDRIES.	2574	98
TO BILLS PAYABLE	For the following Invoices per Dom. I. B.: From P. McHugh & Co., Aug. 10	1006.64		
	From N. Casey & Bro. Aug. 11	660.73		
		1667.37		
" J. DUFFY & CO.	Inv. of Aug. 10, per For. I. B.	221.11		
" C. SHIELDS & CO.	" " 15, "	246.36		
" J. A. KNIGHT	" " 25, "	440.14	2574	98

QUEBEC, AUGUST 31, 1871.

BILLS RECEIVABLE	TO MERCHANDISE.	3438	48
	Sales for the month, per Sales Book:		
	L. Beaudry, Aug. 1, $699.06		
	F. Peters & Co., " 12, 273.52		
	Hazel & Foy, " 14, 869.38		
	Stein & Co., " 20, 1425.48		
	C. E. Lawson, " 28, 171.04	3438	48
	"		
CASH	TO SUNDRIES.	8872	18
	Receipts per Cash Book:		
TO MDSE.	Total Sales for Cash 1062.50		
" S. R. BOYCE	Rec'd on acct., $600; $1440 2040.00		
" LOAN	Return from J. E. Lawrence 800.00		
" BILLS REC'BLE	Received on Notes, $1264; $800; $1425.48 3489.48		
" W. E. GRAY	In full of acct. 1480.20	8872	18
	"		
SUNDRIES	TO CASH.	7620	85
	Disbursements per Cash Book:		
MDSE.	Purchases, etc., for Cash 2372.06		
EXPENSE	As per Items, $15.25; $7.50; $25; $50 97.75		
LOAN	Lent J. E. Lawrence 800.00		
A. J. HALL	Paid on private acct. 80.00		
R. P. DAVIS	In full of acct. 175.00		
INTEREST	Per Items, $15.60; $50.44 66.04		
T. J. COLSTON	Paid on private acct. 100.00		
BILLS PAYABLE	Redeemed note favor G. H. Shills 3800.00		
C. S. MITCHELL	Paid on private acct. 130.00	7620	85

QUEBEC, SEPTEMBER 30, 1871.

MDSE.		TO SUNDRIES.	3567	95
TO BILLS PAYABLE	Invoice per Dom. I. B. From Bell & Archer, Sept. 20	1926.14		
" W. MEADE & CO.	Inv. of Sept. 10, per F. I. B.	1453.19		
" J. BAILEY & SON	" " 20, "	188.62	3567	95
	"			
BILLS RECEIVABLE		TO MDSE.	7602	98
	Sales for the month, as per S. B.:			
	A. M. Rooney & Co., Sept. 1,	1432.89		
	S. D. Higgins " 5,	527.00		
	J. F. Nestor " 8,	752.57		
	S. R. Woods " 15,	908.29		
	Lee & Strang " 27,	3303.71		
	T. Ross & Co. " 28,	578.52		
	A. R. Jacob " 30,	100.00	7602	98
	"			
CASH		TO SUNDRIES.	3952	89
	Receipts, per Cash Book:			
TO MDSE.	Total Sales for Cash	2490.25		
" INTEREST	Disc't on note fav. A. G. C.	29.75		
" BILLS REC'BLE	Rec'd on notes	1432.89	3952	89
	"			
SUNDRIES		TO CASH.	2719	24
	Disbursements, per Cash Book:			
MDSE.	Purchases, etc., for Cash	304.39		
EXPENSE	Per Items, $17.50; $75; $12; $20.75	125.25		
LOAN	Lent L. Morgan	600.00		
BILLS PAYABLE	Disc'td note fav. A. G. Cook	1500.00		
INTEREST	Disc't on A. M. R. & Co.'s note	49.60		
R. A. HUDON	Paid on private acct.	140.00	2719	24

Dr. **Balances of their Resources**

	Resources.		
CASH,	Balance on hand.	$9865	48
MDSE.,	Balance on hand, per Inventory.	7816	61
STORE RENT,	Advance payment.	1600	00
BILLS REC'BLE,	Balance on hand.	8183	09
LOAN,	Balance due them.	600	00
T. J. COLSTON,	Balance in their favor.	100	00
		$28165	18

Dr. **Balances of their Losses**

	Losses.			
EXPENSE,	Loss.		$ 223	00
INTEREST,	Loss.		85	89
A. J. HALL'S	⅓ net gain,	$1661.05		
C. S. MITCHELL'S	⅓ net gain,	1661.05		
R. A. HUDON'S	⅓ net gain,	1661.05		
	NET GAIN,		4983	15
			$5292	04

BALANCE SHEET, SET IV.

and Liabilities. *Cr.*

	Liabilities.			
BILLS PAYABLE,	Balance outstanding.		$ 6793	51
J. DUFFY & Co.,	Balance in their favor.		221	11
C. SHIELDS & Co.,	Balance in their favor.		246	36
J. A. KNIGHT,	Balance in his favor.		440	14
W. MEADE & Co.,	Balance in their favor.		1453	19
J. BAILEY & SON,	Balance in their favor.		188	62
A. J. HALL'S,	Share of capital,	$6310.75		
C. S. MITCHELL'S,	Share of capital,	6260.75		
R. A. HUDON'S,	Share of capital,	6250.75		
	NET CAPITAL.		18822	25
			$28165	18

and Gains. *Cr.*

Gains.			
MERCHANDISE,		$ 5292	04
		$ 5292	04

BOOK-KEEPING

BY

SINGLE ENTRY.

REMARKS.

Though we have introduced Double Entry Book-keeping before Single Entry, yet, we admit that books may be kept by single entry by those unacquainted with the principles of double entry; but the mere keeping of accounts is not all that is required. We gave the precedence to the method by double entry, as it is conceded to be greatly superior to that by single entry. In fact, the simplest settlement of Partnership accounts involves the principles of double entry; and, if the commonest English education includes a knowledge of Arithmetic, Mensuration, and even of Algebra and Geometry, it ought surely to include a knowledge of accounts sufficient to make a partnership settlement between two mechanics.

The following set in Single Entry Book-keeping, though short, exhibits such a variety of transactions as is necessary to an illustration of it.

The principles of Single Entry are so easy of comprehension as scarcely to need explanation. Accounts are kept only for *persons*, who alone have accounts in a "Ledger," and become debtors and creditors as they owe us or we owe them.

The principal books of entry are a "Day Book" and a "Ledger." Besides these, there are other books termed "Auxiliaries," varying, as in Double Entry, in number and form according to the business.

All transactions requiring a debit or credit to any person with whom you have dealings, are entered in the Day Book. The form of entry is very simple, thus: "Paul O'Neil Dr. To 5 yds. Linen @ 25 cts.," or "Peter Howard Cr. By Cash on %, $8.00;" in every case specifying the details which constitute the debit or credit. This is the only book from which posts are made to the Ledger.

REMARK.—The Day Book being derived from the "Auxiliary Books," might simply give the abridgement of the transactions. As the Cash Book is the only auxiliary one we open in this short single entry set, we write in the Day Book the details, that would otherwise be shown in the omitted auxiliaries.

QUEBEC, JULY 5, 1871. 1

1	C. I. LANE, St. Mary, *Cr.*		
	By 45 bbls. Extra Flour, @ $6.25	$ 281	25
	8		
1	J. N. KIROUAC, *Cr.*		
	By 60 barrels Rye Flour, @ $5.30, and at 3 months' credit	318	00
	10		
1	R. J. VINCENT, Levis, *Cr.*		
	By 700 lbs. Butter, @ 15 cents	105	00
	Dr.		
	To my note of this day @ 1 month in full of %	105	00
	11		
2	L. P. CLARK *Dr.*		
	To Cash lent to him	200	00
	"		
1	C. I. LANE *Dr.*		
	To Cash on %	81	25
	12		
2	G. N. ROLLAND, Montreal, *Dr.*		
	To Freight & Cartage of 30 bbls. Green Apples, @ $4, received from him to be sold for his %	6	50

2	S. FRASER & Co. *Cr.*		
	By 60 bbls. Oatmeal, @ $6.30		$ 378 00
	Dr.		
	To Cash on %	$ 50.00	
	" my note of this day, at 2 mos., for	120.00	170 00
	13		
1	C. I. LANE, *Dr.*		
	To my note at 30 days, for	$180.00	
	" Cash in full of %	20.00	200 00
	15		
3	J. Gleason, *Cr.*		
	By 18 bags Java Coffee, 1044 lbs. net, @ 16 cts., received per steamer Florida		167 04
	16		
2	L. P. CLARK, *Cr.*		
	By Cash, in full for loan of 11th inst.	$200.00	
	" " lent him this day	75.00	275 00
	17		
3	S. J. PIERCE, *Dr.*		
	To 12 bags Java Coffee, 696 lbs., @ 20 cts.		139 20
	18		
2	L. P. CLARK, *Dr.*		
	To Cash, in full for loan of 16th inst.		75 00
	20		
3	S. J. PIERCE, *Cr.*		
	By his note, at 40 days, on % for his purchase of 17th inst., for	$100.00	
	" Cash in full of %	39.20	139 20
	21		
3	A. T. HUGHES *Dr.*		
	To 50 bbls. Rye Flour, @ $6.10		305 00

2	S. FRASER & Co. *Dr.*		
	To my note in their favor, at 40 days, drawn for the balance of their acct., $208, with interest at 6 %,	$210 08	
	Less interest added	2.08	208 00
	23		
3	A. T. HUGHES, *Cr.*		
	By Cash on %		120 00
	25		
1	C. I. LANE, *Cr.*		
	By 40 bbls. Fancy Flour, @ $7		280 00
	26		
2	G. N. ROLLAND, *Cr.*		
	By 30 bbls. Green Apples, sold for his % for Cash, per Account Sales rendered him		180 00
	Dr.		
	To Storage on 30 bbls.	$1.00	
	" Commission on $180, at 4 %	7.20	8 20
	27		
1	C. I. LANE, *Dr.*		
	To his Draft on us, at 10 days sight, in favor of L. Water, accepted		200 00
	28		
3	A. T. HUGHES, *Cr.*		
	By his Draft at 8 days sight, on J. Delorme, accepted		105 00
	30		
2	G. N. ROLLAND, *Dr.*		
	To 700 lbs. Butter, @ 18 cts.	$126.00	
	" Cash in full of %	39.30	165 30

CASH BOOK,—SINGLE ENTRY.

1871			*Dr.* $	c.	*Cr.* $	c.
July	1	Amount of Cash on hand at commencing business	3000	00		
"	4	Paid for 300 bu. Wheat, @ 72 cts.			216	00
"	6	" for Freight and other expenses			2	60
"	11	" L. P. Clark as a loan			200	00
"	"	" C. I. Lane on %			81	25
"	12	" for Freight and Drayage on an Invoice of apples from G. N. Rolland			6	50
"	"	" S. Fraser & Co. on %			50	00
"	13	" C. I. Lane on %			20	00
"	15	" for Freight and Cartage			3	40
"	16	Rec'd of L. P. Clark, for loan of 11th, inst.	200	00		
"	"	" of L. P. Clark as loan	75	00		
"	17	" for sale of 12 bbls. extra Flour, @ $7	84	00		
"	"	Paid for Advertising, etc.			2	50
"	18	" L. P. Clark for loan of 16th			75	00
"	20	Rec'd of S. J. Pierce, for balance of %	39	20		
"	22	Paid for Stationery			6	30
"	23	Rec'd of A. T. Hughes on %	120	00		
"	26	" for Sales of Rolland's apples	180	00		
"	"	Paid G. N. Rolland in full of %			39	30
"	31	*Balance on hand*			2995	35
			3698	20	3698	20
Aug	1	Balance from July 31, 1871.	2995	35		

Dr. **C. I. Lane.** **Cr.**

1871						1871					
July	11	To Cash	1	$ 81	25	July	5	By Mdse.	1	$ 281	25
"	13	" My Note at 30 days	2	180	00	"	25	"	3	280	00
"	"	" Cash	2	20	00						
"	27	" his Draft on myself	3	200	00						
"	31	" *Balance*		80	00						
				$ 561	25					$ 561	25
						Aug.	1	By Balance		$ 80	00

Dr. **J. N. Kirouac.** **Cr.**

1871						1871					
July	31	*To Balance*		$ 318	00	July	8	By Mdse.	1	$ 318	00
						Aug.	1	By Balance		$ 318	00

Dr. **R. J. Vincent.** **Cr.**

1871						1871					
July	10	To my Note at 1 month	1	$ 105	00	July	10	By Mdse.	1	$ 105	00

Dr. **L. P. Clark.** **Cr.**

1871						1871					
July	11	To Cash	1	$ 200	00	July	16	By Cash	2	$ 275	00
"	18	" "	2	75	00						
				$ 275	00					$ 275	00

Dr. **G. N. Rolland.** **Cr.**

1871						1871					
July	12	To Freight and Cartage	1	$ 6	50	July	26	By am't of Sales	3	$ 180	00
"	26	" Storage	3	1	00						
"	"	" Commission	3	7	20						
"	30	" Mdse	3	126	00						
"	"	" Cash	3	39	30						
				$ 180	00					$ 180	00

Dr. **S. Fraser & Co.** **Cr.**

1871						1871					
July	12	To Cash	2	$ 50	00	July	12	By Mdse.	2	$ 378	00
"	"	" my Note at 2 months	2	120	00						
"	22	" " " at 40 days	2	208	00						
				$ 378	00					$ 378	00

Dr. **J. Gleason.** **Cr.**

1871						1871					
July	31	*To Balance*		$ 167	00	July	15	By Mdse.	2	$ 167	04
								By Balance		$ 167	04

Dr. **S. J. Pierce.** **Cr.**

1871						1871					
July	17	To Mdse.	2	$ 139	20	July	20	By his Note at 10 days	2	$ 100	00
						"	"	" Cash	2	39	20
				$ 139	20					$ 139	20

Dr. **A. T. Hughes.** **Cr.**

1871						1871					
July	21	To Mdse.	2	$ 305	00	July	23	By Cash	3	$ 120	00
						"	28	" his Draft on Gleason	3	105	00
						"	31	" *Balance*		80	00
				$ 305	00					$ 305	00
Aug.	1	To Balance		$ 80	00						

STATEMENT

SHOWING THE CONDITION OF THE BUSINESS

On the 31st of July.

	$	c.	$	c.
Resources.				
1. *From Ledger Accounts.*—Balance due by				
A. T. Hughes	80	00		
2. *From Cash Book.*—Balance of Cash on hand	2995	35		
3. *From Bill Book.*				
S. J. Pierce's Note, due September 1st	100	00		
A. T. Hughes' Draft, due August 8	105	00		
4. *From Inventory.*—Merchandise on hand.	1188	93	4469	28
Liabilities.				
1. *From Ledger Accounts.*—Balances due to				
C. I. Lane	80	00		
J. N. Kirouac	318	00		
J. Gleason	167	04		
2. *From Bill Book.*				
Note favor R. J, Vincent, due Aug. 12	105	00		
" " S. Fraser & Co., " Sept. 13	120	00		
" " C. I. Lane " Aug. 15	180	00		
" " S. Fraser & Co., " Sept. 1st	210	08	1380	12
Present worth or net capital			3089	16
My capital at commencing business was			3000	00
Net gain realized July 31st			89	16

CHANGING SINGLE, TO DOUBLE ENTRY.

Let the Books of the preceding Set in Single Entry, be changed to Double Entry.

Preparatory Statement.

	$	c.	$	c.
Statement of the Resources and Liabilities taken from page 124.				
Resources.				
Personal Accounts Receivable (already posted)	80	00		
Cash, per Cash Book	2995	35		
Notes Receivable on hand, per Bill Book	205	00		
Merchandise, per Inventory	1188	93	4469	28
Liabilities.				
Personal Accounts Payable (already posted)	565	04		
Bills Payable, outstanding, per Bill Book	815	08		
Capital at commencing business	3000	00	4380	12
Net gain realized in business			89	16

From this schedule, it will be evident that when the above resources and liabilities are duly entered upon the Ledger, they will lack just the amount of net gain ($89.16) of balancing. Opening in the Ledger an account on my name, I enter to its credit, my capital at commencing business, and the net gain, $89.16, as properly belonging to it. I shall have the commencement of a Double Entry Ledger.

I now complete this process by opening accounts in the extended Ledger with the additional resources: Cash, Bills Receivable, Merchandise, and Bills Payable. These new accounts constitute the "change," and will serve, very clearly, to show the exact difference in the two systems, so far as the Ledger is concerned.

Dr. **D. R. Marshall.** *Cr.*

					1871 July	1	By Capital	$3000	00
					"	31	" Net gain	89	16

Dr. **Cash.** *Cr.*

1871 July	31	On hand	$2995	35					

Dr. **Bills Receivable.** *Cr.*

1871 July	31	Notes on hand	$ 205	00					

Dr. **Merchandise.** *Cr.*

1871 July	31	On hand	$1188	93					

Dr. **Bills Payable.** *Cr.*

					1871 July		Notes outstanding	$ 815	08

PRACTICAL EXERCISES IN SINGLE ENTRY.

MEMORANDUM I.

August 1st, 1871, I, B. N. Lindsay, commence business with a capital, in Cash, of $500.—Paid in Cash for Paints, Oils, Glass, Brushes, etc., $105.32.—Lettering Tin Sign for H. Young, $1.75, and sold him 1 Light Stained Glass, $5.—**2**. Paid in cash for 2 Books Gold Leaf, 75 cts., for making Step Ladders, $5, for Muslin, 37 cts.—Bronzing Candelabra for J. Wilson & Co., $2.—**3**. Received of T. Morin for Glazing 27 Lights 10 by 12, at 25 cents, in cash $6.75.—Bronzing of a Valve for L. S. Rogers, 50 cents, and sold him 6 lbs. Blue Paint at 25 cents, $1.50; 18 lbs. White Paint at 12½ cents.—**4**. Paid in cash as follows: to W. Boyd, 3 days' Work at $1.50; to H. Teakle, 2 days' Work at 75 cts.—The Grand Trunk R. R. Co. owes me, viz.: for Lettering Sign 60 feet, at 20 cts. per foot; for Gilding Border of Sign, $2; for Putting up Sign, $2.50.—Due Geo. N. Harper 4 days' Labor, at $1.50. Paid him on %, in Cash $3.—**6**. The Grand Trunk R. R. Co. owes me as follows: for Lettering Sign 15 ft., at 20 cents; for making Board and putting up, $4.25.—I owe J. O'Farrell for making Sign for the Grand Trunk R. R. Co., $3.—Rec'd of John Ray for painting Sign as per agreement, in cash, $30.—**7**. Rec'd of H. Young in full of %, in cash, $6.75.—**8**. Bo't of A. Feigh & Co., Paints, Oils, etc., as per Bill, amounting to $52.64. Paid them in part in Cash, $25.—Bo't of W. G. McLean, Glass and Putty, amounting to $17.50; Paints and Oils, amtg. to $122.50.—**9**. Glazing of 1 Light 11 by 18, 37 cts., for L. S. Rogers.—**11**. Paid A. Feigh & Co. as follows: in cash, on account, $10; and Glazing of 10 Lights 10 by 18, at 25 cts.—I owe Geo. N. Harper 5½ days' Labor at $1.50. Paid him on %, in cash, $9.—Paid H. Teakle, in cash, 5½ days' Work at 75 cts.—**13**. The City Hall owes me, viz.: for Glazing 4 Lights 10 by 16, at 25 cts., $1; for Painting 3 Rooms, 2 Coats, as per agreement, $15.—**15**. Gave A. Feigh & Co. an order on the Grand Trunk R. R. Co., for $15.14.—**17**. Rec'd cash for 37 lbs. Blue Paint at 25 cts.—**18**. I owe Geo. N. Harper 3½ days Labor at $1.50. Paid him cash in full of %, $7.50.—Rec'd cash for 2 Lights 10 by 20, Stained Glass, at $4.50.—Paid H. Teakle, in cash, 6 days' Work at $1.—**20**. Rec'd cash of T. Brown for 120 lbs. White Paint at 11 cts.—Paid cash for Repairs of Shop, $2.12; for making Transparency, Frame, $4.38.—**22**. Sold Hardy & Galt Stained Glass for Transparency, Frame, &c. for $40.—**24**. Received cash as follows: of F. J. Glover, for Stained Glass, as per agreement, $25; of C. Harris for painting Church, $210.—**25**. H. Young owes me, viz.: for Glazing New Sash at Manufactory, as per agreement, to wit, 56 Lights 11 by 15, at 9 cts.; 139 Lights 10 by 12, at 7 cts.; 1920 Lights 8 by 10, at 3 cts.; for 20 Lights, Old Glazing 11 by 17, at 35 cts.; for Painting 110 Window Frames, at 45 cts.; for Painting Reception Room, $5.25.—Paid cash as follows: to L. Jones 6 days' Work, at $1.50; to H. Walter, 5½ days' Work, at $1.50; to H. Teakle, 5 days' Work, at $1; to P. Landry, 4½ days' Work, at 75 cts.—**27**. Bo't of L. S. Rogers, viz.: 1 Black Cloth Dress Coat, $15; 1 Pair Black Cassimere Pants, $6.—Paid P. Gardner, Carpenter, for Sundry Jobs, as per Bill, in

cash $22.50.—**28**. Paid cash for 10 gals. Linseed Oil, at $1.62½.—**29**. Rec'd cash for Tin Sign, $10. Paid cash for Tin and Japanning, $4.25.—**30**. Hardy & Galt owe me for Painting office, as per agreement, $30.—Paid H. Teakle, in cash, 6 days' Work, at $1.—**31**. Paid for Rent of Shop one month, in cash, $16.67.

Balances of the Resources and Liabilities.

Resources.			*Liabilities.*		
Cash, balance on hand	$540	08	L. S. Rogers	16	38
Stock of tools, as per Inventory-Book	75	00	J. O'Farrell	3	00
			W. G. McLean	140	00
Stock of paints, etc.	65	50			
H. Young	134	12		159	38
J. Wilson & Co.	2	00	*Balance.*—My net capital	751	93
Grand Trunk R. R. Co.	8	61			
City Hall	16	00			
Hardy & Galt	70	00			
	911	31		911	31

Thus:

My net capital, on Sept. 1st, is $751.93
" " " at commencing business was only 575.00

My *gains in business* have been $176.93

MEMORANDUM II.

Sept. 1, 1871, I commence business with the following resources: Cash, $801.34; Merchandise, $5120; Bills Receivable, $1385.60; E. S. Burroughs owes me, on %, $167.04; L. N. Veldon, $120.98; T. A. Maguire, $96.40; C. N. David, $50.64.—I owe as follows: On Notes, $350; to Poston & Co., on %, $51.12; to Garneau & Roy, $180.88.—Paid L. Davis, for repairs on the Store, $38.74. —Sold C. N. David, on credit, 2 bbls. Flour, at $7.25.—**2**. Sold L. N. Veldon, on %, 24 gals. of Sperm Oil, at $1.50; and 50 lbs. Powdered Sugar, at 10 cts. per pound.—**3**. Bo't of Poston & Co., on %, 8 boxes Havana Sugar, 3284 lbs., net weight, at 7½ cts.—Paid in cash, for a Set of Account Books, $20.50.—**4**. T. A. Maguire has paid me $30, on his old account.—Sold D. S. Raymond, on account, 60 lbs. Crushed Sugar, at 10 cts.; and 100 lbs. Brown Havana Sugar, at 8¼ cts.—**6**. Bo't of Garneau & Roy, on % Goods amounting to $406.58. Paid them $300 in cash.—**8**. Bo't of W. C. Lord, for cash, Merchandise amounting to $290.40.—L. N. Veldon has paid me $40 on %. —**8**. C. N. David has been painting in the store 9 days, at $1.25, for

which I am to give him credit. And I sell him 2 lbs. Coffee at 25 cts.; and 8 lbs. Crushed Sugar, at 12½ cts.—**9**. Sold E. S. Burroughs, on account. 2 bbls. of Flour at $7, and 2 bbls. of New Orleans Sugar, weighing 468 lbs., weight of casks 40 lbs., net weight 428 lbs., at 6 cts. per lb.—**10**. Paid cash for provisions, for my house, $14.16.—Sold L. N. Veldon 2 bbls. of Flour at $7, and 4 lbs. of Ning Yong Tea, at 45 cts. Bo't of him, 4 cords of Maple Wood for the Store, at $5.—**11**. The amount of cash rec'd for Sales to this date, is $649.50.—Sold T. A. Maguire 2 tubs of Butter, 120 lbs.—(tubs 17 lbs.,) 103 lbs. net, at 22 cts. a lb.; and 33½ lbs. Cheese, at 13 cts.—**13**. E. S. Burroughs has given me his Note for $200, payable in 30 days, in part payment of his account.—Paid cash for Desk, Chairs, Stove, etc., per bill, $53.—Sold D. S. Raymond 2 bbls. of Flour, at $7.25, and 20 gallons of Sperm Oil, at $1.50. He has presented his bill for "bakery" for the past 6 months, amounting to $33.05, and has paid the balance of his account, $25.70, in cash.—**14**. Paid Poston & Co. on %, in cash, $200.—Rec'd of S. Baldwin the amount of his Note: principal $700, with interest for three months, $10.50.—Bought of R. Colton & Son a bill of Mdse. amounting to $246.90, for which I gave my Note, at 2 months from date.—**15**. L. N. Veldon has given me an Order on Walter & Lewis for Mdse., to the amount of $80; and they have delivered me the goods.—**16**. Sold J. Grant 2 barrels of Flour, at $7.50; 2 cwt. of Fish, at $4.25; 20 gals. Molasses, at 35 cts., and 2 lbs. of Black Tea, at 45 cts.—Rec'd of C. N. David on %, in cash, $20.—**17**. W. McKay brought an Order from E. S. Burroughs, requesting me to sell goods to McKay, to the amt. of $36.92, and to charge the same to his (Burroughs') account; and I have delivered the goods.—Paid my Note No. 1, for $350, with interest, 6 months and 15 days, $11.38.—Sold D. S. Raymond 2 tubs of Butter, 112 lbs. net, at 21½ cts.—**18**. Bo't Mdse. of Poston & Co., on acct., amounting to $334.38.—Rec'd of T. A. Maguire, on %, in cash, $34.—Paid cash for 2 Cords of Maple Wood, for the Store, $13.—The Cash Sales this week amount to $414.76.—**20**. Sold L. N. Veldon 2 bbls. of N. O. Sugar, 416 lbs. net, at 5½ cts.—Lent L. Shea $1000, taking his Note at 60 days, for same.—**21**. Rec'd of C. N. David $14.50 on %, in cash.—**22**. Delivered to S. A. Boyd, on T. A. Maguire's Order, goods amounting to $24.—**23**. E. S. Burroughs has paid me $50, in cash, on %.—**24**. J. Grant has sold me 12 bbls. of Apples, at $2.12½, and paid the balance of his acct. in Cash, $5.90.—Paid Garneau & Roy $100, on %.—**25**. Paid a bill for Clothing, for myself, $47.90.—Sold L. N. Veldon 2 bbls. of Molasses, containing 60 gals., at 28 cts., and took his Note, on demand, for the balance of his account, $77.46.—The Butter which I sold to T. A. Maguire on the 11th inst., not proving as good as I thought it was, I have made him a deduction of $3.—**27**. C. F. Joly's Note, No. 2, was paid in to-day, amounting to $685.60.—Gave Poston & Co. E. S. Burroughs' Note, dated 13th inst. for $200; and also my Note, at 60 days from this date, for $80.—The Cash Sales last week amounted to $590.32.—**28**. T. A. Maguire has passed a Note to me, which he had previously received of U. Ryan, for $50.—**29**. Gave Garneau & Roy my Note, on demand, for the balance due them, $187.46.—**30**. Paid

Poston & Co. $100 on account, in cash.—Paid my Clerk's salary for the month, in cash, $60.—Cash Sales for the week, $338.96.—Having taken an Inventory of the goods in the store, I find the amount to be $5086.41.. I have Notes against various persons, amtg. to $1127.46. I owe Notes amounting to $514.36.

On September 30th, my Net Capital is $7528.73, and my Net Gain, $ 368.73.

MEMORANDUM III.

October 1, 1871, W. S. Drum, Cabinet-Maker, associates with himself T. A. Graham;—Drum transferring to the firm such portion of his resources and liabilities as is mutually agreed upon, and Graham investing their equivalent in cash. The parties are to share alike in gains and losses.

W. S. Drum invests in the business, as follows: Cash, 100; Sundry Notes which he holds against others, per B.-B., $700; E. Miles' balance of account, his favor, $111.50; J. R. Nesbitt's balance of account, his favor, $74.80; Materials and Unfinished Work, as per Inventory, $713; Stock of Furniture, as per Inventory, $420.86; Stock of Tools, as per Inventory, $302.40. W. S. Drum owes; viz., Sundry Notes, as per B.-B., amtg. to $842; L. McIntyre & Co., bal. of acct., $134; N. Percy & Son, bal. of acct., $150.40. T. A. Graham, invests in the business, in cash, $1296.16.—**2**. Bo't for cash of C. Vallee, Planks, as per Bill, $151.20.—**3**. Sold E. Miles 2 Hair Cloth, Mahogany Sofas, at $20. Rec'd from the same on account, in cash, $120.—**4**. Sold Mrs. C. Nelson, on acct.; viz., 18 Mahogany Chairs, Cane Seats, at $1.25; 12 Mahogany Chairs, Hair-Cloth Seats, at $3; 4 Cherry Dining-Tables, at $6; 2 Maple French Bedsteads, at $4.25; 2 Maple Low-Post Bedsteads, at $2.75.—**5**. Sold P. McGee on acct, per wife, 2 doz. Windsor Chairs, at $12; 1 doz. Windsor Chairs, for $15; 1 doz. Windsor Chairs, for $10; 2 Spring-Seat Black Walnut Sofas, at $21.—**6**. Paid for Wages, per Time-Book, in cash, $15.—**8**. Sold for cash, 2 Bureaux, Mahogany Veneered, at $22. Paid as follows: A. Patry, for repairs of Shop, in cash, $103; S. Jones, for Painting Shop, in cash, $44; L. McIntyre & Co., in full of acct., in cash, $134; for Glazing 2 Lights of Glass, cash, 76 cts.—**9**. Rec'd cash for B. Motley's Note, Drum's favor, $250.—Bo't of N. Percy & Son, Lumber, for $270. Gave in payment our Note at 30 days, in full of all acct.—Sold E. Miles, per daughter, on acct., 2 Black Walnut Footstools, at $1.50.—Sold C. T. Renaud, on acct., 6 Patent Pivot Chairs for Office, at $5.—**11**. Sold for cash, 2 Arm-Chairs for Office, $10.—Sold E. Miles, per wife, on acct., 2 Black Walnut Extension Dining-Tables, at $40.—**13**. Sold P. D. Flood, on acct., 4 Children's High Dining-Chairs, Mahogany, at $2.—Sold Miss Anna Roy, on acct.; viz., 6 doz. Windsor Chairs, at $11; 2 Rocking Chairs, Second-Hand, at $9.—Paid cash for Wages, $75.—**15**. Sold for cash 2 Pints of Varnish, $1.—E. Miles assumes P. McGee's account, transferred to him, for $91.—**16**. Bought of L. McIntyre & Co., Paints, Varnish, Brushes, etc., as per Bill, amtg. to $350.52. Paid to them cash, in part, $100.—**17**. Received for Staining Cupboard, in cash,

$1·50.—**18**. Sold P. D. Flood, on acct., 2 Black Walnut Book-Cases, at $15.—Sold for cash, 2 Reclining Chairs, at $25.—**19**. Sold for cash 2 Ottomans, at $7.—Sold E. Miles, per son, on account, 2 Hat-Stands, Mahogany Veneered, at $10.—**20**. P. D. Flood owes us for Varnishing 1 Table, $1.—Sold for Cash 1 French Bedstead, Birds'-Eye Maple, $9.—Paid cash for Wages, per Time-Book, $76.24.—**22**. Sold for Steamboat Aurora; viz., 6 Mahogany Sofas, at $15; 4 Tete-a-Tetes, at $16; 2 Rocking Chairs, at $12; 8 Arm Chairs, at $8; 4 doz. Chairs, at $30; 28 Cherry Wash-Stands, at $2; 28 Looking-Glasses, at $2. Rec'd cash in part payment, $300.—Paid cash for Drum's Note, F. Walter's favor. Face of the Note, $400. Discount from date, to Dec. 6th, amounts to $3.—**23**. Paid W. S. Drum on acct., in cash, $20.—Bo't of J. King, on account, Lumber, per Bill, $192.80.—**24**. Paid cash as follows: Bill of Varnish, $27; Gas Bill, $28.50; Drayage of Furniture, $1.—**26**. Sold E. Miles, per wife, on acct., 2 Mahogany Bureaux, with Glass, at $25.—**27**. Paid cash for Wages, as per Time-Book, $73.30.—**29**. Sold for cash; viz., 4 Mahogany Rocking Chairs, Plush covered, at $12.50; 2 Birds'-Eye Maple French Bedsteads, at $9; 2 doz. Cane-Seat Walnut Chairs, at $24; 2 doz. Child's High Chairs, at $1.25; 2 doz. Child's Rocking Chairs, at $1.25.—Paid cash as follows: for Bill of Veneering, $51.20; to T. A. Graham, on acct., $50; for Repairs of Lock, 50 cts.—Sold to Miss Anna Roy, on acct., 1 Mahogany Bureau, $38.—**30**. Sold P. D. Flood, on acct., 2 Black Walnut Centre Tables, at $15. Miss Anna Roy has returned the Mahogany Bureau sold her on the 29th instant, because it was too large for the room, amt. $38.—Rec'd of E. Miles as follows: his Note at 60 days for $150, and cash to bal. acct., $125.50.—**31**. Rec'd cash for Repairing Tables and Chairs, $8.—Paid cash for Rent of Shop, $80; for Wages, as per Time-Book, $73.50.

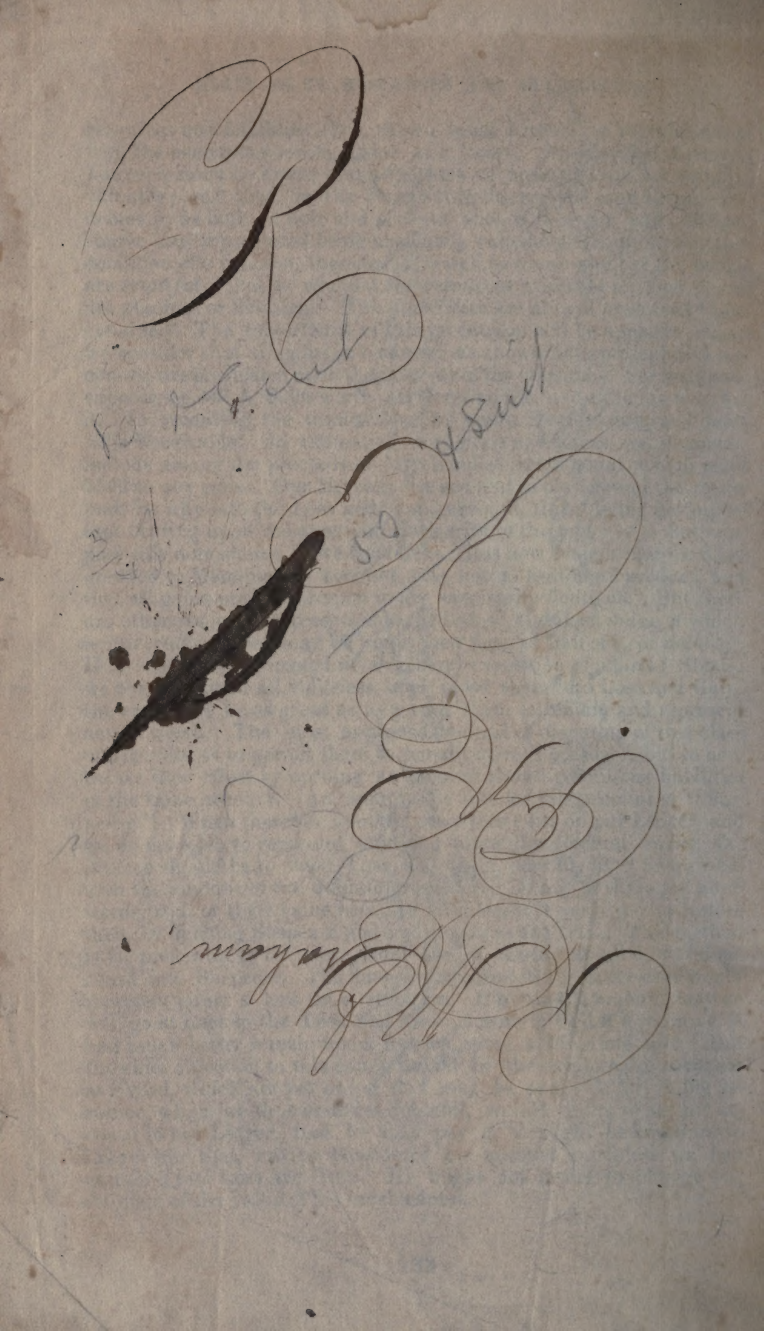
The Stock of furniture on hand am'ts,	per Inv. Book.,	to	$1553.50
" " " Materials and unfinished work,	"	"	479.53
" " " tools, depreciated by use	"	"	283.90
The amount of Bills Rec. in possession of the firm,	per B. B.,		600.00
" " " Bills Payable outstanding,	"	"	862.40

The Firm's *net capital* is $3083.52, and its *net gain*, $561.20 of which ½, $280.60 for each partner.

HINTS AS TO RESOURCES AND LIABILITIES.

The terms resources and liabilities are used extensively in this treatise, and their importance in properly defining the condition of the business has been forcibly set before the student. He has been taught that certain Ledger Accounts are used to show resources, and certain others to show liabilities, and that the correspondence between the

resources and liabilities thus shown must agree in a certain sense, with the accounts showing gains and losses. Any careful observer, however, must be aware that all classes of resources are not equally valuable; and that, in the course of trade, persons may become indebted to us both on note and account who will *never* pay; the resource thus represented being absolutely valueless. In estimating the condition of a concern, therefore, it is well to know whether the books are *truthful;* that is whether the *resources* exhibited on their pages are absolute or fictitious. The *liabilities* are always presumed to be genuine.) The importance of this precaution will be apparent when we consider that all gains in business, as shown by representative accounts, are predicated upon the integrity of the resources. For instance, suppose we sell A, $300 worth of Merchandise, and take his note for it. In recording the transaction, we credit Merchandise, and debit Bills Receivable. In estimating our gains and losses, we, of course include among the proceeds of Merchandise this amount, which adds $300 to our gains. Our Merchandise account is closed, and the result finds its way into the Loss and Gain account, thus having an important bearing upon the apparent prosperity of the business. But suppose this note should prove *worthless*. It is now evident that the $300 credited to Merchandise account was not a legitimate product, and that all gains predicated upon it are necessarily fictitious. But there are other resources represented in the Ledger, the exact value of which is *uncertain*,—they may be worth their face, or half of it, or *nothing*. How shall they be treated in a general exposition of affairs? Should we consider them all valueless, and close them into Loss and Gain, the error may be as great as to permit them to remain and represent actual worth. The most approved method of disposing of this class of accounts, is to permit them to remain upon the Ledger, but to neutralize their effect by opening an account showing fictitious liabilities of the same account. An appropriate title for this account is "Suspense." When therefore doubtful resources exist on our Ledger, and we do not wish to represent anything more than actual *gains*, the process should be to debit Loss and Gain, and credit "Suspense" with the amount of the doubtful resources. If any of these are afterwards paid, or their value becomes tangible, it is very easy to restore them by debiting Suspense and crediting Loss and Gain. This method is far preferable to the more usual one of *closing* up all doubtful account into Suspense. The Suspense account in the latter case would represent either a loss or a resource. If a loss the amount may as well go at once to the Loss and Gain account; and if a resource, it had much better remain under its own more appropriate title. But the chief objection to this course would be the exhibitng of accounts as closed, which are yet owing and may be paid. If Mr. A, for instance, whom we thus consider doubtful, should desire to see his account in our Ledger, that he may pay it, it might be awkward to inform him that, having considered his account worthless we had carried it into Loss and Gain. He might not desire to change our estimate of the value of his indebtedness.

$$\left(\frac{31}{37}\times\frac{40}{1}\times\frac{9}{24}\times\frac{11}{16}\right)\div\frac{5}{4}\times\frac{39}{40}\times\frac{15}{16}=$$

$$\frac{31}{37}\times\frac{\overset{10}{40}}{1}\times\frac{\overset{3}{9}}{24}\times\frac{11}{16}\times\frac{4}{5}\times\frac{39}{40}\times\frac{15}{16}=\frac{930}{13}=71\frac{7}{13}$$